# Multi User Interference in Power-Unbalanced Ultra Wide Band systems: Analysis and Verification

Guerino Giancola, Luca De Nardis, Maria-Gabriella Di Benedetto
INFO-COM Department, University of Rome La Sapienza
Via Eudossiana, 18 – 00184 Rome, Italy
giancola@infocom.uniroma1.it, lucadn@newyork.ing.uniroma1.it, gaby@acts.ing.uniroma1.it

*Abstract:* **Validity of the Standard Gaussian Approximation (SGA) for modeling Multi User Interference (MUI) in Impulse Radio Ultra Wide Band (IR-UWB) systems which do not implement power control is investigated. Analysis focuses on the case of UWB systems adopting binary Pulse Position Modulation (2PPM) with a Time Hopping (TH) code division multiple access scheme. Theoretical predictions are compared vs. simulation outputs in order to quantify limitations of the SGA hypothesis.**

*Keywords:* Ultra Wide Band, Multi User Interference, Standard Gaussian Approximation, Power Control.

## I. INTRODUCTION

During the last years, significant results were achieved regarding the use of Impulse Radio Ultra Wide Band (IR-UWB) radio for personal communication systems [1],[2]. In particular, an increased interest is witnessed towards the application of UWB in the context of next-generation self-organizing wireless Local Area Networks (WLANs), where UWB presents promising potentials in terms of capacity, flexibility and power consumption [3], [4]. In such a context, it is important to evaluate system performance in the presence of multiple asynchronous UWB devices sharing the same channel [2],[5], i.e. when the system is affected by Multi User Interference (MUI).

In this paper, multiple access performance of a UWB system is evaluated by assuming propagation over a flat AWGN channel (multipath-free), and by evaluating MUI through the Standard Gaussian Approximation (SGA). The SGA is based on the hypothesis that MUI contributions can be treated as an additive Gaussian noise with uniform power spectrum over the frequency band of interest.

Recently it was suggested that the validity of the SGA increases with the number of interfering users [2], and that it cannot adequately predict the impact on Bit Error Rate (BER) for low values of the user bit rate [5] and low values of pulse repetition frequency [6]. The analyses presented in [5], in particular, refer to the case of perfect power control, and show that the SGA leads to more optimistic predictions of BER in comparison with results obtained by simulation. The hypothesis of perfect power control does not hold, in general, for self-organizing (ad-hoc) WLAN scenarios, because of the increased complexity of distributed algorithms and limitation in scalability. The aim of this work is thus the extension of the results of [5] to the case of systems without power control (power-unbalanced). Analysis focuses on UWB systems implementing binary Pulse Position Modulation (2PPM) with a Time Hopping (TH) code division multiple access (CDMA) scheme.

The paper is organized as follows. Section II contains a description of the adopted system model and derives the analytical expression for BER through the SGA. Section III compares theoretical and simulation results, and discusses the validity of the SGA in the case of power-unbalanced UWB systems. Finally, Section IV contains the conclusions.

## II. SYSTEM MODEL AND BER COMPUTATION

The system model examined in this paper consists of $N_u$ asynchronous devices sharing the same channel and generating the same bit rate $R_b$. The transmitted UWB signal of the $n$-th user can be expressed as follows:

$$s_{TX}^{(n)}(t)=\sum_{j=-\infty}^{\infty}\sqrt{E_{TX}^{(n)}}\,p_0(t-j\cdot T_s-c_j^{(n)}\cdot T_c-\varepsilon\cdot d_j^{(n)}) \qquad (1)$$

where $p_0(t)$ is an energy-normalized waveform representing the basic pulse, and $E_{TX}^{(n)}$ is the energy which is transmitted for each single pulse. According to Eq.(1), the UWB signal consists of a train of pulses which are transmitted with an average repetition time equal to $T_s$. The $j$-th pulse is characterized in addition by two time shifts. The first, $c_j^{(n)}\cdot T_c$, is due to the TH code and the second, $\varepsilon\cdot d_j^{(n)}$, to the 2PPM modulation.

In the $c_j^{(n)}\cdot T_c$ term $c_j^{(n)}$ and $T_c$ are the $j$-th coefficient of the TH sequence of the $n$-th user and the chip time, respectively. Each user is provided with a different TH code in order to avoid catastrophic collisions at the receiver. Each TH code consists in $N_p$ independent and identically distributed random variables, each characterized by a probability of $1/N_h$ to assume one of the integer values in the range $[0, N_h-1]$.

In the $\varepsilon\cdot d_j^{(n)}$ term $\varepsilon$ is the basic shift introduced by PPM and $d_j^{(n)}$ is the binary data value (i.e. 1 or 0) which is conveyed by the single $j$-th pulse. In case of $N_s$ per bit, Eq.(1) rewrites:

$$s_{TX}^{(n)}(t)=\sum_{j=-\infty}^{\infty}\sqrt{E_{TX}^{(n)}}\,p_0(t-j\cdot T_s-c_j^{(n)}\cdot T_c-\varepsilon\cdot b_{\lfloor j/N_s\rfloor}^{(n)}) \qquad (2)$$

where $\lfloor x\rfloor$ is the inferior integer part of $x$ and $b_x^{(n)}=b^{(n)}(x\cdot T_b)$ is the $x$-th bit of the $n$-th binary user data flow $b^{(n)}(t)$. We assume that the bits $b_x^{(n)}$ are independent, equiprobable, and identically distributed random variables with 50% probability of being 0 or 1.

In order to avoid pulse overlapping one must also impose:

$$T_c \cdot N_h \le T_s \quad and \quad T_s \cdot N_s \le T_b \qquad (3)$$

where $T_b$ is the bit interval. Moreover, if $T_M$ denotes the time duration of a pulse, one also has:

$$T_M \le T_C - \varepsilon \qquad (4)$$

We consider a general flat channel model (multipath-free). The impulse response of the channel for the link between the $n$-th user and the reference receiver is thus given by:

$$h^{(n)}(t) = \alpha^{(n)} \cdot \delta(t - \tau^{(n)}) \qquad (5)$$

where $\alpha^{(n)}$ and $\tau^{(n)}$ are the path gain and the time delay of the $n$-th user, respectively. The channel output is further corrupted by thermal noise $n(t)$, modelled as AWGN with double-sided spectral density $N_o/2$.

The received signal writes:

$$r(t) = \sum_{n=1}^{N_u} \sum_{j=-\infty}^{\infty} \sqrt{E_{RX}^{(n)}} \cdot p_0(t - j \cdot T_s - c_j^{(n)} \cdot T_c - \varepsilon \cdot b_{\lfloor j/N_s \rfloor}^{(n)} - \tau^{(n)}) + n(t) \qquad (6)$$

where:

$$E_{RX}^{(n)} = \left[\alpha^{(n)}\right]^2 E_{TX}^{(n)} \qquad (7)$$

We label the reference transmitter with $n$=1, and we focus the attention on the transmission of the reference bit $b^{(1)}$ corresponding to $t \in [0,T_b]$, (i.e $j \in [0,N_s\text{-}1]$). Equation (6) re-writes:

$$r(t) = r_u(t) + r_{mui}(t) + n(t) \qquad (8)$$

where $r_u(t)$ and $r_{mui}(t)$ are the signal contribution and the multi user interference at the receiver input:

$$r_u(t) = \sum_{j=0}^{N_s-1} \sqrt{E_{RX}^{(1)}} \cdot p_0(t - j \cdot T_s - c_j^{(1)} \cdot T_c - \varepsilon \cdot b^{(1)} - \tau^{(1)}) \qquad (9)$$

$$r_{mui}(t) = \sum_{n=2}^{N_u} \sum_{l=-\infty}^{\infty} \sqrt{E_{RX}^{(n)}} \cdot p_0(t - j \cdot T_s - c_j^{(n)} \cdot T_c - \varepsilon \cdot b_{\lfloor j/N_s \rfloor}^{(n)} - \tau^{(n)}), \quad with \quad t \in [0,T_b] \qquad (10)$$

We assume perfect synchronization between the reference transmitter and the receiver. Consequently, we assume with no loss in generality that $\tau^{(1)}$=0. In addition, we model the delays $\tau^{(n)}$ with $n \ne 1$ as independent and identically uniformly distributed random variables between 0 and $T_s$.

We adopt a single user correlation receiver implementing soft decision detection. In this case, the output of the receiver for the reference bit is given by [1]:

$$Z = \int_0^{T_b} r(t) \cdot m(t) dt \qquad (11)$$

where $m(t)$ is defined as follows:

$$m(t) = \sum_{j=0}^{N_s} \left[p_0(t - j \cdot T_s - c_j^{(1)} \cdot T_s) - p_0(t - j \cdot T_s - c_j^{(1)} \cdot T_s - \varepsilon)\right] \qquad (12)$$

The receiver must decide if the reference bit $b^{(1)}$ is 0 or 1 based on the observation of $Z$. In particular, the decision procedure can be expressed as follows:

$$\begin{cases} Z > 0 \Rightarrow \hat{b} = 0 \\ Z < 0 \Rightarrow \hat{b} = 1 \end{cases} \qquad (13)$$

where $\hat{b}$ represents the estimated bit. When introducing $r(t)$ as derived in Eq.(8) into Eq.(11), we obtain:

$$Z = Z_u + Z_{mui} + Z_n \qquad (14)$$

where $Z_u$, $Z_{mui}$ and $Z_n$ are, respectively, the useful contribution, the MUI contribution and the thermal noise contribution.

Assuming that data bits are equiprobable, the BER for the examined system is given by:

$$BER = \Pr\left(Z < 0 \mid b^{(1)} = 0\right) \qquad (15)$$

The reference literature on UWB ([1],[2]) evaluates BER through the SGA, i.e. by modelling MUI as an additive white Gaussian noise process with zero mean and variance equal to the MUI variance. In other words, both $Z_n$ and $Z_{mui}$ are assumed to be zero-mean gaussian random variables with variance $\sigma_n^2 = E\{|Z_n|^2\}$ and $\sigma_{mui}^2 = E\{|Z_{mui}|^2\}$, respectively, where $E\{\cdot\}$ is the expected value operator.

Under the above assumption, the BER in Eq.(15) can be evaluated as for the PSK case and writes:

$$BER \approx Q(\sqrt{SNR}) \qquad (16)$$

where:

$$SNR = \left[\frac{1}{SNR_{mui}} + \frac{1}{SNR_n}\right]^{-1} \qquad (17)$$

and where $SNR_{mui} = [Z_u(b=0)]^2/\sigma_{mui}^2$ and $SNR_n = [Z_u(b=0)]^2/\sigma_n^2$ are the signal to MUI ratio and the signal to thermal noise ratio, respectively.

Based on [2], we derive the following expression for $SNR_{mui}$:

$$SNR_{mui} = \frac{N_s \cdot T_s \cdot [1 - R_0(\varepsilon)]^2}{\sigma_M^2 \cdot \sum_{n=2}^{N_u} \frac{E_{RX}^{(n)}}{E_{RX}^{(1)}}} \qquad (18)$$

where $R_0(t)$ is the autocorrelation function of the basic pulse $p_0(t)$, and $\sigma_M^2$ is a term which depends on both $p_0(t)$ and the value of $\varepsilon$, and it is expressed by:

$$\sigma_M^2 = \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} p_0(t + \varphi) \cdot [p_0(t) - p_0(t - \varepsilon)] dt\right]^2 d\varphi \qquad (19)$$

Note that:

$$T_s = \frac{\gamma_R}{N_s \cdot R_b} \tag{20}$$

where $R_b=1/T_b$ is the bit rate and $\gamma_R$ the fraction of the bit period $T_b$ over which the transmitter can generate the pulses ($\gamma_R \leq 1$). Equation (18) can be thus rewritten as follows:

$$SNR_{mui} = \frac{\gamma_R \cdot [1 - R_0(\varepsilon)]^2}{\sigma_M^2} \frac{1}{R_b \cdot \sum_{n=2}^{N_u} \frac{E_{RX}^{(n)}}{E_{RX}^{(1)}}} \tag{21}$$

Equation (21) highlights that the MUI can be controlled by reducing the bit rate $R_b$ which is assigned to all users.

With reference to $SNR_n$, [5] shows that:

$$SNR_n = \frac{N_s \cdot E_{RX}^{(1)}}{N_0} \cdot [1 - R(\varepsilon)] = \frac{E_b}{N_0} \cdot [1 - R(\varepsilon)] \tag{22}$$

where $E_b$ is the useful energy at the receiver during $T_b$.

By combining Eqs.(21) and (25) one obtains the BER evaluated under the SGA:

$$BER = Q\left( \sqrt{ \left( \left\{ \frac{E_b}{N_0} \cdot [1 - R_0(\varepsilon)] \right\}^{-1} + \left\{ \left( \frac{[1 - R_0(\varepsilon)]^2 \cdot \gamma_R}{\sigma_M^2} \right) \Big/ R_b \cdot \sum_{n=2}^{N_u} \frac{E_{RX}^{(n)}}{E_{RX}^{(1)}} \right\}^{-1} \right)^{-1} } \right) \tag{23}$$

which in presence of perfect power control simplifies in:

$$BER = Q\left( \sqrt{ \left\{ \frac{E_b}{N_0} \cdot [1 - R_0(\varepsilon)] \right\}^{-1} + \left\{ \left( \frac{[1 - R_0(\varepsilon)]^2 \cdot \gamma_R}{\sigma_M^2} \right) \Big/ R_b \cdot (N_u - 1) \cdot \right\}^{-1} } \right) \tag{24}$$

Figure 1 shows the theoretical BER in Eq.(27) as a function of the ratio $E_b/N_o$ and for different values of the bit rate $R_b$. The second derivative gaussian waveform was considered for $p_0(t)$ [1]. It is easy to verify the increasing effect of the MUI on system performance when considering higher $R_b$ values.

## III. Simulation

In section II, performance of an IR-UWB system adopting 2PPM with a TH-CDMA scheme was evaluated through the SGA, i.e. by considering the overall interference as an additive zero-mean white gaussian random process. Under this hypothesis, theoretical BER is expressed in Eqs.(23) and (24) for power-unbalanced and power-balanced systems, respectively. The validity of SGA for evaluating MUI has been discussed for both conventional SS and UWB systems [5]-[7]. As regards the UWB case, it was suggested that the validity of the SGA increases with the number of interfering users, while the SGA does not drive to adequate predictions of BER for low values of user bit rate as well as pulse repetition frequency. The analyses presented in [5], in particular, show that in a power-balanced UWB system the SGA leads to more optimistic predictions of the BER floor in comparison with the results obtained through simulation. We extend below the analysis of [5] to the case of power-unbalanced UWB systems.

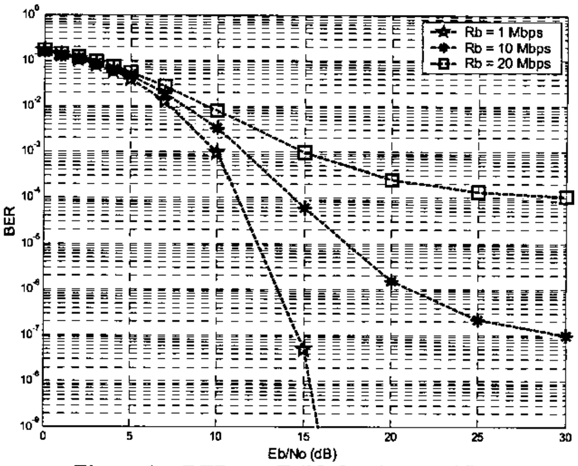


Figure 1 – BER vs. $E_b/N_o$ for the specific case of $N_u$=15 and $\gamma_R$=1.

We consider the signal format presented in Eq.(2) with $T_s=T_c \cdot N_h$ and $T_b=T_s \cdot N_s$, i.e. $\gamma_R=1$. The frame time $T_s$ is set to $10ns$, and the chip time $T_c$ to $1ns$, i.e. $N_h=10$. Non-periodic TH codes are taken into account, i.e. $N_p$ is always equal to the total number of transmitted pulses. The adopted pulse $p_0(t)$ is the energy-normalized version of the second derivative gaussian pulse as suggested in [1], and characterized by a time duration $T_M=0.7ns$. The PPM is $\varepsilon=0.156ns$ is considered. The number of users is fixed to $N_u=15$ for all the simulations. All users are assumed to generate a constant bit rate binary data flow at rate $R_b$ and transmit using the same power. Interfering users are randomly located inside a circular area with radius $R_{max}$ with centre in the reference receiver. The following path-loss model is considered for emulating power attenuation on the $n$-th link [8]:

$$\left[\alpha^{(n)}\right]^2 = A^{(n)} = \frac{s}{\left[d^{(n)}\right]^{2.4}} \tag{31}$$

where $d^{(n)}$ is the distance between the $n$-th transmitter and the reference receiver, and $s$ a constant term which was tuned in order to verify $A=-31dB$ at a distance of $1m$. Three scenarios corresponding to different $R_{max}$ values are considered for the simulation. The first scenario (*case A*) is characterized by $R_{max}=5m$, with fixed distance $d_0=3m$ between the receiver and the reference transmitter (Fig.2.a). The second scenario (*case B*) is characterized by a lower spatial density of users, since $R_{max}=7m$ and $d_0=4m$ (Fig.3.a). The third scenario (*case C*), has the lower value of user spatial density with $R_{max}=9m$ and $d_0=5_m$ (Fig.4.a).

Figure 2.b compares theoretical and measured BER for different $R_b$ values ($R_b=11Mbps$, $R_b$=22Mbps, and $R_b=44Mbps$) in the scenario corresponding to *case A*, and shows a good agreement between theoretical performance (dotted lines) and simulation results (solid lines).

The same comparison is shown in Fig.3.b for *case B*. Here we observe a marked discrepancy between theoretical and measured performance for the lower $R_b$ values. In the case of $R_b=11Mbps$, in particular, the measured and the predicted BER floors differ of about one degree of magnitude. The gap decreases when considering $R_b$=22*Mbps*, while complete agreement between theory and simulation is observed in the case of $R_b$=44*Mbps*.

The same trend emerges from Fig.4.b with reference to *case C*. Here we verify that the theoretical approach poorly matches the prediction of the BER floor for the lowest $R_b$ value. In the case of $R_b$=22*Mbps* and $R_b$=44*Mbps* we observe a better agreement between theoretical and measured BER values, but the discrepancy between theory and simulation is higher in comparison with previous scenarios. We conclude that the SGA hypothesis suffers a lack of accuracy in evaluating MUI when considering scenarios with a low spatial density of interfering users, i.e. when the receiver is more sensitive to the near-far effect.

## IV. CONCLUSION

In this paper, multiple access capabilities of power-unbalanced UWB systems were discussed. Analysis focused on the case of impulse radio UWB systems adopting 2PPM with TH-CDMA. An analytical expression for the BER was derived under the SGA, i.e. by modeling the MUI as an additive zero-mean white Gaussian noise. Theoretical and simulation results were then compared. Different scenarios with decreasing spatial densities of the interfering users were taken into account. As expected, the analysis demonstrated that the validity of the SGA for evaluating the MUI does not only depend on the number of users and user bit rates. When considering power-unbalanced systems, the accuracy of the Gaussian model also depends on the spatial density of the interfering users. The lower the spatial density, the higher the bit rate which is required for guaranteeing the accuracy of the SGA.


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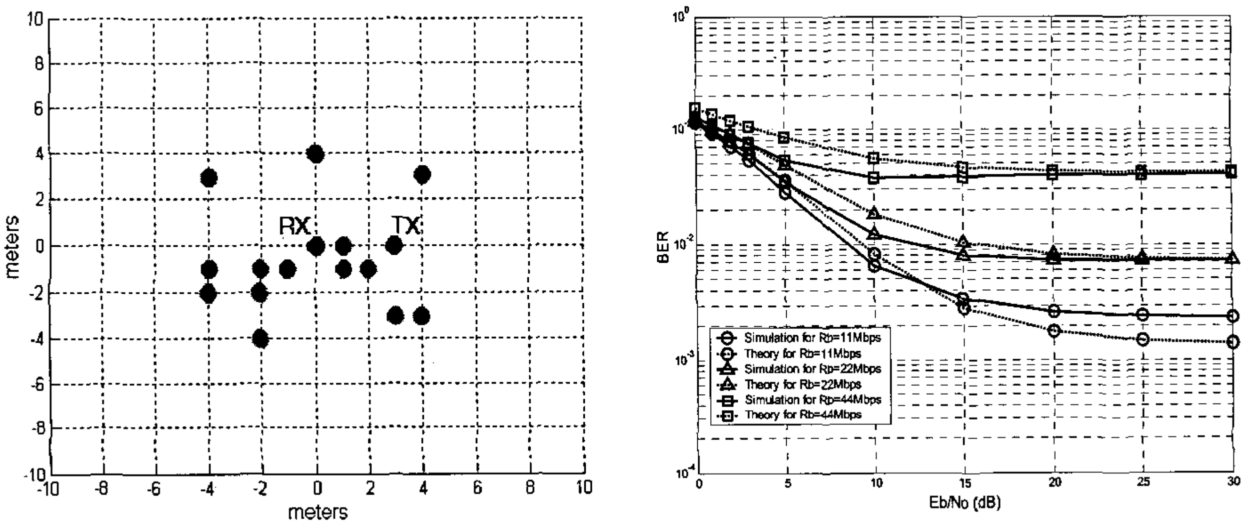
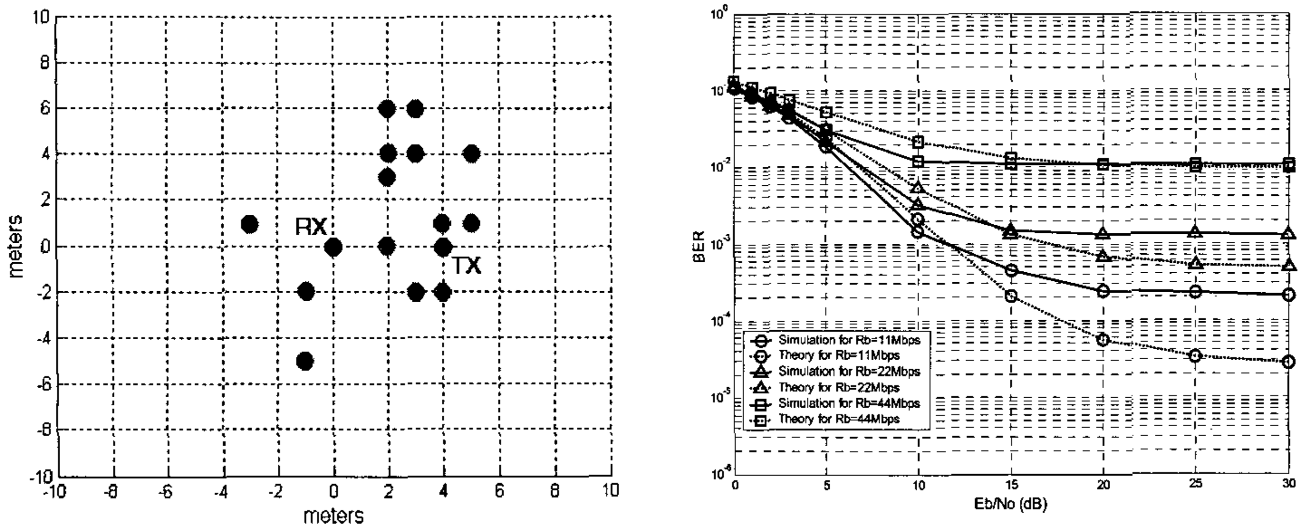


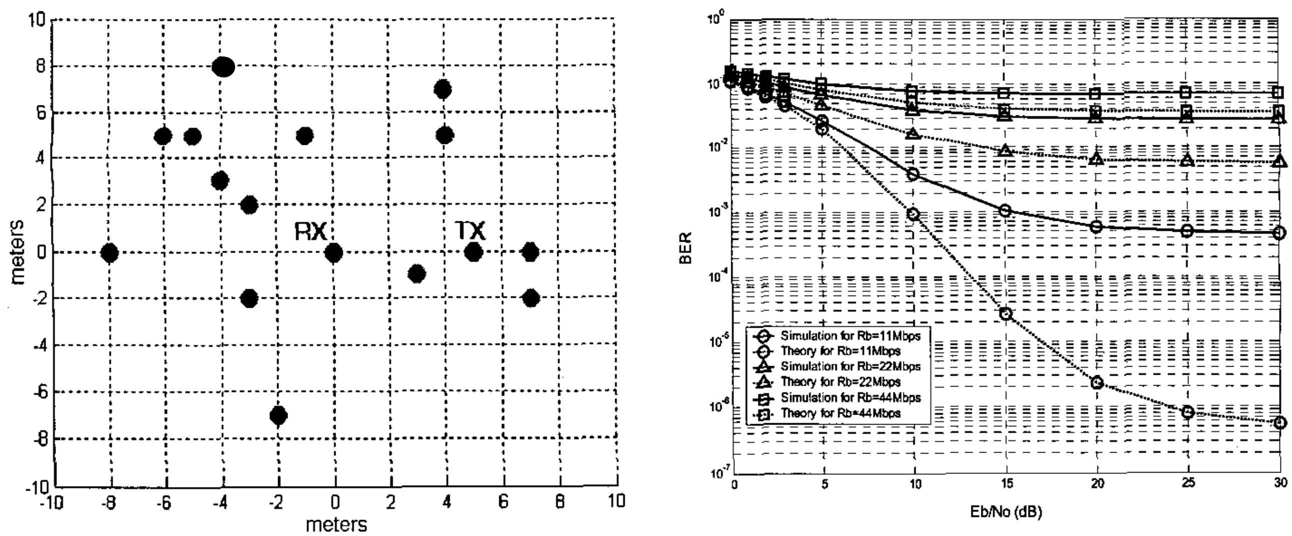

(a) (b)

Figure 2 – Spatial distribution of the users (a) and BER vs. $E_b/N_o$ (b) for *case A* ($R_{max}$=5*m*)




(a) (b)

Figure 3 – Spatial distribution of the users (a) and BER vs. $E_b/N_o$ (b) for *case B* ($R_{max}$=7*m*)




(a) (b)

Figure 4 – Spatial distribution of the users (a) and BER vs. $E_b/N_o$ (b) for *case C* ($R_{max}$=9*m*)