Synchronization in OFDM mobile satellite systems: LEO vs GEO

Maria-Gabriella Di Benedetto, Cristiano Terra

University of Rome 'La Sapienza' - Dipartimento INFOCOM
Via Eudossiana, 18 - 00184 Rome-Italy
tel: 39-6-41585863 - fax: 39-6-4873300 - email: gaby@acts.mapei.uniroma1.it

Abstract
OFDM is a potential candidate in the design of future generation mobile systems; it offers flexibility in bandwidth management, and potential good protection against the severe degradations induced by the channel on signals characterized by large bandwidths. OFDM is adopted in the MEDIAN project as the modulation-access technique of a wireless LAN, which at the network layer is ATM. The possibility of interfacing local with satellite networks seems attractive; in order to do so, OFDM performance over the satellite channel is investigated in the present paper. Results of analyses on synchronization issues show that while satellite-OFDM is in all cases a feasible hypothesis, LEO systems are favored with respect to GEO, due to the lower delays they introduce.

Introduction
The analysis of traditional digital modulation schemes imbedded in multi-carrier access based systems such as OFDM has resulted in continuous and growing interest in the scientific community. OFDM by itself is not in contrast with other modulation techniques; it can be used over QAM, PSK, QPSK, etc., since any modulation technique can be adopted to transmit signals at the different carriers. Rather, OFDM offers the opportunity of increasing bandwidth efficiency by letting signals overlap in frequency. OFDM can be viewed as a sophisticated Multi-Carrier (MC) technique which makes good management of the available bandwidth. This flexibility feature implies that the occupied band can be easily varied according to the data rate. This is an important factor in systems providing multimedia services. In addition, OFDM offers the great advantage of allowing a good encoding of the information. In the case of rapidly varying channels, such as the mobile channel, the above is a challenging property. The reason for this appealing characteristic is the possibility of selectively protect the information making thus fading effects less severe.

The results listed above have made OFDM a potential candidate in the design of future generation mobile systems. For example, OFDM has been selected in the project ACTS-MEDIAN to be the modulation-access technique of a wireless LAN. In the MEDIAN system, the network layer is ATM, and the data rates considered are in the order of 150 Mbit/s (net data rate). The MEDIAN system design shows that OFDM is an interesting solution for Wireless ATM. It offers the possibility of transmitting very high data rates with good information protection against the severe distortions on the very large signal bandwidth.

The above consideration is an additional reason for investigating OFDM in the satellite case. Harmonizing the techniques adopted in different environments leads undoubtedly to better interworking and better system design. However, when using OFDM over the satellite channel, a problem arises which is not present in the indoor environment: large delay, and consequently difficult synchronization. In the indoor environment, the delays are very small due to the small distances to be covered and thus, synchronization can be reasonably solved. In the satellite channel, which is typically characterized by important delays, different users must synchronize to different carriers and errors can be tolerated only to a limited extent. Needless to say, the amount of error which can be tolerated depends upon the design of the system in terms of guard bands; the larger the guard bands, the larger the synchronization errors can be. However, the larger the guard bands, the lower is the advantage of using OFDM over a more traditional FDM technique.

In the present paper, OFDM synchronization in a satellite system is examined. The synchronization technique will be described in section 2 after a summary of the OFDM technique (section 1). The simulation of an OFDM mobile satellite system was implemented by inserting the OFDM module in a more general system in which the access method was a PRMA-based technique. The simulation experiment will be described in section 3. Finally, results of simulation will be reported in section 4. We will show how OFDM behaves in satellite systems with different guard bands. Since the extension of the guard band will be found to be strictly related to the acceptable delay, we shall show results of simulation of LEO systems which have low delays and compare the requirements to the case of GEO systems which have significant higher delays. Finally, we shall attempt to give a conclusive discussion on the hypothesis of using OFDM over the LEO/GEO-satellite channel.

1. Description of the OFDM technique over the satellite mobile channel
As highlighted by Weinstein and Elbert (1971) the modulation in an OFDM system can be implemented by digitally-based FFT circuits. The scheme of an OFDM system which uses DSP techniques is shown in Fig. 1. OFDM allows the transmission of several signals over several carrier frequencies. A encoder generates a group of N symbols out of a group of K bits. These symbols are indicated by \( a_m,k \) with \( m = 0, \ldots, N - 1 \) where \( k \) indicates

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the block number. Each symbol belongs to an alphabet $A$ of $L$ elements. Thus, each symbol carries $\gamma = \log_2 L$ bits. The duration of each block is $T$. Each symbol modulates a sub-carrier using a modulator (for example, QAM), as shown in Fig. 2. The carrier frequencies are spaced by $\Delta f$ and distributed around $f_p$. Consequently, one can write:

$$f_m = m \cdot \Delta f - \frac{N}{2} \cdot \Delta f \quad \text{with } m = 0, \ldots, N-1.$$  

\hspace{1cm} (1)

![Block diagram of an OFDM system using DSP techniques.](image)

Figure 1 – Block diagram of an OFDM system using DSP techniques.

The binary rate at the coder output is $N \cdot \gamma / T$ where $T$ must be shorter than the $g_T(t)$ of the QAM modulator. The complex envelope for the output of the OFDM modulator, $s(t)$, is:

$$s(t) = g_T(t) \sum_{m=0}^{N-1} c_m \exp[j2\pi f_m \cdot t] \cdot g_m = \sum_{m=0}^{N-1} c_m g_m(t)$$  \hspace{1cm} (2)

which is periodic with period $T_m = 1/\Delta f$, except for $g_T(t)$. In order to evaluate the bandwidth of $s(t)$, suppose that $g_T(t)$ is rectangular with duration equal to $T$. The Fourier transform of $s(t)$ is:

$$S(f) = \sum_{n=0}^{N-1} c_n G_T(f - f_n)$$  \hspace{1cm} (3)

which has an infinite band. If we assume that $G_T(f)$ has an effective band limited to $\pm M \cdot \Delta f$ (M integer) out of which the amplitude of the spectrum can be neglected, then $S(f)$ can be limited to $(N + 2M) \cdot \Delta f$. If over $N$ symbols, the first and the last M are “neutral” (corresponding to the origin of the QAM constellation) then the bandwidth of $S(f)$ is: $B = N \cdot \Delta f$ although the useful band is: $B = (N - 2M) \Delta f$. Note that if $B$ is the bandwidth of $s(t)$, then the sampled version of $s(t)$ is:

$$s_k = s \left( t = \frac{k}{N \cdot \Delta f} \right) = \exp(-jk\pi) g_T \left( \frac{k}{N \cdot \Delta f} \right) \sum_{m=0}^{N-1} c_m \exp \left[ j2\pi \frac{m \cdot k}{N} \right] = (-1)^k \cdot g_T \left( \frac{k}{N \cdot \Delta f} \right) \cdot C_k$$

$$C_k = \sum_{m=0}^{N-1} c_m \exp \left[ j2\pi \frac{m \cdot k}{N} \right] \quad \text{for } k = 0, \ldots, N-1$$  \hspace{1cm} (4)

Eq. 4 is the discrete inverse Fourier transform of $c_m = \{c_0, \ldots, c_{N-1}\}$ of period $N$. Thus, the N modulators can be
replaced by an $\text{FFT}^{-1}$ circuit on the $N$ complex $C_k$ derived by the $\alpha=\infty$ association in the QAM. Once the $C_k$ are computed, one obtains the $s(t)$ sample weighted by the $g_T(t)$ sample.

As for the demodulator, it is necessary to have a set of functions $\psi_k(t)$ orthogonal to the $\varphi_k(t)$:

$$\int_0^T \psi_m(t) \cdot \psi_k^*(t) dt = \begin{cases} 0 & 0 \leq m = k \leq N - 1 \\ 1 & m \neq k \end{cases}$$ \hspace{1cm} (5)

If the $\psi_k(t)$ exist, and indicating by $r(t)$ the received signal, one has:

$$\int_0^T \xi(t) \cdot \psi_k^*(t) dt = \sum_{m=0}^{N-1} \sum_{T} \zeta_m \int_0^T \varphi_m(t) \cdot \psi_k^*(t) = \zeta_k$$ \hspace{1cm} (6)

The receiver is a bank of correlators. There must be block synchrony; $\psi_k(t)$ must be temporally aligned to $\varphi_k(t)$.

As regards the application of OFDM to satellite systems, the Satellite Mobile Channel (SMC) can be represented by the Rice channel model with parameter $k=P/B_d$ (ratio between the direct and the diffuse component). As well known, the SMC includes the case of Gaussian channel ($k=\infty$) and of Rayleigh channel with one path ($k=0$). In the case of SMC, it is appropriate to have $g_T(t)$ and $g_R(t)$ as the raised cosine.

In a satellite multi-users mobile communication system, the different signals in one OFDM block can be thought to be associated to a single mobile unit; these signals are characterized by different delays and Doppler frequencies, and are also affected by different temporal and frequency offsets which can cause interference with users located in the same coverage area. Consequently, there is a strong need for synchrony between mobile stations. This problem will be analyzed in the next section. If $U$ indicates the number of mobile stations served by the satellite, the $i^{th}$ transmitted signal ($i=0,...,U-1$) has the following base-band expression:

$$s_i(t) = \sum_{k=-\infty}^{\infty} x_i(t-dt_i-kT) \cdot \exp\left[j2\pi \cdot (df_i + f_{ci} - f_0) \cdot (t-dt_i)\right]$$ \hspace{1cm} (7)

$$x_i(t) = g_T(t) \cdot \exp\left(-j2\pi \cdot \frac{N}{2} \cdot \Delta f \cdot t\right) \cdot \sum_{n=0}^{N-1} c_n \cdot \exp\left(j2\pi \cdot n \cdot \Delta f \cdot t\right)$$ \hspace{1cm} (8)

Since a raised cosine is used (with roll-off $\alpha$), then $\Delta f = (1+\alpha) / T$, and since the OFDM signal bandwidth is $B=\text{N} \cdot \Delta f$ the carrier of the $i^{th}$ channel is $f_{ci}=f_0+i \cdot B$. The terms $dt_i$ and $df_i$ represent the temporal and frequency offsets of the $i^{th}$ user with respect to the mobile user $i=0$. At the $i^{th}$ channel output, the received signal is:

$$r_i(t) = \left[2 \cdot P_s \cdot s_i(t) + \frac{\text{P}_d}{\text{S}} \cdot s_i(t-dt_i) \cdot \exp\left(-j2\pi \cdot f_{ci} \cdot t\right) \cdot \xi_i(t) + n_i(t)\right]$$ \hspace{1cm} (9)

i.e. the sum of the direct component, of the diffuse component, and of gaussian white zero-mean noise, with power spectral density $N_0$. The diffuse component takes into account the delay $t_{di}$ and the Doppler shift $f_{di}$.

Focusing on user 0, after filtering through $g_R(t)$, the received signal is:

$$r(t) = \sum_{i \in U} \left[2 \cdot P_s \cdot G_{s_i}(t) + \frac{\text{P}_d}{\text{S}} \cdot G_{d_i}(t) \cdot \xi_i(t) + n_i(t)\right]$$ \hspace{1cm} (10)

$$G_{s_i}(t) = \sum_{k=-\infty}^{\infty} \left[ \exp\left(-j2\pi \cdot \frac{N}{2} \cdot \Delta f \cdot t\right) \cdot \sum_{n=0}^{N-1} c_n \cdot \exp\left(j2\pi \cdot n \cdot \Delta f \cdot t\right) \right] \cdot g_T(t-kT)$$ \hspace{1cm} (11)

$$G_{d_i}(t) = \sum_{k=-\infty}^{\infty} \left[ \exp\left(-j2\pi \cdot \frac{N}{2} \cdot \Delta f \cdot t\right) \cdot \sum_{n=0}^{N-1} c_n \cdot \exp\left(j2\pi \cdot n \cdot \Delta f \cdot t\right) \right] \cdot g_T(t-kT)$$ \hspace{1cm} (11)

$$g_{s_i}(t) = \left[ g_R(t-dt_i) \cdot \exp\left[j2\pi \cdot (f_{ci} - f_0 + df_i) \cdot (t-dt_i)\right] \right] g_R(t)$$ \hspace{1cm} (11)

$$g_{d_i}(t) = \left[ g_R(t-dt_i-t_{di}) \cdot \exp\left[j2\pi \cdot (f_{ci} - f_0 + df_i - f_{di}) \cdot (t-dt_i-t_{di})\right] \right] g_R(t)$$ \hspace{1cm} (11)

As for $n_i(t)$, it is a gaussian zero-mean noise, with autocorrelation

$$R_{n_i}(\tau) = N_0 \cdot \left|G_R(f)\right|^2 \cdot \exp\left(j2\pi \cdot f \cdot \tau\right) df$$ \hspace{1cm} (12)

Note that the shape of the received signal for user 0 includes the contributions of the other units.

2. Multi-user synchronization in OFDM

Synchronization issues vary in the up- or down-link. Having a good synchronization in the down-link is easier since the different signals all originate at the same source. In the up-link, the signals originating in the different units have different delays due to multipath. In addition, the spectra are widened and shifted by the Doppler.

Consider the up-link. Suppose that the OFDM channels are in a band $W$ between the frequencies $f_0$ and $f_0 + W$. The algorithm described was first presented by Wei and Schlegel (1995); The bandwidth $W$ is divided into $Q$ sections which are interspaced by a "guard interval" $B_G$. Since the OFDM signal bandwidth is $B=Q \cdot \Delta f$, one has (in each section) $N_s = (W/Q) - 2R_G / B$ channels, one for each mobile station. The separation of the spectrum into
sections can be obtained by using pilot tones \((f_{q_0}, f_{q_1}, \ldots, f_{q_Q})\) used for synchronization. Each section is identified by two adjacent tones. The satellite broadcasts \((f_{q_0}, f_{q_1}, \ldots, f_{q_Q})\) and the mobile aligns its transmission to the carrier of its OFDM channel.

In case of a non-distorting channel and absence of Doppler shifts, if the \(n^{th}\) channel of the \(n^{th}\) section is assigned to the \(j^{th}\) mobile unit, then the carrier frequency of the \(j^{th}\) unit is given by:

\[
f_{c_j} = \frac{\left( f_{q_{n+1}} - f_{q_n} \right)}{W} \cdot i + f_{q_n} + B_G
\]

If the \(N_n\) users of the \(n^{th}\) section are capable of accurately estimating the synchronization tones which delimit the \(n^{th}\) section, then they are synchronized to their carrier frequencies.

In the case of a real channel, multipath fading and Doppler effects affect the estimation of the synchronization tones. Note that due to the relative speed \(v_j\) of the \(j^{th}\) station a frequency \(f\) sent by the satellite, is received as:

\[
f'_j = f \cdot (1 \pm \frac{v_j}{c})
\]

in which \(f(v_j, /c)\) represents the Doppler shift \(f_d\).

We examine now the method used by the mobile to estimate the pilot tones and correct the Doppler. The \(j^{th}\) mobile receives the synchronization tones relative to section \(n, f_{q_n, n+1}\) (while the satellite transmits \(f_{q_n, n+1}\)):

\[
f'_{q_{n+1}} - f'_{q_n} = \left( f_{q_{n+1}} - f_{q_n} \right) \cdot \left( 1 \pm \frac{v_j}{c} \right)
\]

from which one has:

\[
\left( 1 \pm \frac{v_j}{c} \right) = \frac{Q}{W} \left( f'_{q_{n+1}} - f'_{q_n} \right)
\]

Thus, the mobile can compute the Doppler \((1 \pm v_j/c)\) and derive the correct synchronization tones which are:

\[
f'_k = f_{q_k} \left( 1 \pm \frac{v_j}{c} \right) = f_{q_k} \left( 1 \pm \frac{v_j}{c} \right)
\]

If in the up-link, the carrier associated to the \(j^{th}\) mobile, \(f_{c_j}\), is associated to the \(m^{th}\) channel of the \(n^{th}\) section, then the mobile unit generates the carrier \(f'_{c_j}\) by using the estimated synchronization tones \(f'_{q_k}\) and one has:

\[
f'_c = \left( f_{q_{n+1}} - f_{q_n} \right) + f'_{q_n} + B_G
\]

On the satellite side, the satellite receives the following carrier:

\[
f_{c_j} = f_{c_j} \left( 1 \pm \frac{v_j}{c} \right) = \frac{\left( f_{q_{n+1}} - f_{q_n} \right) + f_{q_n} + B_G}{W} = \frac{\left( f_{q_{n+1}} - f_{q_n} \right) + f_{q_n} + B_G}{Q \cdot B}
\]

This last expression is equivalent to the one examined in the case of an ideal channel, if \(i=m\). Thus, since the terrestrial mobile unit derives its transmission carrier frequency from the synchronization tones received in the down-link, it is possible to correct the offset due to the Doppler if \(f_{q_0}, f_{q_1}, \ldots, f_{q_Q}\) are correctly recovered.

3. **Description of the simulated system**

The OFDM synchronization has been simulated by writing a C software running on a Sun Workstation Spark 5. In the simulator, the satellite acts as the supervisor. It uses a bandwidth. The coverage area can be thought as composed by \(Q\) coverage sub-areas to which the entire resource is allocated (\(N\) channels). These channels are allocated on a bandwidth which is: \(B_N = W/Q\)

Since the satellite acts as the supervisor of the connection, it not only allows synchronization of the users but also checks the state of the different channels which can be either occupied by a transmitting user, or non-occupied. In the first case, the channel cannot be used until the user stops sending messages and disconnects. In the second case, the satellite must decide whether to assign the channel to the users making a connection request.

The resource management acted by the satellite is of a connection-oriented type based on FDMA. In order to limit collisions, a permission probability parameter is introduced. This parameter represents the probability of obtaining a channel when requested. If more than one user asks for connection simultaneously, some of the users will be allowed to access the channels, and others will not. Each user generates a casual number between 0 and 1. This
number is compared to a permission probability threshold (typically 0.3). Only those users who emitted a number lower than 0.3 will be allowed to make a request for the resource. If more than one user has succeeded in request, and only one channel is non-occupied, then the satellite might decide not to assign the channel to any of them. This is a typical case of collision. In the simulator, the state of the channels is checked by the satellite every 6 msecs, which implies that a given channel (if there is a request) will not stay non-occupied for a very long time.

Figure 3 – System model

The mobile users are supposed to be represented by statistical models of their activity. It is hypothesized that a user might be in a silence mode or conversation mode for a given average period of time and that the transition probabilities between two states are negative-exponentially distributed. If T_s and T_t indicate the durations of the silence and conversation modes, the transition probabilities are given by: 

\[ p_{ss} = 1.0 - e^{-T_s/T_s} \]

\[ p_{st} = 1.0 - e^{-T_t/T_t} \]

where T_s=6 msecs, T_t=11.3 minutes, T_t=2 minutes. The ratio T_t/(T_s+T_t) indicates the average activity per user. With the selected times the activity is about 15%.

As shown in Fig. 3, the system model has 4 states. In order to communicate, the user must send a channel request. If the channel is obtained, the user goes in the reservation mode, and then tries to synchronize. An important parameter is T_{sync} which is the maximum time gap which the user can use in order to synchronize.

Figure 4 – Simulation results in the case of LEO

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4. Results of simulation trials
4.1 Simulation: the case of LEO

Results are presented in terms of average synchronization probability. This parameter is strongly dependent upon the guard bands extension. In addition, it also depends upon the synchronization time allowed (the parameter $T_{syn}$). In the simulations, we have assumed that the ratio between number of users and number of channels be 5 to 1. The total number of users has been supposed to be 10000 while the number of channels is 2000. The average silence time is 2 minutes; the average conversation time is 12 minutes. The available bandwidth is 20 MHz in each direction. The waiting time before channel is assigned is 60 msecs. The LEO system considered assumes a distance between the satellite and the mobile of 780 km. The maximum mobile speed is 100 m/s. The satellite speed is 7462 m/s. The simulation considers the worst case, i.e. the satellite and the mobile travel in opposite directions. Results are presented in Fig. 4 in terms of average probability of correct synchronization as a function of used bandwidth and of maximum synchronization time $T_{sync}$. The results show that when $T_{sync}$ is 600 msecs, the probability of correct synchronization is over 98% when the used bandwidth is below 89.2%. This data can be considered as a threshold, and a good dimensioning of the system would select an extension of the guard band which leads to <89.2% of bandwidth utilization and a maximum synchronization time equal to 600 msecs.

4.2 Simulation: the case of SECOMS

The same hypotheses as in the LEO case are made in terms of number of users and channels. However, the distance between the satellite and the mobile is much bigger. The round-trip time is now 0.6 sec compared to the 12 msecs of the LEO case. Thus, fewer synchronization attempts are possible in the same $T_{sync}$. Results are reported in Fig. 5 which shows that in order to have similar performance to the LEO case, the GEO system must have $T_{sync}$=24 sec which is an unaffordable time. The conclusion is that the used bandwidth must be decreased, to keep $T_{sync}$ to reasonable values. Figure 5 shows that for $T_{sync}$=720 msecs the performance is adequate, with more than 95% of correct synchronization when the used bandwidth is below 88.48%. OFDM is still a valuable choice but does not present the great advantage as in LEO systems.

![Figure 5 - Simulation results in the case of SECOMS](image)

5. Conclusions

In this paper, the design of a satellite system using OFDM was examined. Results of simulation trials implementing a multi-user synchronization procedure show that in the case of LEO systems, the OFDM solution offers great advantage over traditional FDM, without introducing severe limitations due to synchronization errors. In the case of GEO systems, such as SECOMS, OFDM is still a valid alternative to FDM; however, propagation delays impose a limit which reduces the utilization of the available bandwidth.

References
