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Analysis of Cognitive Radio Dynamics*

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15.1 Introduction

The cognitive concept of a radio capable of adapting to the environment and of adjusting its operation as a function of both external and internal unpredictable events forms the conceptual basis for the design of future wireless communication systems. Designing and developing smart wireless devices able to sense the environment, and to modify accordingly spectral shape and other features of radiated signals is extremely appealing [1,2]. In particular, by defining and developing technologies that can enable a radio device to adapt its spectrum according to the operating environment, that is, to be aware of the scenario in which it operates, design innovation is taking its first step toward conceiving wireless networks that cooperatively coexist with other wireless networks and devices. The coexistence principle is intrinsic to innovative technologies such as ultra wide band (UWB) radio [3], although the concept has a rather broader acceptation.

Cognitive radio focuses on improving efficiency in the use of the wireless resource and applies basically to the behavior of one node. By introducing cognitive principles in the logic of the wireless network one extends the cognitive concept to rules of interaction between nodes. In order to optimize the design one must therefore model the set of wireless nodes as a social network forming one single entity.

Time scale is particularly important when conceiving adaptive mechanisms that should allow the update of system configuration. System operation is ruled by clocks, that must be tuned according to the granularity that is requested by a specific operation. In the case of spectrum sensing, for example, it might be desirable to force the system to being continuous, in order to incorporate the capability of detecting sudden and unpredicted changes in the environment. Conversely, it might be desirable for other operations to be ruled on different time scales.

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The problem that must be analyzed is therefore related to asynchronicity of phenomena that would force a node to change its state of operation with respect to input–output dynamics of the node. Mobility is an example of such phenomena. Other types of perturbations, such as atmospheric changes, have the additional complex feature of being unpredictable. Continuous and discrete dynamics must be integrated in the mathematical model that describes the node. Hybrid systems (see, e.g., [4]) offer a challenging framework for formalizing such a complex system [5].

In this chapter, we propose a model that generalizes those proposed in [6] and [7] for UWB networks, to the case of a self-organizing network of nodes that operate under the coexisting principle. We formalize the model by using hybrid systems that offer in fact the analytical framework for modeling complex systems where continuous dynamics and discrete processes tightly interact.

The chapter is organized as follows. In Sect. 15.2, we define the problem and set the basis and main assumptions. Section 15.3 contains the rules for governing resource in the network, in terms of computing transmission power levels of nodes. Section 15.4 describes the application of hybrid system modeling to the system under consideration that incorporates the cognitive radio concept. In particular, we show how the proposed model represents the behavior of each node and of the population of nodes that form the network. In Sect. 15.4, we present some concluding remarks highlighting open problems that may be formally stated and analyzed using the proposed hybrid model.

15.2 Problem Statement and System Description

We consider the formation of a self-organizing network of nodes that adopt a multiple access scheme in which coexistence is foreseen, that is signals originating from different users share in principle a same resource in terms of time and frequency. Users separation is obtained by appropriate coding. Code division multiple access (CDMA) as well as time hopping multiple access (THMA) are possible access schemes. In general terms, the multiple access scheme may be based on any coding scheme that allows resource sharing while providing acceptable system performance at the receiver. In such a context, the receiver is supposed to operate in a correlation mode, that is to be capable of sensing the presence of a useful signal by appropriate synchronization in encoded time instances. The dominant noise is interference noise, with a dominant component formed by multi-user interference (MUI).

Call P_{TX} the average transmitted power. This power is upper-bounded by a maximum power $P_{\max}$ that can be determined from recommendations on emission levels as well as technological limitations. T_{b} is the bit repetition period. The impulse response of the pulse shaper is indicated by $p_w(t)$. Furthermore, we indicate by $v_{a_j}(t)$ the data-modulated multi-pulse signal made of the sum of N_{S} shifted, and eventually amplitude modified, versions of $p_w(t)$, where N_{S} is the number of chips forming one bit of the data sequence $\{a_j\}$. Chip duration is thus $T_{\mathrm{S}} = T_{\mathrm{b}}/N_{\mathrm{S}}$. Note that $p_w(t)$ has a direct impact on the power spectral density of transmitted signals and that therefore by selecting a specific pulse shape one may adapt spectral features

of radiated emissions as a function of the environment, i.e., for example specific interference patterns. An important hypothesis that is fundamental in our model is the possibility of selecting one pulse shaper among many [8]. We therefore assume W different pulse shapes $p_w(t)$, with $w = 1, \ldots, W$.

A general flat additive white Gaussian noise (AWGN) channel model is assumed. The impulse response for the channel between a reference transmitter TX and a reference receiver RX is indicated by $h(t) = \alpha\delta(t-\tau)$, and is characterized by a constant amplitude gain α and a constant delay τ. The signal at RX input writes:

$$\begin{aligned} r(t) &= \alpha\sqrt{P_{\mathrm{TX}}T_{\mathrm{S}}}\sum\nolimits_j v_{a_j}\left(t - jT_{\mathrm{b}} - \tau\right) + n(t) \\ &= \sqrt{P_{\mathrm{RX}}T_{\mathrm{S}}}\sum\nolimits_j v_{a_j}\left(t - jT_{\mathrm{b}} - \tau\right) + n(t) \end{aligned} \tag{15.1}$$

where $P_{\mathrm{RX}} = \alpha^2 P_{\mathrm{TX}}$ is the average received power and $n(t)$ is the cumulative noise at the receiver input. It is well known that, under the above conditions, single-user reception is optimal when the receiver is composed of a coherent correlator followed by a maximum likelihood detector [3]. The output of the correlator within a bit period T_{b} is indicated by Z. It is on the basis of Z that the ML detector takes a decision, that is, we suppose decision is taken on a bit period, i.e., based on N_{S} pulses forming one multi-pulse (soft detection). According to this scheme, the received signal is thus cross-correlated with a correlation mask $m_w(t)$ that is matched with the train of pulses representing one bit. The correlator output Z is given by

$$Z = \int_{\tau}^{\tau+T_{\mathrm{b}}} r(t)\, m_w(t-\tau)\,\mathrm{d}t. \tag{15.2}$$

The decision variable Z in (15.2) is compared against a zero-valued threshold according to the following rule: when $Z > 0$, decision is "0," while when $Z < 0$, decision is "1," or vice versa. Then, for independent and equiprobable transmitted bits, given a transmitted bit $b_0 = 0$ the average bit error rate (BER) is:

$$\mathrm{BER} = \mathrm{Prob}\ \{\ Z < 0 | b_0 = 0\ \}. \tag{15.3}$$

It is essential to set a hypothesis on the capability of the system to synchronize. By analyzing (15.2), we notice that a necessary condition for the receiver to properly function is that the correlator mask must be aligned with $s(t-\tau)$. In other terms, the receiver must be capable of estimating the delay introduced by propagation over the channel and synchronize with the received signal. In order to achieve this function one can suppose for example to send within each data packet a specific sequence to be used for synchronization purposes and that is known by both the transmitter and the receiver [9]. In any case, system performance depends on the accuracy that can be achieved in synchronization. This accuracy is in turn related to signal to noise ratio and in particular to the receiver ability of extracting one single pulse from noise. Detecting the first pulse of the synchronization trailer for example might be crucial for correct operation. One system specification is thus the level of signal to noise ratio that can be achieved on the single pulse. We call SNR_0 the minimum signal to noise ratio over a single pulse that is required by system specification. A condition for correct reception is therefore that signal to noise ratio on the pulse be $\mathrm{SNR}_\mathrm{p} > \mathrm{SNR}_0$ [10].

The topology of the network is a star, that is, nodes communicate through the network controller. We suppose that the node that has the role of network controller also implements the cognitive radio paradigm. The role of controller can be played by any of the devices in the network but in our analysis we suppose that the controller is the node that starts the network by activating a beacon on a broadcast channel.

Multiple access is based on code division either in the time domain (time hopping) or amplitude-based (direct sequence). As a consequence, system performance is limited by multi-user interference (MUI). We suppose that the set-up of a link between a node and the coordinator occurs on a dedicated channel that is identified by a specific code.

The received signal of (15.1) that incorporates MUI can be expressed as follows:

$$r(t) = r_\mathrm{u}(t) + n_\mathrm{e}(t) + n_\mathrm{mui}(t) \tag{15.4}$$

where $r_\mathrm{u}(t)$ is the useful received signal, $n_\mathrm{mui}(t)$ accounts for MUI, and $n_\mathrm{e}(t)$ incorporates thermal noise and external interference introduced by wireless belonging to coexisting networks. In the present case, the decision variable at the output of the correlator is made of three terms: a useful contribution Z_u, external noise Z_e, and MUI Z_mui and writes $Z = Z_\mathrm{u} + Z_\mathrm{e} + Z_\mathrm{mui}$. Signal to noise ratio SNR at the correlator output for one link is thus:

$$SNR = \frac{E_\mathrm{u}}{\eta_\mathrm{e} + \eta_\mathrm{mui}} \tag{15.5}$$

where E_u is the received useful energy per bit for the reference link, and η_e and η_mui are the variance of Z_e and Z_mui. If all signals are received with same power,

$$E_\mathrm{u} = (N_\mathrm{S})^2 P_\mathrm{RX} T_\mathrm{S} \tag{15.6}$$

and

$$\eta_\mathrm{e} = N_\mathrm{S}\, \eta_\mathrm{p}(w) \tag{15.7}$$

$$\eta_\mathrm{mui} = N_\mathrm{S} \sigma_\mathrm{m}^2(w)(N-1)P_\mathrm{RX} \tag{15.8}$$

in which $\sigma_\mathrm{m}^2(w)$ is a term that depends upon the mask shape of the correlator and $\eta_\mathrm{p}(w)$ is noise variance on one pulse. Note that according to (15.7) and (15.8), both noise and interference depend on pulse shaping. If $R_\mathrm{b} = 1/T_\mathrm{b}$ is the bit rate for the link under examination, then the signal to noise ratio on the reference link can be obtained by combining (15.6), (15.7), (15.8), and (15.5) as follows:

$$\mathrm{SNR} = \frac{N_\mathrm{s}^2 T_\mathrm{S} P_\mathrm{RX}}{N_\mathrm{s}\eta_\mathrm{p}(w) + \sigma_\mathrm{m}^2(w) N_\mathrm{s}\,(N-1)\,P_\mathrm{RX}} = \frac{1}{R_\mathrm{b}} \frac{P_\mathrm{RX}}{\eta_\mathrm{p}(w) + \sigma_\mathrm{m}^2(w)\,(N-1)\,P_\mathrm{RX}}. \tag{15.9}$$

The BER of (15.3) can be expressed in a closed form as a function of SNR of (15.9) if $\eta_\mathrm{e}(t)$ and $\eta_\mathrm{mui}(t)$ have statistical properties that are known. If $\eta_\mathrm{e}(t)$ and $\eta_\mathrm{mui}(t)$ can be modeled as white Gaussian random processes, the relationship between BER and SNR becomes

$$\mathrm{BER} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{\mathrm{SNR}}{2}}\right) \tag{15.10}$$

where $\mathrm{erfc}(x)$ is the complementary error function of x. Based on (15.9) the signal to noise ratio on the pulse SNR_p is

$$\mathrm{SNR}_\mathrm{p} = \frac{T_\mathrm{S} P_\mathrm{RX}}{\eta_\mathrm{p}(w) + \sigma_\mathrm{m}^2(w)\,(N-1)\,P_\mathrm{RX}}. \tag{15.11}$$

Note that since we are considering an active connection, the following relationship holds:

$$\mathrm{SNR}_\mathrm{p} \geq \mathrm{SNR}_0. \tag{15.12}$$

15.3 Rules by Which a Node Communicates with the Coordinator

We assume that data flows are grouped into packets and that each packet is segmented into MAC frames. These frames are transmitted over the radio interface. The MAC frames have standard format and each of the frames is composed of a header and a payload. The header typically contains the MAC address as well as the synchronization trailer.

The system supports the best-effort data sources which do not require the use of a minimum value for the transmission rate and are not bound by maximum delay specifications. In other terms, best-effort sources do not require any quality of service guarantee and are allowed to transmit, i.e., are admitted in the network, if their presence does not disturb the operation of other sources that cannot tolerate end-to-end delay D (s) greater than specific values, and that require at least a F percentage of packets to reach destination within D. These different sources, called quality of service sources are characterized by the characteristics of the traffic that they generate and by their required specifications D and F. We suppose that the traffic generated by the sources in the network is shaped by a standard dual leaky bucket (DLB) [11] that functions as an interface between the source and the system and that outputs traffic described by the four following parameters: the peak rate p (bits/s), the average rate r (bits/s), the token buffer dimension b (bits) and the maximum packet size M (bits). Rates p and r do not account for the overhead introduced at the MAC and physical layers, and can thus be lower than binary rate R_b of (15.9).

In this section we illustrate the principles of operation of a network of N nodes, and we define the basic principles of the admission control function. The admission control function for UWB nodes proposed in [6] can be generalized for the present analysis case. We summarize here the rules for power assignment for convenience of the reader. We refer to [6] for a full coverage of the topic and full description of the admission control function that includes the rules by which both best-effort and quality of service nodes compute their rate of transmission. As indicated above, a star topology is taken into consideration. As a consequence the analysis is focused on the uplink connections.

Different pulses $p_w(t)$, with $w = 1, ..., W$ can be used for shaping the spectra of radiated signals and adapt this shape to the features of the channel. It is the controller that has the capability of sensing the channel, and therefore, of appropriately selecting the "best" impulse response of the pulse shaper. The controller node computes $\eta_{\mathrm{p}}(w)$ of (15.7) and $\sigma_{\mathrm{m}}^2(w)$ of (15.8) for all possible pulse shapes that is $w = 1, ..., W$, and based on $\eta_{\mathrm{p}}(w)$ and $\sigma_{\mathrm{m}}^2(w)$, estimates $P_{\min}(w)$ as follows:

$$P_{\min}(w) = \frac{\eta_{\mathrm{p}}(w)}{T_{\mathrm{S}}} \left(\frac{1}{\mathrm{SNR}_0} - \frac{\sigma_{\mathrm{m}}^2(w)}{T_{\mathrm{S}}} (N-1) \right)^{-1} . \qquad (15.13)$$

The above equations provide the minimum power that the controller must receive from each node in order to comply with (15.12). The "best" pulse shaper can be thus defined as the one that provides the lowest $P_{\min}(w)$ value. As a consequence, each node j must use a transmission power P_j that can be computed as follows:

$$P_j = P_{\min}(w^*)\, A_j \qquad j = 1, ..., N \qquad (15.14)$$

where A_j is the attenuation characterizing the link between node j and the controller.

15.4 Admission Control by Hybrid Modeling

Hybrid system formalism offers the framework for modeling the behavior of self-organizing networks. Thanks to this formalism, we can characterize self-organizing network dynamics as a discrete finite-state automaton where, for each state, state-specific rules of operation govern the evolution of the network itself. In this section, we first illustrate the fundamental principles of hybrid system modeling. We then describe the application of hybrid system modeling to the system under consideration that incorporates the cognitive radio concept.

15.4.1 Basic Principles of Hybrid Modeling

Hybrid systems are dynamical systems where continuous and discrete dynamics are embedded together to propositional logic. Continuous and discrete variables interact and determine the hybrid system evolution. The hybrid state of a hybrid system is made of two components: The discrete state belonging to a finite set $\boldsymbol{Q}$ and the continuous state belonging to a linear subspace of $\mathbb{R}^n$. The evolution of the discrete state is governed by an automaton, while the evolution of the continuous state is given by a dynamical system controlled by a continuous input and subject to continuous disturbances. Whenever a discrete transition occurs, the continuous state is instantly reset to a new value. Even if the intuitive notion of hybrid system is simple, the combination of discrete and continuous dynamics and the mechanisms that govern discrete transitions create serious difficulties in defining its operation precisely. Other complexity stems from the continuous state reset that occurs when the system undergoes a discrete transition. This is why we need formal definitions of the variables that characterize a hybrid system as well as of their evolution in time, as will be defined below:

1. The state variable of a hybrid system $\mathcal{H}$ is made of two components: the discrete state q and the continuous state x. The discrete state belongs to a finite set $\boldsymbol{Q} = \{q_i, i \in J\}$, $J = \{1, 2, ..., N\}$, $N \in \mathbb{N}$ and the continuous state takes value in $\mathbb{R}^n$. The set $\Xi = \boldsymbol{Q} \times \mathbb{R}$ is the hybrid state space of $\mathcal{H}$ and its elements $\xi = (q, x) \in \Xi$ are the hybrid states.
2. The control input variable of $\mathcal{H}$ is made of two components: the discrete control input σ_c and the continuous control input u. The discrete control input belongs to a finite set $\boldsymbol{\Sigma}^c$ and the continuous control input to the set $\mathbb{R}^m$, $m \in \mathbb{N}$. We assume that the input functions $\boldsymbol{u} : \mathbb{R} \to \mathbb{R}^m$ are piecewise continuous.
3. The disturbance variable of $\mathcal{H}$ is made of two components: the discrete disturbance σ_d and the continuous disturbance input d. The discrete disturbance takes value in a finite set $\boldsymbol{\Sigma}^d$ and the continuous disturbance in the set $\mathbb{R}^r$, $r \in \mathbb{N}$. We assume that the disturbance functions $\boldsymbol{d} : \mathbb{R} \to \mathbb{R}^r$ are piecewise continuous.
4. The output variable of $\mathcal{H}$ is made of two components: the discrete output p and the continuous output y. The discrete output is assumed to belong to a finite set $\boldsymbol{P}$ and the continuous output to the set $\mathbb{R}^s$, $s \in \mathbb{N}$. The continuous output functions $\boldsymbol{y} : \mathbb{R} \to \mathbb{R}^s$ are assumed to be piecewise continuous.

The evolution of the discrete state q of hybrid system $\mathcal{H}$ depends on the initial discrete state as well as on the discrete input σ_c, the discrete disturbance σ_d, and the continuous state x, and is driven by events forcing discrete states to jump. There are three types of discrete transitions:

1. Switching transition, forced by a discrete disturbance $\sigma_d \in \boldsymbol{\Sigma}^d$
2. Invariance transition, determined by the continuous state x reaching some regions of the continuous state space; events inducing invariance transitions are assumed to belong to the finite set $\boldsymbol{\Sigma}^i$ and are internally generated by the hybrid system
3. Controllable transition, determined by a discrete control input $\sigma_c \in \boldsymbol{\Sigma}^c$

We denote by $\boldsymbol{\Sigma}$ the set of all events causing discrete transitions of discrete states. A relation represents the collection of all discrete transitions $e = (q, \sigma, q\prime) \in E \subset \boldsymbol{Q} \times \boldsymbol{\Sigma} \times \boldsymbol{Q}$ taking the discrete state from q to $q\prime$ if the event $\sigma \in \boldsymbol{\Sigma}$ occurs. The evolution of the continuous state x depends on the initial continuous state and on the evolution in time of the continuous input u, the continuous disturbance d, and the discrete state q. The continuous state and output evolution between two consecutive discrete transitions is modeled by a dynamical system $S(q)$ that is assumed to be linear for simplicity and governed by the following equations:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Dd(t) \\ x(t) &\in \mathbb{R}^n, u(t) \in \mathbb{R}^m, d(t) \in \mathbb{R}^r, t \geq 0. \end{aligned} \tag{15.15}$$

During its evolution in time, the hybrid state $\xi = (q, x)$ has to satisfy the so-called invariance condition $x \in \mathrm{Inv}(q)$, where $\mathrm{Inv}(.)$ is called the invariance map. Whenever a discrete transition $e = (q, \sigma, q\prime) \in E$ occurs, the hybrid state $\xi = (q, x^-)$ has to satisfy the so-called guard condition $x^- \in G(e)$, where $G(.)$ is called the guard map and the continuous state instantly jumps from $x^- \in \mathbb{R}^n$ to a new value $x^+ \in R(e, x^+)$, where $R(.,.)$ is called the reset map.

15.4.2 Hybrid Modeling of Self-Organizing Networks

We describe now how hybrid system formalism can be used for modeling the behavior of self-organizing networks. Different models are possible, depending on the role of the discrete state. In the model proposed in [6], each discrete state of the automaton corresponds to the presence in the network of N active nodes and one controller. In each discrete state, the system receives different inputs ranging from RF stimuli from the environment, that are processed by the controller, to indicators of the attenuation that is present over the N active links. These attenuation indicators are used by the active nodes for evaluating potential transmission parameters as well as their capability to comply with the above.

We generalize here the hybrid model proposed in [7] for UWB networks. Each discrete state of the automaton corresponds to an operation mode, described by the waveform used for pulse shaping. In addition, one particular state of the automaton corresponds to the admission control mode, where the controller evaluates the possibility of admitting a new node in the network. A transition to this state takes place when a new node is asking for admission in the network and, for simplicity, we assume that the control procedure requires a negligible time to be performed. The automaton is represented in Fig. 15.1 for the very simple case of two waveforms, namely w_1 and w_2.

A continuous variable $N(t)$, the current number of active nodes that are allowed to transmit data over the wireless channel, is associated to each discrete state of the automaton. This variable is reset to a new value whenever a transition occurs, as described in the next subsection.

In each state, the system receives different inputs ranging from radio frequencies (RF) stimuli from the environment, to indicators of the attenuation that is present over the active links. These attenuation indicators are used by the active nodes for evaluating at time t both potential transmission parameters as well as their capability to comply with the transmission constraints that are communicated by the controller through a time-dependent set of parameters named $\boldsymbol{K}(t)$. This time-varying set of parameters $\boldsymbol{K}(t)$, is formed as follows:

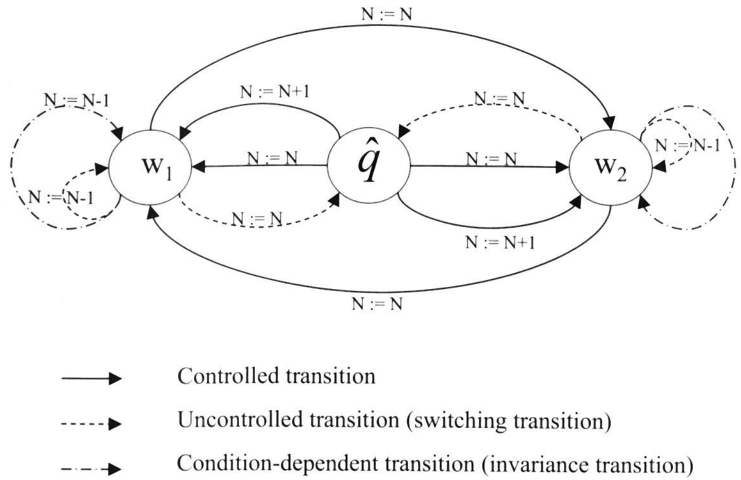

Fig. 15.1. Hybrid model.

1. The waveform w^* that must be used for pulse shaping. Different pulse shapes can be selected for transmitting data over the wireless channel; w^* is the one that better adapts with the environment, as well as with thermal noise and multi-user interference (MUI) patterns.
2. The power level $P_{\min}(w^*)$ that is required at the controller in order to comply with the requirement of a given signal to noise ratio threshold.
3. The noise level $\eta_{\mathrm{p}}(w^*)$ that is currently measured at the coordinator.
4. The MUI weight $\sigma^2_{\mathrm{m}}(w^*)$.
5. The number of active nodes N.

Within the above set, the first two parameters w^* and $P_{\min}(w^*)$ can be considered as constraints that are imposed to the nodes in the network. The noise level $\eta_{\mathrm{p}}(w^*)$ can be interpreted as a continuous disturbance. The MUI weight $\sigma^2_{\mathrm{m}}(w^*)$ and the number of active nodes N are information characterizing the current system state. The time-varying set of parameters $\boldsymbol{K}(t)$ is evaluated at the coordinator. We suppose that the signal containing the above information is sent by the controller at a fixed power level that is pre-determined and known by all nodes.

Each active node j receives the signal conveying $\boldsymbol{K}(t)$ and, on the basis of received power level, can estimate the attenuation A_j characterizing its path to the coordinator. Node j determines both power and rate to be used in its future transmissions (see Sect. 15.3). We assume for now that the possible variations in the environment that reflect in $\boldsymbol{K}(t)$ are tolerable by all nodes.

In the next subsection, we formally describe the network dynamics using the hybrid systems formalism.

15.4.3 The Hybrid Model

A hybrid model $\mathcal{H}$ for the network can be defined by introducing the tuple $\mathcal{H} = (\Xi, \boldsymbol{\Sigma}, S, E, R)$:

- $\Xi = \boldsymbol{Q} \times \mathbb{R}$ is the hybrid state space; $\boldsymbol{Q} = \boldsymbol{W} \cup \{\widehat{q}\}$, $\boldsymbol{W}$ is the finite set of waveforms that can be used for pulse shaping; the state $\widehat{q}$ represents the fact that a candidate node is waiting for admission in the network.
- $\boldsymbol{\Sigma}$ is the set of discrete inputs; $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}^c \cup \boldsymbol{\Sigma}^d \cup \{\widehat{\sigma}\}$, where $\boldsymbol{\Sigma}^c$ is the set of discrete controls and $\boldsymbol{\Sigma}^d$ is the set of discrete disturbances and

$$\begin{aligned} \boldsymbol{\Sigma}^c &= \boldsymbol{\Sigma}^c_{\boldsymbol{W}} \cup \boldsymbol{\Sigma}^c_a \\ \boldsymbol{\Sigma}^c_{\boldsymbol{W}} &= \{\sigma_{ij}, i, j \in J\} \\ \boldsymbol{\Sigma}^c_a &= \{\mathrm{OK}, \mathrm{NO}\} \\ \boldsymbol{\Sigma}^d &= \{\sigma_{\mathrm{a}}, \sigma_{\mathrm{l}}\}. \end{aligned}$$

  The discrete controls σ_{ij} model the decision taken by the coordinator to commute from pulse shape w_i to pulse shape w_j. The discrete controls $\{\mathrm{OK}, \mathrm{NO}\}$ correspond, respectively, to the decision taken by the coordinator to accept or not to accept a candidate node in the network. The discrete disturbances σ_{a} and σ_{l} represent, respectively, the request by some candidate node to enter the network and the event that a node leaves the network. Finally, the discrete input $\widehat{\sigma}$ is an endogenous signal that is generated when changes in the environment and in radio propagation are no more compliant with node requirements.
- S is a map associating to every discrete state in $\boldsymbol{Q}$, a dynamical system.
  - If the discrete state is $w_i \in \boldsymbol{W}$, then the dynamical system $S(w_i)$ is described by the equations:

$$\dot{N}(t) = 0$$

$$\boldsymbol{K}_i(t) = \begin{pmatrix} w_i \\ p_i(t) \\ \eta_{\mathrm{p}}(w_i) \\ \sigma^2_{\mathrm{m}}(w_i) \\ N(t) \end{pmatrix}$$

  where

$$p_i(t) = P_{\mathrm{m}}(w_i, t)$$

$$P_{\mathrm{m}}(w_i, t) = \frac{\eta_{\mathrm{p}}(w_i)}{T_{\mathrm{S}}}\left(\frac{1}{\mathrm{SNR}_0} - \frac{\sigma^2_{\mathrm{m}}(w_i)}{T_{\mathrm{S}}}(N(t) - 1)\right).$$

  $N(t)$ is the continuous state at time t, i.e., the number of current nodes in the network; $\boldsymbol{K}_i(t)$ is the output, where $p_i(t)$ is the power that the coordinator must receive from the $N(t)$ active nodes in order to comply with the requirements of a threshold SNR_0, when the pulse shape is w_i, and T_{S} is the pulse repetition period. $\eta_{\mathrm{p}}(w_i)$ and $\sigma^2_{\mathrm{m}}(w_i)$, represent the discrete state dependent disturbances. The initial value for the continuous state is $N(0) = 1$ (at the beginning, the coordinator is the only node of the network). The value of the state is reset whenever a transition occurs.
  - If the discrete state is $\widehat{q}$, then $S(\widehat{q})$ is described by the equations

$$\dot{N}(t) = 0$$

$$\widehat{\boldsymbol{K}}(t) = \begin{pmatrix} w^* = \arg\min_{w \in \boldsymbol{W}} \widehat{P}_{\min}(w) \\ \widehat{p}(t) \\ \eta_{\mathrm{p}}(w^*) \\ \sigma^2_{\mathrm{m}}(w^*) \\ N(t) + 1 \end{pmatrix}$$

  where

$$\widehat{p}(t) = \widehat{P}_{\mathrm{m}}(w^*, t)$$

$$\widehat{P}_{\mathrm{m}}(w^*, t) = \frac{\eta_{\mathrm{p}}(w^*)}{T_{\mathrm{S}}}\left(\frac{1}{\mathrm{SNR}_0} - \frac{\sigma^2_{\mathrm{m}}(w^*)}{T_{\mathrm{S}}} N(t)\right).$$

  $N(t)$ is the continuous state at time t, $\widehat{\boldsymbol{K}}(t)$ is the output produced by the coordinator at time t. The state $\widehat{q}$ corresponds to the control admission mode and the role of the output $\widehat{\boldsymbol{K}}(t)$ is therefore discussed in Sect. 15.4.4 describing the admission control algorithm.
- $E \subset \boldsymbol{Q} \times \boldsymbol{\Sigma} \times \boldsymbol{Q}$ is a collection of transitions.

$$E = E^c \cup E^d \cup E^{\mathrm{inv}}$$

  where

$$\begin{aligned} E^c &= E^c_{\boldsymbol{W}} \cup E^c_{\mathrm{a}},\ E^d = E^d_{\mathrm{a}} \cup E^d_{\mathrm{l}} \\ E^c_{\boldsymbol{W}} &= \{(w_i, \sigma_{ij}, w_j), \sigma_{ij} \in \boldsymbol{\Sigma}^c_{\boldsymbol{W}}, w \in \boldsymbol{W}\} \\ E^c_{\mathrm{a}} &= \{(\widehat{q}, \sigma, w), \sigma \in \boldsymbol{\Sigma}^c_{\mathrm{a}}, w \in \boldsymbol{W}\} \\ E^d_{\mathrm{a}} &= \{(w, \sigma_{\mathrm{a}}, \widehat{q}), w \in \boldsymbol{W}\} \\ E^d_{\mathrm{l}} &= \{(w, \sigma_{\mathrm{l}}, w), w \in \boldsymbol{W}\} \\ E^{\mathrm{inv}} &= \{(w, \widehat{\sigma}, w), w \in \boldsymbol{W}\}. \end{aligned}$$

  - The transitions in E^c are controlled (in Fig. 15.1 these transitions are represented by solid arrows).
  - A transition (w_i, σ_{ij}, w_j) in $E^c_{\boldsymbol{W}}$ models the decision, taken at some time t by the coordinator, of commuting from pulse shape w_i to pulse shape w_j, for transmitting data over the wireless channel. This transition takes place when w_j is the pulse shape that, at time t, better adapts to the time-varying environment, as well as thermal noise and MUI patterns, or when $p_i(t) > P_{\mathrm{m}}(w_j, t)$. In the latter case, w_j is such that $P_{\mathrm{m}}(w_j, t) \leq P_{\mathrm{m}}(w_{\mathrm{h}}, t)$, $\forall w_{\mathrm{h}} \in \boldsymbol{W}$.

- The transitions in E_a^c occur when the coordinator decides to accept or not to accept a candidate node in the network.
- The transitions $\{(w, \sigma_a, \widehat{q}), w \in \boldsymbol{W}\}$ are not controlled (switching transitions) and represent the request of entering the network by some candidate node.
- The transitions $\{(w, \sigma_l, w), w \in \boldsymbol{W}\}$ (dashed arrows in Fig. 15.1) are not controlled and represent the fact that a node could leave the network because its activity is terminated for reasons that range from no more data packets to transmit, to node failure, to power exhaustion.
- The transitions in E^{inv} (dot-line arrows in Fig. 15.1) occur because changes in the environment (as sensed by the coordinator) and in radio propagation (as perceived by the active nodes) are no more compliant with node's requirements. Then, the node leaves the network.

For simplicity, we assume that simultaneous transitions are not allowed.

- $R : \Xi \times E \to \Xi$ and

$R((q_i, x), e) = (q_h, x)$,	$e = (q_i, \sigma, q_h) \in E_{\boldsymbol{W}}^c$
$R((q_i, x), e) = (q_h, x + 1)$,	$e = (q_i, \text{OK}, q_h) \in E_a^c$
$R((q_i, x), e) = (q_h, x)$,	$e = (q_i, \text{NO}, q_h) \in E_a^c$
$R((q_i, x), e) = (\widehat{q}, x)$,	$e = (q_i, \sigma, \widehat{q}) \in E_a^d$
$R((q_i, x), e) = (q_i, x - 1)$,	$e = (q_i, \sigma, q_i) \in E_l^d$
$R((q_i, x), e) = (q_i, x - 1)$,	$e = (q_i, \sigma, q_i) \in E^{\text{inv}}$

15.4.4 The Control Algorithm

The main control objective is to maximize the number of active nodes in the network, while preserving transmission requirements. In fact, when a node asks for admission, i.e., when the current discrete state at time t is $\widehat{q}$, the coordinator evaluates the possibility of admitting the new element in the network, by computing a hypothetical set of parameters $\widehat{\boldsymbol{K}}(t)$. The use of this information is twofold. First, it serves to the current active nodes in order to check whether constraints for transition are compatible with their specifications and informs the coordinator. Willingness to transition of all nodes is a necessary condition for transition. Second, the information in $\widehat{\boldsymbol{K}}(t)$ is used by the candidate node for evaluating its willingness to join the network. A candidate node that listens to $\widehat{\boldsymbol{K}}(t)$ must agree in accepting those constraints for the transition to take place. The two conditions above correspond to guard conditions that must be satisfied in order for the transition $(\widehat{q}, \text{OK}, w^*) \in E_a^c$ to take place (where w^* is the first component of $\widehat{\boldsymbol{K}}(t)$). If the above conditions are not fulfilled, then a transition $(\widehat{q}, \text{NO}, w_h)$ in the set E_a^c takes place, where $w_h = \arg\min_{w \in \boldsymbol{W}} P_m(w)$. Therefore, a new node is admitted only if none of the current active nodes is forced to leave the network as a result of its admission.

At each time, the network is controlled so that the power level is minimum with respect to the number of active nodes, possible choices of pulse shaping, and environmental parameters. As a consequence, the described control strategy minimizes the energy consumption in the network, which is a beneficial effect in wireless communication.

Conclusion

Based on hybrid system formalism we described self-organizing network dynamics as a discrete finite-state automaton where, for each state, state-specific rules of operation govern the evolution of the network itself. By doing so, we modeled network of radio devices that must coexist with severely interfered environments, and therefore must control their behavior and adapt to ever-changing operating conditions in order to favor coexistence. In the proposed model, this is achieved by introducing cognitive mechanisms in the analysis process that is used by nodes for determining whether changes in the global network state are appropriate.

Several benefits are obtained by introducing the hybrid system model for the design of cognitive networks:

- The formal description resulting from the adoption of the hybrid system formalism allows a better understanding of some important properties of the system. As an example, it is possible to characterize the trade-off that exists between the complexity of a real-time and precise scanning of the external environment vs. the improvement in system efficiency that is achieved when the nodes can rapidly adapt themselves to the varying conditions of the operating scenario. Based on this trade-off, we could investigate the existence of suboptimal but computational-efficient strategies, where the capability of the nodes to adapt to the external environment is limited and depends upon the current state of the automaton.
- Using the hybrid system model, it is possible to optimize the distribution of functional specifications among the different components of the system. For example, we can analyze how system performance is affected whenever some of the functionalities that are related to active nodes are related to the coordinator and vice versa.
- The hybrid formalism may help to predict in which states the automaton will spend most of the time or the maximum number of nodes of the network. This information is of fundamental importance for network designers.
- The characterization of the wireless network as a hybrid system facilitates the analysis of the stability [8] of the overall system. This task is by no means trivial since we assume that the nodes dynamically adapt transmission parameters and rules of operation to external stimuli.

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