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# Performance Analysis of a DCQPSK-OFDM Modem Using Analog BB Filters

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**Abstract:** Results of a performance analysis on a DCQPSK-OFDM system using analog BB filters is presented. The above system configuration is typically affected by a mismatch of the I and Q paths. As regards the transmitter side, theoretical analysis show that the mismatch can be represented by a mismatch transfer function. The maximum amplitude/phase mismatch of the BB filters paths and the maximum phase error of the in-phase and in-quadrature carriers are reported.

## Introduction

In this paper, a performance analysis on a DCQPSK-OFDM system is carried out. In this system, the reference axis for each submodulator is constituted by the phase of the previous (in frequency) submodulator<sup>1</sup>. In a system such as the Median system which is characterized by a high system clock, analog base-band filters must be used, and the effects produced by the mismatch between the in-phase (I) and in-quadrature (Q) filters must be analyzed. Section 1 describes the modem operations of a full-digital QPSK/OFDM modem. In section 2, the add-on necessary to realize a DC scheme are reported and the error probability is evaluated. In section 3, the effects of the mismatch between I and Q baseband paths are examined. Finally, section 4 contains the applications and section 5 the conclusions.

## 1. Description of a QPSK OFDM modem.

The Orthogonal Frequency Division Multiplex is a digital modulation scheme in which several digitally modulated signals are frequency multiplexed. Each modulator uses the same modulation method (QAM, CPSK or DCPSK), the same symbol period  $T_s$ , the same rectangular data pulse, and carries  $\beta$  bits/symbol. The subcarrier frequencies of the modulators are spaced by  $\Delta f$  around the carrier frequency  $f_p$ ; moreover, the symbol period must be greater than  $T_o = 1/\Delta f$ , and is usually written as  $T_s = T_o + T_g$ , in which  $T_g$  is a *guard time* which is introduced to cope with defective symbol synchronization and multipath. Other important parameters of the OFDM modem are the number  $N$  of available subcarriers (a power of two) and the number  $N_u = M_1 + M_2 < N$  of effectively used subcarriers ( $M_1$  is the number of subcarriers at frequencies above the system carrier,  $M_2 - 1$  is the number of subcarriers below the system carrier). A full-digital implementation of the OFDM scheme, based on the use of IFFT/FFT processors ([1],[2], Fig.1), produces a sequence of  $N + N_g$  samples for each OFDM symbol, taken *exactly* at frequency  $B$ , where  $N_g = T_g \cdot B$  is the number of samples belonging to the prefix. Figure 2 represents the BB equivalent of the overall analog connection, in which an eventual mismatch between the in-phase and in-quadrature paths of the transmitter is highlighted.

In a QDCPSK-OFDM system, the samples of the complex envelope of the transmitted signal are generated by an IFFT processor from a sequence of  $N_u < N$  constellation points  $\{c_{-M_2-1}, \dots, c_0, \dots, c_{M_1}\}$ . The sequence  $\{c_m\}$  is generated by a segmenter & differential encoder described in the next section, from the input binary data sequence. In the present section, we suppose

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<sup>1</sup> This scheme is used in "burst mode" transmissions, in which previous (in time) OFDM symbols can be transmitted by different users.

$H_{BTI}(f) = H_{BTQ}(f) \equiv H_{BT}(f)$  and  $\phi = 0$ ; therefore, the received complex envelope, and its Fourier Transform are given by:

$$y(t) = \sum_{k=-N_t}^{N-1} d_k \cdot g(t-k/B) \Leftrightarrow Y(f) = G(f) \cdot \sum_{k=-N_t}^{N-1} d_k \cdot e^{-j2\pi f \frac{k}{B}} \quad (1)$$

where:

$$g(t) \equiv \pi(t) * h(t) \Leftrightarrow G(f) = \frac{1}{B} \frac{\sin(\pi f / B)}{\pi f / B} \cdot H_{BT}(f) \cdot H_R(f) \equiv G_T(f) \cdot H_{BR}(f) \quad (2)$$

$$\pi(t) = \text{rect waveform from each DAC} \Leftrightarrow \frac{1}{B} \frac{\sin(\pi f / B)}{\pi f / B} \equiv \Pi(f)$$

$$d_k = \text{IFFT of } \{D_m\} = \sum_{m=0}^{N-1} D_m e^{j2\pi \frac{mk}{N}} \text{ periodic of period } N \quad D_m = \begin{cases} c_m & 0 \leq m \leq M_1 \\ 0 & M_1 < m \leq N - M_2 \\ c_{m-N} & N - M_2 < m \leq N - 1 \end{cases} \quad (3)$$

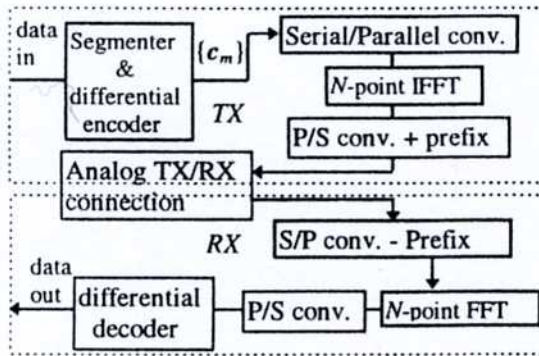


Fig.1 Block diagram of an OFDM modem.

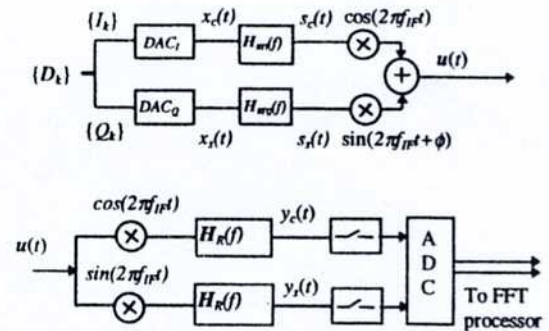


Fig.2 Block diagram of the baseband equivalent of the analog TX/RX connection.

The signal  $y(t)$  is sampled at frequency  $B = N \cdot \Delta f$ . If the duration of the impulse response  $g(t)$  is lower than  $T_g$ , the  $m$ -th FFT output, i.e. an estimate of the transmitted constellation point  $c_m$ , is given by:  $\hat{c}_m = G(m \cdot \Delta f)$ ,  $-M_2 < m \leq M_1$ . Therefore, the estimate of  $c_m$  is biased by  $\beta_m = G(m \Delta f)$ . In a CPSK

$c_m$



scheme, the phase of  $\beta_m$  may destroy the information carried by the modulation; this can be avoided by using a differentially coherent scheme<sup>2</sup>, as described in the next section.

## 2. Error probability in a QDCPSK/OFDM system.

It is hypothesized that an additive gaussian noise with a symmetrical power density  $N_o$  is present at the channel output. Therefore, a complex zero mean gaussian process  $n(t)$ , with symmetric power spectrum  $P_N(f) = 4N_o$  is added to the complex envelope of the received signal  $y(t)$ . This noise produces an additive zero mean gaussian complex noise sample  $n_k$  on each FFT output, with variance ([1]):

$$\psi_m^2 = \frac{B}{N} P_N(m \cdot \Delta f) = 4N_o |H_R(f_m)|^2 \Delta f \quad (5)$$

The data sequence is grouped into dibits and coded using a 4 points constellation represented in Fig.3, to produce a sequence  $\{p_k\}$  which is then differentially encoded to generate a sequence  $\{q_k\}$  such that  $q_k^* \cdot q_{k-1} = p_k$ . Each term of  $\{q_k\}$ , which belongs to a set of 8 values equally spaced along the unitary circle, is multiplied by a precorrection factor  $\gamma_k = 1/G_T(f_k)$  INCRUSTAR which equalizes the SNR on each subcarrier, to produce the transmitted constellation sequence  $c_k = q_k/G_T(f_k)$ . Therefore, the received signal power, which is half the power of its complex envelope, is given by:

$$P_R = \frac{1}{2} \sum_{k=-\infty}^{\infty} E\{|d_k|^2\} |G_T(f_k)|^2 \cong \frac{1}{2} \sum_{k=-M_2}^{M_1} |\gamma_k|^2 |G_T(f_k)|^2 = \frac{N_u}{2} \quad (6)$$

Finally, we indicate by SNR the ratio between the received power and the noise power in a frequency band equal to the on-air symbol rate  $f_s = N_u/T_s = N_u/T_o(1+T_g/T_o) = N_u\Delta f/(1+T_g/T_o)$ . From (6), one has:

$$SNR = \frac{P_R}{2N_o f_s} = (1 + T_g / T_o) \frac{1}{4N_o \Delta f} \quad (7)$$

If the differential co/decoding is taken into account, the estimate of the transmitted constellation point  $p_m$  on the  $m$ -th subcarrier is:

$$\begin{aligned} \hat{p}_m &= c_{m-1}^* c_m = (H_R^*(f_{m-1}) \cdot q_{m-1}^* + n_{m-1}^*) (H_R(f_m) \cdot q_m + n_m) \\ &\cong H_R^*(f_{m-1}) \cdot H_R(f_m) \cdot p_m + H_R(f_m) \cdot q_m \cdot n_{m-1}^* + H_R^*(f_{m-1}) \cdot q_{m-1}^* \cdot n_m \equiv \beta_m p_m + v_m \end{aligned} \quad (8)$$

Equation (8) shows that the estimate of  $p_m$  is affected by an additive gaussian noise  $v_m$  and by a multiplicative bias  $\beta_m = H_R^*(f_{m-1}) \cdot H_R(f_m) \equiv b_m \cdot e^{j\varphi_m}$ , as in a coherent system, but now  $\varphi_m$  is usually small. Since  $n_m$  and  $n_{m-1}$  are independent and since  $|q_m|=1$ , from (8) the variance of  $v_m$  is:  $\sigma_v^2 = (|H_R(f_m)|^2 + |H_R(f_{m-1})|^2) \cdot 4N_o |H_R(f_m)|^2 \Delta f$ . Finally, since  $p_m = (\pm 1 \pm j) / \sqrt{2}$ , the bit error rate on the  $k$ -th constellation point is:

<sup>2</sup> In effect, a CPSK scheme requires the use of an equalization procedure, but it is prohibitive in a multi-user burst-mode operations.

$$P_m = \frac{1}{4} \operatorname{erfc}(z_{m+}) + \frac{1}{4} \operatorname{erfc}(z_{m-}) \quad (9)$$

$$z_{m\pm}^2 = \left\{ \frac{b_m (\cos \varphi_m \pm \sin \varphi_m)}{\sqrt{2} \sigma_v} \right\}^2 = \frac{|H_R(f_{m-1})|^2 \cdot (\cos \varphi_m \pm \sin \varphi_m)^2}{8N_o \Delta f \{ |H_R(f_m)|^2 + |H_R(f_{m-1})|^2 \}} = \frac{(\cos \varphi_m \pm \sin \varphi_m)^2}{8N_o \Delta f} = \frac{\operatorname{SNR} \cdot (\cos \varphi_m \pm \sin \varphi_m)^2}{4(1 + T_g / T_o)}$$

### 3. Effect of LP transmitter filters mismatch on the I and Q paths.

In Fig.2, the input sequence to be transmitted is represented by two real sequences  $\{I_{-N_g}, \dots, I_{N-1}\}$  and  $\{Q_{-N_g}, \dots, Q_{N-1}\}$  which are D to A converted, filtered and cos/sin modulated. We indicate by :

$$G_{TI}(f) = \frac{1}{B} \frac{\sin(\pi f / B)}{\pi f / B} H_{BTI}(f) \cdot H_T(f) \Leftrightarrow g_{TI}(t) \quad (10)$$

$$G_{TQ}(f) = \frac{1}{B} \frac{\sin(\pi f / B)}{\pi f / B} H_{BTQ}(f) \cdot H_T(f) \Leftrightarrow g_{TQ}(t) \quad (11)$$

the equivalent LP filters of the I and Q paths, respectively. The RF signal can be represented by:

$$\begin{aligned} s(t) &= s_c(t) \cdot \cos(2\pi f_p t) - s_s(t) \cdot \sin(2\pi f_p t + \phi) = s_I(t) \cdot \cos(2\pi f_p t) - s_Q(t) \cdot \sin(2\pi f_p t) \\ s_I(t) &= s_c(t) - \sin \phi \cdot s_s(t) \\ s_Q(t) &= \cos \phi \cdot s_s(t) \\ s_c(t) &= \sum_{k=-N_g}^{N-1} I_k \cdot g_{TI}(t - k / B) \\ s_s(t) &= \sum_{k=-N_g}^{N-1} Q_k \cdot g_{TQ}(t - k / B) \end{aligned} \quad (12)$$

or by the Fourier Transform of its complex envelope  $s(t) = s_I(t) + js_Q(t)$ :

$$S(f) = G_{TI}(f) \sum_{k=-N_g}^{N-1} I_k \cdot e^{-j2\pi f k / B} + je^{j\phi} G_{TQ}(f) \sum_{k=-N_g}^{N-1} Q_k \cdot e^{-j2\pi f k / B}$$

<sup>3</sup> Any difference between the true DAC's output pulse waveform and the ideal *rect* function is included in the  $H_{BTI}$  and  $H_{BTQ}$  transfer functions.

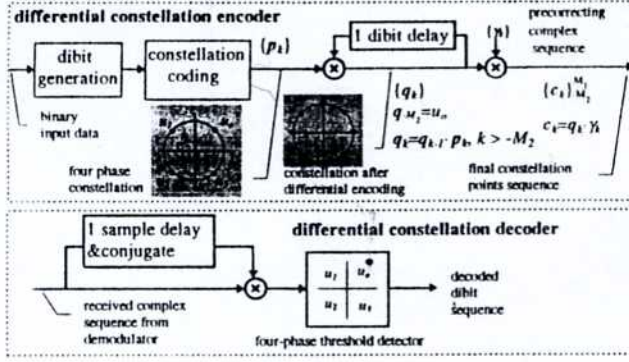


Fig.3 Block diagram of the differential co/decoder

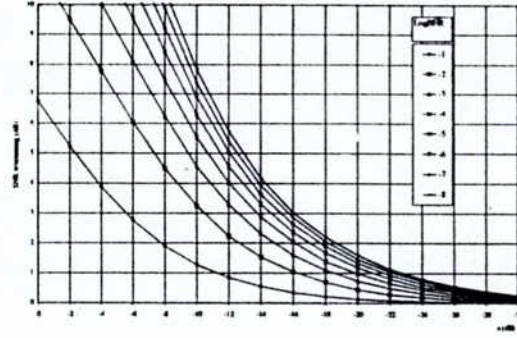


Fig.4 SNR worsening versus a (dB), for different values of  $\log_{10}(\text{Bit Error Probability})$  evaluated @  $a = 0$ .

If one sets:

$$\begin{cases} G_{TI}(f) \equiv G_T(f) + G_e(f) \\ G_{TQ}(f) \equiv G_T(f) - G_e(f) \end{cases} \Rightarrow \begin{cases} G_T(f) = (G_{TI}(f) + G_{TQ}(f))/2 \\ G_e(f) = (G_{TI}(f) - G_{TQ}(f))/2 \end{cases} \quad (13)$$

one obtains:

$$\begin{aligned} S(f) &= G_T(f) \sum_{k=-N_g}^{N-1} (I_k + jQ_k) \cdot e^{-j2\pi fk/B} + G_e(f) \sum_{k=-N_g}^{N-1} (I_k - jQ_k) \cdot e^{-j2\pi fk/B} = \\ &= G_T(f) \sum_{k=-N_g}^{N-1} d_k \cdot e^{-j2\pi fk/B} + G_e(f) \sum_{k=-N_g}^{N-1} d_k^* \cdot e^{-j2\pi fk/B} \end{aligned} \quad (14)$$

The previous equations show that the mismatch between the  $I$  and  $Q$  LP paths adds a conjugate component to the transmitted sequence. Therefore, the  $m$ -th outputs of the FFT processor due to the first and second component, are given by:

$$c_m^{(I)} = H_R(f_m) \cdot G_T(f_m) \cdot c_m = H_R(f_m) \cdot G_T(f_m) \cdot \gamma_m \cdot q_m \quad (15)$$

$$c_m^{(Q)} = H_R(f_m) \cdot G_e(f_m) \cdot c_m^* = H_R(f_m) \cdot G_e(f_m) \cdot \gamma_{-m}^* \cdot q_{-m}^* \quad (16)$$

respectively<sup>4</sup>, and if we keep the ideal choice  $\gamma_k = 1/G_T(f_k) = \gamma_{-k}^*$ <sup>5</sup>, from the linearity of the demodulation process, the true  $m$ -th output is:

<sup>4</sup> We have also used the following relation:  $\sum_{k=0}^{N-1} D_k^* e^{-j2\pi km/N} = \left( \sum_{k=0}^{N-1} D_k e^{j2\pi km/N} \right)^* = d_{-m}^* = d_{N-m}^*$

<sup>5</sup> We have supposed that the IF/RF transmitting filters exhibit an arithmetic symmetry around the carrier frequency.

