Fundamental Limits for Energy Detection with Noise Power Estimation

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Abstract - The energy detector (ED) is the most common algorithm used in spectrum sensing thanks to its simple implementation and the fact that it does not require any knowledge about the signal to be detected. However practical implementations require knowledge of the noise power level in order to properly set the threshold. The noise uncertainty model adopted in recent literature is based on a worst case approach that always generate the SNR wall phenomenon. We propose an analytical study of the ED with estimated noise power and prove that the presence of the SNR wall is determined by the asymptotical properties of the variance of the estimator. This result is confirmed by the study of the maximum likelihood noise power estimator case, that can be adopted in practical implementations based on two-step sensing schemes.

Design curves and SNR wall

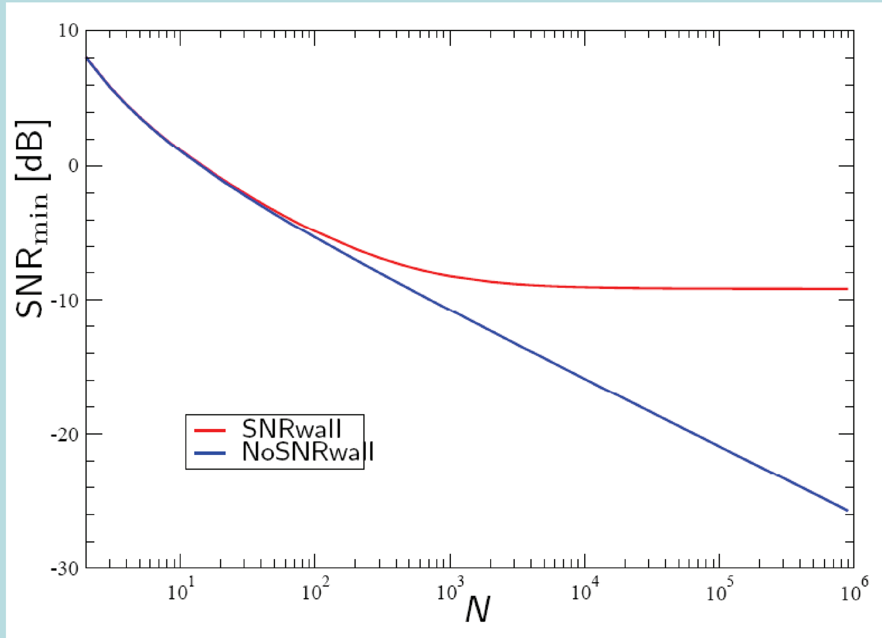
The design of any detector requires to satisfy a probablity of false alarm (P_{FA}) constraint and a probability of detection (P_D) constraint, defined by the *target performance pair*

$$\left(P_{FA}^{\mathrm{DES}}, P_D^{\mathrm{DES}}\right).$$

From the expressions of P_D and P_{FA} it is possible to obtain the design curve

$$\mathrm{SNR}_{\min} = \psi\left(P_{FA}^{\mathrm{DES}}, P_D^{\mathrm{DES}}, N, \dots\right).$$

If the design curve presents a minimum value of SNR under which detection reaching the target performance pait is not possible, than the SNR wall phenomenon occurrs.



Ideal ED

$$\Lambda(\mathbf{y}) \triangleq \frac{1}{2\sigma^2}\cdot\frac{1}{N}\sum_{i=0}^{N-1}|y_i|^2$$

Hp: perfect knowledge of the noise power level

Corresponding design curve:

$$\mathrm{SNR}_{\min,\,\mathrm{ED}} = \frac{\mathrm{Inv}\tilde{\Gamma}\left(N, P_{FA}^{\mathrm{DES}}\right)}{\mathrm{Inv}\tilde{\Gamma}\left(N, P_{D}^{\mathrm{DES}}\right)} - 1$$

where $\mathrm{Inv}\tilde{\Gamma}(\cdot,\cdot)$ is the inverse of the gamma regularized function $\tilde{\Gamma}(a,x) = \frac{1}{\Gamma(a)}\int_x^{+\infty} x^{a-1}e^{-x}dx$

$$\lim_{N\to\infty} \mathrm{SNR}_{\min,\,\mathrm{ED}} = 0$$

For the ideal ED, SNR wall does not occur

Typical noise uncertainty model

Hp: uncertain noise power value constrained into a fixed range $\tilde{\sigma}^2 \in \left[\sigma^2_{\min}, \sigma^2_{\max}\right]$.

Evaluate P_D and P_{FA} in the corresp. worst cases

$$P_D = \tilde{\Gamma}\left(N, \frac{\sigma^2_{\max}}{\sigma^2_t} N\xi\right)$$

$$P_{FA} = \tilde{\Gamma}\left(N, \frac{\sigma^2_{\min}}{\sigma^2} N\xi\right)$$

Design curve:

$$\mathrm{SNR}_{\min,\,\mathrm{BWB}} = \frac{\sigma^2_{\max}}{\sigma^2_{\min}}\cdot\frac{\mathrm{Inv}\tilde{\Gamma}\left(N, P_{FA}^{\mathrm{DES}}\right)}{\mathrm{Inv}\tilde{\Gamma}\left(N, P_{D}^{\mathrm{DES}}\right)} - 1$$

$$\lim_{N\to\infty} \mathrm{SNR}_{\min,\,\mathrm{BWB}} = \frac{\sigma^2_{\max}}{\sigma^2_{\min}} - 1 > 0$$

This model gives always the SNR wall

ED with estimated noise power (ENP-ED)

$$\Lambda_{\mathrm{g}}(\mathbf{y}) \triangleq \frac{1}{2\hat{\sigma}^2}\cdot\frac{1}{N}\sum_{i=0}^{N-1}|y_i|^2 \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \xi$$

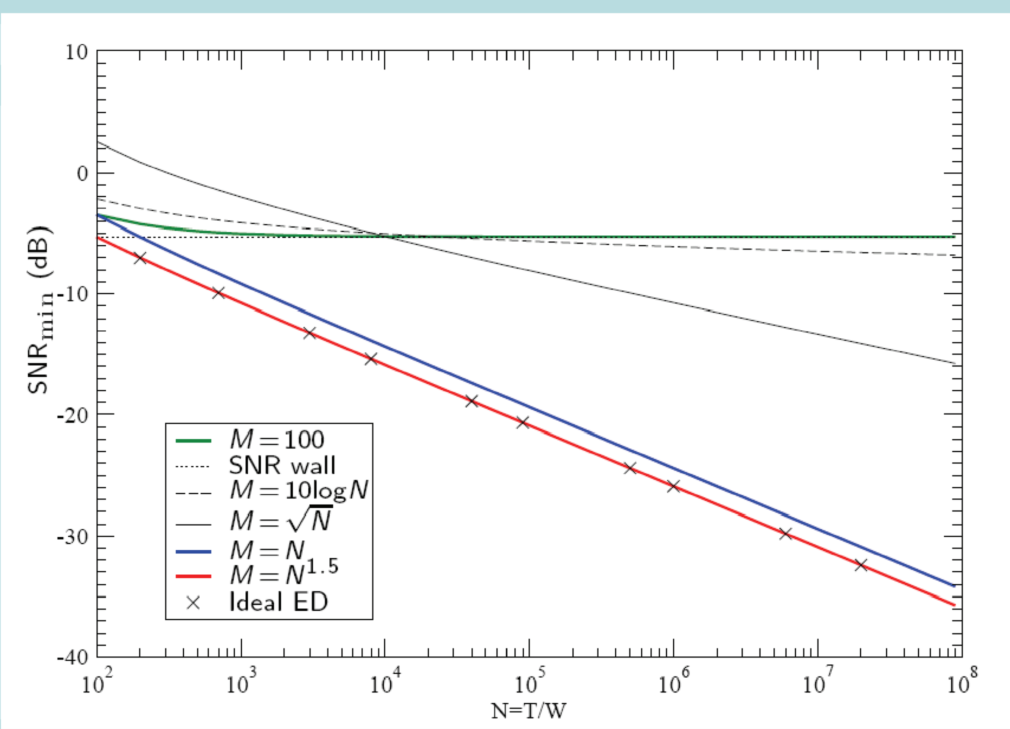
*The SNR wall phenomenon is not caused by the presence of an uncertainty on σ^2 itself, but it is rather due to an **insufficient refinement** of the **noise power estimation**.*

Existence of the SNR wall (Theorem)

For the energy detection with noise power estimation, assuming that $\hat{\sigma}^2$ is an unbiased asymptotically Gaussian estimator of the noise power σ^2, an arbitrary $\left(P_{FA}^{\mathrm{DES}}, P_D^{\mathrm{DES}}\right)$ pair can be achieved by increasing the observation interval (i.e., increasing the number of collected samples N) for all values of SNR, if and only if the variance of the noise power estimator, $var\left(\hat{\sigma}^2\right)$, is o (1) for $N \to \infty$.

Moreover, if $var\left(\hat{\sigma}^2\right)$ is $\Theta(1)$ for $N \to \infty$ there exists a minimum SNR (SNR wall), under which it is impossible to reach the desired $\left(P_{FA}^{\mathrm{DES}}, P_D^{\mathrm{DES}}\right)$ pair; in this case, by defining $\alpha = Q^{-1}\left(P_{FA}^{\mathrm{DES}}\right)$, $\delta = Q^{-1}\left(P_{D}^{\mathrm{DES}}\right)$, $\phi = var\left(\hat{\sigma}^2/\sigma^2\right)$ the minimum SNR for $N \to \infty$ converges to

$$\mathrm{SNR}^{(\infty)}_{\min} = \frac{1-\delta\sqrt{\phi}}{1-\alpha\sqrt{\phi}} - 1$$



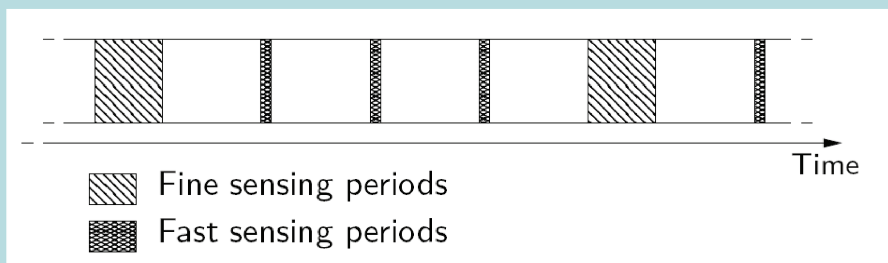
Then if $var\left(\hat{\sigma}^2\right)$ can be reduced while the ED observation time increases, the SNR wall can be avoided

ENP-ED with ML noise power estimator

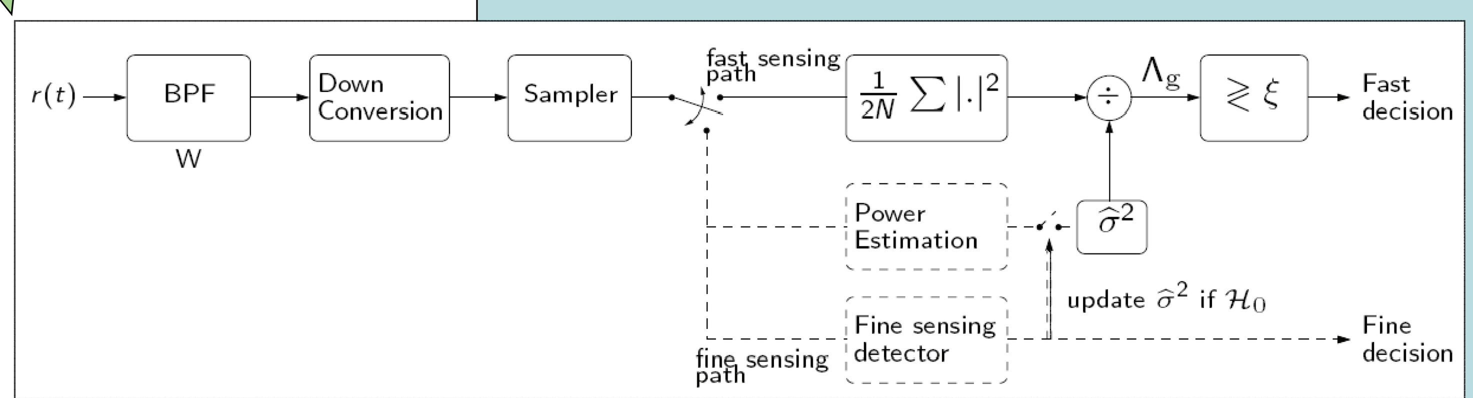
$$\hat{\sigma}^2_{\mathrm{ML}} = \frac{1}{2M}\sum_{i=1}^{M}|n_{-i}|^2 \qquad var\left(\hat{\sigma}^2_{\mathrm{ML}}\right) = \frac{\sigma^4}{M}$$

ENP-ED with ML noise power estimator is based on the observation of M noise only samples and can be adopted in two step sensing schemes.

Two-step sensing scheme



High performance algorithms requires generally long observation intervals to perform spectrum sensing, with an impact on the efficiency of the CR communications.



Therefore they can be used in *less frequent fine sensing periods*, while constant monitoring of the spectrum is performed introducing *more frequent fast-sensising periods* in which the ED can be adopted.

Asymptotical behavior of the design curves

If the SNR wall occurs, the design curve tends asymptotically to a straight line (in a bi-logarithmic space) which slope and distance to the ideal ED curve is determined by the asymptotical behavior of $var\left(\hat{\sigma}^2\right)$.

EXAMPLE (Two step sensing scheme):

If $var\left(\hat{\sigma}^2\right) \approx 1/(\lambda N)$ for $N \to \infty$ with $\lambda > 0$, which means that, considering the ML noise power estimator case, we have $M = \lambda N$, the design curve of the ENP-ED tends asymptotically to

$$\mathrm{SNR}^{(\infty)}_{\min}(dB) \approx -5\log_{10} N - 5\log_{10}\left(\frac{1+\lambda}{\lambda}\right) + 10\log_{10}(\alpha-\delta)$$

Note that, increasing λ, the gap with respect to the ideal ED design curve can be reduced.