A feedback code for the Multiple Access Channel (MAC): a case study

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abstract

The capacity region of the Multiple Access Channel (MAC) can be enlarged by using feedback. In this paper, an extension of the deterministic feedback code proposed by Ozarow for the two-user Gaussian Multiple Access Channel (MAC) [7] is presented; the Ozarow's code is extended to the case of U-users ($U \geq 2$) in the specific case in which all users use equal power. Results on system performance in terms of Symbol Error Rate obtained from computer simulation of the proposed algorithm are reported. Moreover, computer simulation of the new code also allows verification and confirmation of theoretical bounds.

1. Introduction

In the single-user Gaussian channel with non-white noise, the use of feedback increases capacity. A factor of two bound on the increase of capacity due to the presence of feedback has been proved [1,2]. Unlike the simple discrete memoryless channel, the use of feedback in the Multiple Access Channel (MAC) increases capacity even in the case of a memoryless channel [3,4,5,6]. In [7], Ozarow found the capacity region for the two users Gaussian MAC with feedback and demonstrated a feedback coding scheme which allows reliable communication at all points in the capacity region.

In the present paper, an extension of the Ozarow's code to the case of several users characterized by equal power is proposed. The paper is organized as follows. Section 2 summarizes the Ozarow's code by briefly reporting the algorithm, the theoretical bounds, and the results of computer simulations of the algorithm in terms of system performance (typically Symbol Error Rate, SER). In section 3, the proposed extension of the above code to several users is described. Results of computer simulation of the proposed algorithm are reported in terms of system performance and intrinsic limitation of the method. Finally, in section 4 our conclusions are presented.

2. Description of Ozarow's code.

Ozarow's code [7] refers to a situation in which two users transmit messages to a central node through an ideal channel. The channel output is corrupted by an additive Gaussian noise z, which, sampled at time k, forms the sequence Z_k of identically distributed zero-mean Gaussian noise variables, with variance σ^2. Each transmitter has a limited power P_i (index i indicates the i-th transmitter) for transmitting a block of n transmission words. Ozarow's code is designed for channels with feedback and leads to an achievable capacity region, C^{FB}, given by:

$$C^{FB} = \bigcup_{0 \leq \rho \leq 1} \left\{ \begin{array}{l} \left(R_1^{FB}, R_2^{FB}\right) 0 \leq R_1^{FB} \leq 1/2 \cdot \log_2\left(1 + \frac{P_1}{\sigma^2}\left(1-\rho^2\right)\right), \\ 0 \leq R_2^{FB} \leq 1/2 \cdot \log_2\left(1 + P_2\left(1-\rho^2\right)/\sigma^2\right) \\ 0 \leq R_1^{FB} + R_2^{FB} \leq \frac{1}{2}\log_2\left(1 + \frac{P_1 + P_2 + 2\sqrt{P_1 P_2}\,\rho}{\sigma^2}\right) \end{array} \right\}$$

where ρ is the correlation coefficient of the two transmitted variables. The above region includes the capacity region of the Gaussian MAC without feedback, which is:

$$C = \left\{ \begin{array}{l} \left(R_1, R_2\right) 0 \leq R_1 \leq \frac{1}{2}\log_2\left(1 + \frac{P_1}{\sigma^2}\right), 0 \leq R_2 \leq \frac{1}{2}\log_2\left(1 + \frac{P_2}{\sigma^2}\right), \\ 0 \leq R_1 + R_2 \leq \frac{1}{2}\log_2\left(1 + \frac{P_1 + P_2}{\sigma^2}\right) \end{array} \right\}$$

Before transmission, the M_i transmission words m_i are mapped into values $\vartheta_i = m_i/(M_i - 1) - 1/2$ and ϑ_i is sent. Thus, ϑ_i is uniformly distributed over M_i equally spaced values in $[-1/2, 1/2]$. For high values of M_i, ϑ_i has variance $1/12$. At step k, the central node computes an estimate $\hat{\vartheta}_i^k$ of ϑ_i with an error $\varepsilon_{i,k} = \hat{\vartheta}_i^k - \vartheta_i$ of variance $\alpha_{i,k}$.

summary of Ozarow's algorithm

*step 1: at instant k=1, transmitter 1, T_1, sends $X_{1,1} = \sqrt{12P_1}\,\vartheta_1$ and transmitter 2, T_2, is silent. The central node receives: $Y_1 = X_{1,1} + Z_1$ and computes $\hat{\vartheta}_1^1 = \frac{Y_1}{\sqrt{12P_1}} = \vartheta_1 + \varepsilon_{1,1}$ where $\varepsilon_{1,1}$ has variance $\alpha_{1,1} = \frac{\sigma^2}{12P_1}$.

*step 2: as in step 1 but referred to T_2.

*from k=3 to k=n: thanks to feedback, both transmitters compute $\hat{\vartheta}_i^k$ and $\varepsilon_{i,k}$. Assume $\varepsilon_{1,k}$ and $\varepsilon_{2,k}$ to be jointly zero-mean Gaussian with correlation coefficient $\rho_k = \frac{E\left(\varepsilon_{1,k}\varepsilon_{2,k}\right)}{\sqrt{\alpha_{1,k}\alpha_{2,k}}}$, the parameters $\alpha_{1,k}, \alpha_{2,k}$, and ρ_k can be computed by the central node and the two transmitters.

*k+1: T_1 sends $X_{1,k+1} = \sqrt{P_1/\alpha_{1,k}}\,\varepsilon_{1,k}$. T_2 sends $X_{2,k+1} = \sqrt{P_2/\alpha_{2,k}}\,\varepsilon_{2,k}\,sgn(\rho_k)$. The channel putput is $Y_{k+1} = X_{1,k+1} + X_{2,k+1} + Z_{k+1}$. The receiver computes the maximum likelihood estimates

$$\hat{\vartheta}_i^{k+1} = \hat{\vartheta}_i^k - \frac{E\left(Y_{k+1}\varepsilon_{i,k}\right)}{E\left(Y_{k+1}^2\right)}Y_{k+1}, \quad \hat{\varepsilon}_i^{k+1} = \hat{\varepsilon}_i^k - \frac{E\left(Y_{k+1}\varepsilon_{i,k}\right)}{E\left(Y_{k+1}^2\right)}Y_{k+1},$$

and $\alpha_{i,k+1} = \alpha_{i,k} - \frac{\left(E\left(Y_{k+1}\varepsilon_{i,k}\right)\right)^2}{E\left(Y_{k+1}^2\right)}$ where:

$$E\left(Y_{k+1}^2\right) = P_1 + P_2 + 2\sqrt{P_1 P_2}\left|\rho_k\right| + \sigma^2,$$

$$E\left(Y_{k+1}\varepsilon_{1,k}\right) = \sqrt{\alpha_{1,k}}\left(\sqrt{P_1} + \sqrt{P_2}\left|\rho_k\right|\right),$$

$$E\left(Y_{k+1}\varepsilon_{1,k}\right) = \sqrt{\alpha_{1,k}}\left(\sqrt{P_1} + \sqrt{P_2}\left|\rho_k\right|\right)sgn\left(\rho_k\right), \quad \text{and}$$

$$E\left(\varepsilon_{1,k+1}\varepsilon_{2,k+1}\right) = E\left(\varepsilon_{1,k}\varepsilon_{2,k}\right) - \frac{E\left(Y_{k+1}\varepsilon_{1,k}\right)E\left(Y_{k+1}\varepsilon_{2,k}\right)}{E\left(Y_{k+1}^2\right)}.$$

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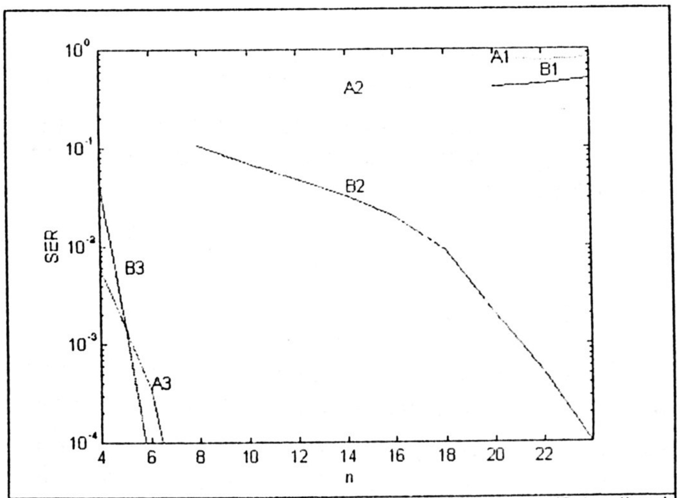

Fig. 1. Simulation results with different (n,R) codes, and SNR_1=10 dB (user1) and SNR_2=20 dB (user2). A1 and B1: user1 R_1=1.6 (A1) and user2 R_2=2.6 (B1), A2 and B2: user1 R_1=1 (A2) and user2 R_2=2.5 (B2), A3 and B3: user1 R_1=0.8 (A3) and user2 R_2=2 (B3).

*the updating expression for the correlation coefficient is

$$\rho_{k+1}=\frac{\rho_k\sigma^2-sgn(\rho_k)\sqrt{P_1P_2\left(1-\rho_k^2\right)}}{\sqrt{\left(P_1\left(1-\rho_k^2\right)+\sigma^2\right)\left(P_2\left(1-\rho_k^2\right)+\sigma^2\right)}}$$

theoretical bounds

Given P_1, P_2 and σ^2, $\{\rho_k\}$ converges to $\rho_k=(-1)^k\rho^*$, with ρ^* solution in (0,1) of

$$\sigma^2\left(\sigma^2+P_1+P_2+2\sqrt{P_1P_2}\,\rho\right)=\left(\sigma^2+P_1\left(1-\rho^2\right)\right)\left(\sigma^2+P_2\left(1-\rho^2\right)\right)$$

An error in the estimate $\hat{\vartheta}_i^n=\vartheta_i+\varepsilon_{i,n}$ occurs if the amplitude of $\varepsilon_{i,n}$ is greater than half the distance between adjacent values of ϑ_i. Now, the probability of error is $P_{e,i}\le Pr\left[\left|\varepsilon_{i,n}\right|>\frac{1}{2\left(M_i-1\right)}\right]$ and given that $M_i=2^{nR_i}$, then

$$P_{e,i}\le 2Q\left[\frac{\sigma^2}{2\sqrt{\alpha_{i,2}\left(\sigma^2+P_i\left(1-\rho^{*2}\right)\right)}}\,2^{\,n\left[\frac{1}{2}\log_2\left(\frac{\sigma^2+P_i\left(1-\rho^{*2}\right)}{\sigma^2}\right)-R_i\right]}\right]$$

Thus, if: $R_i\le\frac{1}{2}\log_2\left(1+\frac{P_i}{2}\left(1-\rho^{*2}\right)\right)$

the probability of error can be made as small as desired by increasing n. Consequently, the capacity is:

$$C=\left\{\begin{array}{l}\left(R_1,R_2\right)0\le R_1\le\frac{1}{2}\log_2\left(1+\frac{P_1}{2}\frac{\left(1-\rho^{*2}\right)}{\sigma^2}\right)0\le R_2\le\frac{1}{2}\log_2\left(1+\frac{P_2}{2}\frac{\left(1-\rho^{*2}\right)}{\sigma^2}\right)\\ 0\le R_1+R_2\le\frac{1}{2}\log_2\left(1+\frac{P_1+P_2+2\sqrt{P_1P_2}\,\rho^*}{\sigma^2}\right)\end{array}\right\}\qquad(1)$$

simulation of Ozarow's code

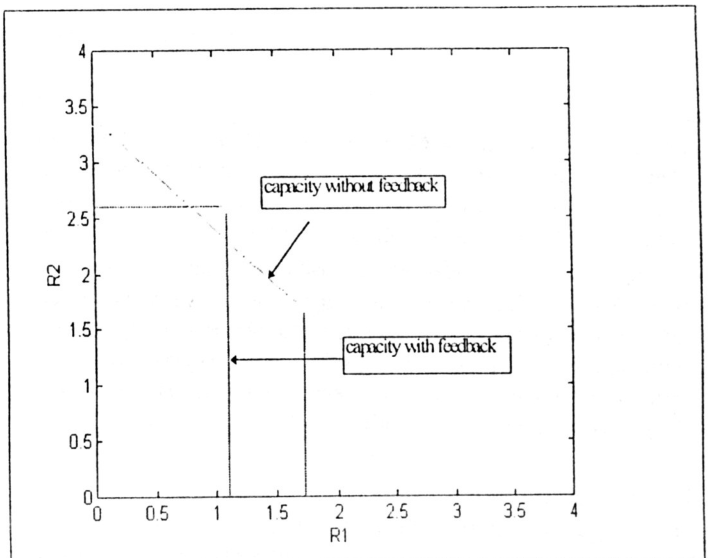

Figure 2. Capacity regions with and without feedback

The equivalent base-band system was simulated by generating for each transmitter a symbol w_i (i=1,2). The symbol error rate (SER) was computed over transmitted 500000 symbols. Figure 1 shows the results of the estimation of the error probability for different code lengths and different rates, with SNR_1=10 dB (user1) and SNR_2=20 dB (user2). Observe that curves A1 and B1 (corresponding to R_1 and R_2 greater than 1.1 and 2.6, respectively) do not decrease when n increases as these values are beyond the limits of (1) ($R_1\le 1.1008$ and $R_2\le 2.6047$). The capacity region is

$$C^{FB}=\left\{\begin{array}{l}\left(R_1^{FB},R_2^{FB}\right)0\le R_1^{FB}\le 1.1008,0\le R_2^{FB}\le 2.6047\\ 0\le R_1^{FB}+R_2^{FB}\le 3.6681\end{array}\right\}$$

. Under the same SNR, the capacity region for the Gaussian MAC without feedback is:

$$C=\left\{\begin{array}{l}\left(R_1,R_2\right)0\le R_1\le 1.7297,0\le R_2\le 3.3291\\ 0\le R_1+R_2\le 3.3972\end{array}\right\}$$

Fig.2 shows the capacity regions with and without feedback.

The TDM transmission of 500000 symbols with the two users having R_1=1 and R_2=2 was simulated for different code lengths n and SNR. The results are reported in Fig. 3 which confirms that when n increases (for equal SNR) SER improves. Curves U1 and U2 correspond to a channel without feedback and an uncoded transmission. At SER=10^{-3} and for user1 (curves A), with n=8, compared to n=6, the code gains about 0.2 dB. With n=6, compared to n=4, the code gains about 1 dB. At SER=10^{-3} and user2 (curves B) with n=8, vs n=6 (n=6, vs. n=4), the code gains about 1 dB (2 dB). The improvement with n is greater than for user1 because user2 has a higher power and thus higher SNR (10 times greater).The gain for the first user (curves U1 and A), ranges from about 4 dB (at SER=10^{-1}) to a very high value (at SER=10^{-4}). The coding gain for the second user (curves U2 and B) is very high for all SER values.

the case of two users with equal power.

With the two users having equal powers (set to one in the simulation), one has $R_1=R_2=R$.

Figure 4 shows the SER for different code lengths and different rates, with SNR=10 dB. We observe that rates greater than 1.3 (curve A) are beyond the limit of (1) which gives $R\le 1.2848$.

In this case, the capacity region is

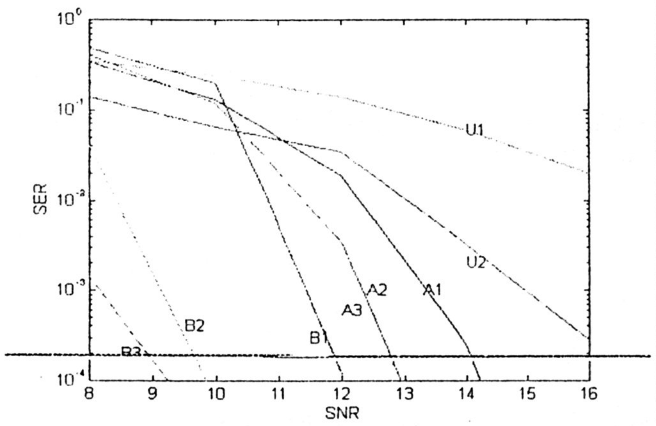

Fig. 3. Simulation results with different SNR, different code lengths n, and $R_1=1$ (first user-curves A), $R_2=2$ (second user-curves B). U1 and U2: uncoded TDM transmission without feedback, A1 and B1: n=4, A2 and B2: n=6, A3 and B3: n=8

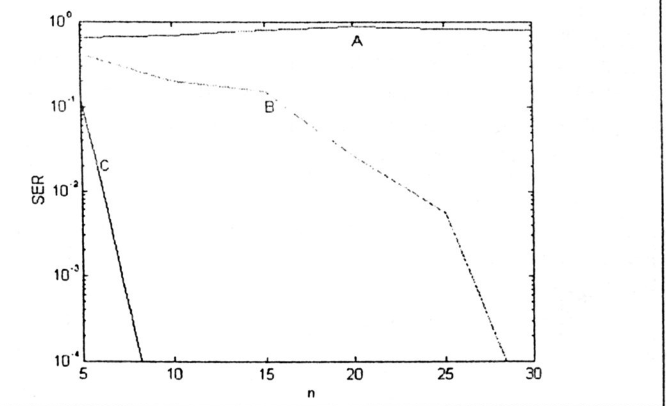

Fig. 4. Simulation results with different (n,R) codes and SNR=10 dB. A: R=1.4, B: R=1.2, C: R=1

$$C^{FB}=\left\{\begin{matrix}\left(R_1^{FB},R_2^{FB}\right):0\le R_1^{FB}\le 1.2875, 0\le R_2^{FB}\le 1.2875\\ 0\le R_1^{FB}+R_2^{FB}\le 2.5688\end{matrix}\right\}$$

Without feedback, the capacity region is

$$C=\left\{\begin{matrix}\left(R_1,R_2\right):0\le R_1\le 1.7297, 0\le R_2\le 1.7297\\ 0\le R_1+R_2\le 2.1962\end{matrix}\right\}.$$

The capacity region with and without feedback is shown in Fig. 5. The TDM of 500000 symbols at R=1 was simulated for different code lengths and SNR (Fig. 6). Curve A shows the results for the channel without feedback and uncoded transmission. Consider SER=10^{-3}. With n=8, compared to n=6, the code gains about 1 dB. With n=12, compared to n=10, the code gains about 0.2 dB. With n=10, compared to n=8, the code gains about 0.2 dB. The coding gain is (as in the case of users with different powers) very high for all SER values.

3. Extension of the Ozarow's code to the case of several users with equal power.

We propose to extend Owarow's code to U users ($U\ge 2$) with equal power P. Under this hypothesis, the correlation coefficient between signals X_{ik} sent by T_i at instant k and X_{jk} sent by T_j at the same time ($i\ne j$) is, for all i and j, ρ_k. At the beginning of a block of length n, T_i selects one m_i among M_i symbols; m_i is coded as described in

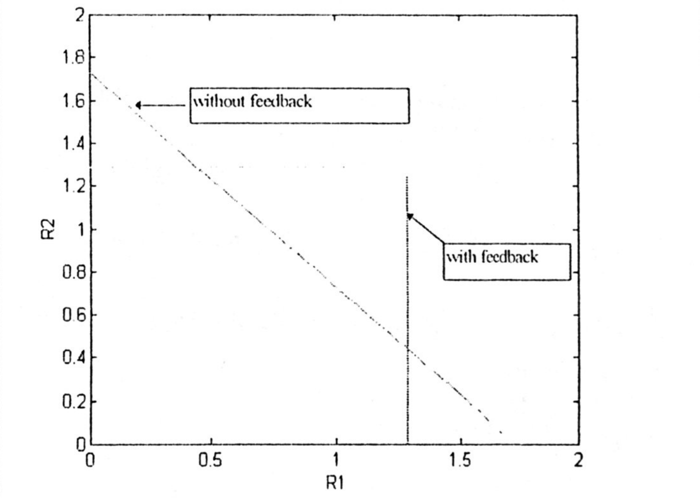

Figure 5. Capacity region achievable with and without feedback

section 2. After the k-th step, the central node computes an estimate $\hat{\vartheta}_i^k$ with error $\varepsilon_{i,k}=\hat{\vartheta}_i^k-\vartheta_i$ and variance $\alpha_{i,k}=var\left(\varepsilon_{i,k}\right)$.

description of the proposed extension of Ozarow's algorithm

*step l=1 to l=U, T_l sends $X_{l,l}=\sqrt{12P}\,\vartheta_l$ while T_j, $j\ne l$ are silent.

The central node receives $Y_l=X_{l,l}+Z_l$ and computes $\hat{\vartheta}_l^l=\vartheta_l+\varepsilon_{l,l}$, where $\varepsilon_{l,l}$ has zero-mean and variance $\alpha_{l,l}=\frac{\sigma^2}{12P}$.

*from step k=U+1 to step k=n: thanks to feedback, each transmitter computes $\hat{\vartheta}_i^k$ and $\varepsilon_{i,k}$. Assume $\varepsilon_{i,k}$ and $\varepsilon_{j,k}$, $i\ne j$, be jointly Gaussian, then $\alpha_{i,k}$, $\alpha_{j,k}$, and ρ_k, can be computed by the central node and the two transmitters.

* $k=l+rj$ $l\le j\le U$ $r=1,2..$: T_i sends $X_{j,k+1}=\sqrt{P/\alpha_{j,k}}\,\varepsilon_{j,k}\,sgn\left(\rho_k\right)$ and $T_{i\ i\ne j}$ sends $X_{i,k+1}=\sqrt{P/\alpha_{j,k}}\,\varepsilon_{i,k}$. The channel output is

$Y_{k+1}=\sum_{i=1}^{U}X_{i,k+1}+Z_{k+1}$. The receiver estimates

$$\hat{\vartheta}_i^{k+1}=\hat{\vartheta}_i^k-\frac{E\left(Y_{k+1}\varepsilon_{i,k}\right)}{E\left(Y_{k+1}^2\right)}Y_{k+1},\ \hat{\varepsilon}_i^{k+1}=\hat{\varepsilon}_i^k-\frac{E\left(Y_{k+1}\varepsilon_{i,k}\right)}{E\left(Y_{k+1}^2\right)}Y_{k+1}\ \text{(i=1,2,...,U}$$

The number of cross-terms in the evaluation of $E\left(Y_{k+1}\right)$ is equal to

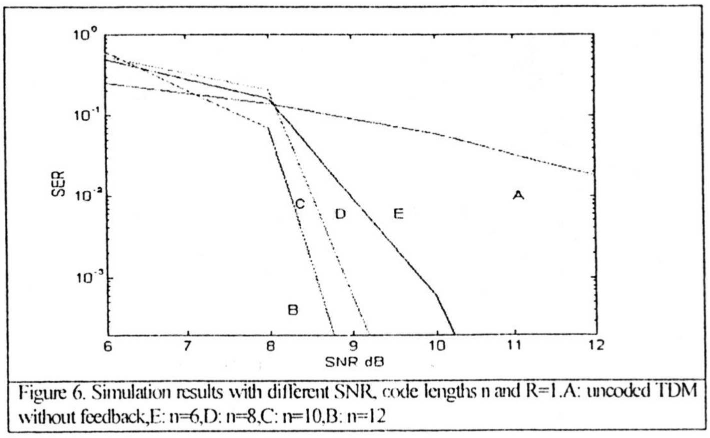

Figure 6. Simulation results with different SNR, code lengths n and R=1. A: uncoded TDM without feedback, E: n=6, D: n=8, C: n=10, B: n=12

the number of elements of the upper (or lower) triangle of the correlation matrix of $X(k)=(X_{1,k}, X_{2,k} \dots X_{m,k})$,

$$(U-1)+(U-2)+\dots+2+1=\frac{U(U-1)}{2};\ E\left(Y_{k+1}^{2}\right)=UP+U(U-1)P\left|\rho_k\right|+\sigma^2=UP\left[1+(U-1)\left|\rho_k\right|\right]+\sigma^2,$$

and $E\left(Y_{k+1}\varepsilon_{i,k}\right)=\sqrt{\alpha_k}\left(\sqrt{P}+(U-1)\sqrt{P}\left|\rho_k\right|\right)$. For the i-th transmitter,

$E\left(Y_{k+1}\varepsilon_{i,k}\right)=\sqrt{\alpha_k}\left(\sqrt{P}+(U-1)\sqrt{P}\left|\rho_k\right|\right)sgn(\rho_k)$. Let $E\left(Y_{k+1}\varepsilon_k\right)=\sqrt{\alpha_k}\left(\sqrt{P}+(U-1)\sqrt{P}\left|\rho_k\right|\right)$.

The updating for α_k is

$$\alpha_k=\alpha_k-\frac{\left(E\left(Y_{k+1}\varepsilon_k\right)\right)^2}{E\left(Y_{k+1}^2\right)}.\ \text{Thus,}$$

$$\alpha_{k+1}=\alpha_k\left[\frac{(U-1)P+\left|\rho_k\right|\left((U-2)(U-1)\right)P-(U-1)^2\rho_k^2P+\sigma^2}{UP+U(U-1)\left|\rho_k\right|P+\sigma^2}\right].$$

*at the end of the n-transmission

$$\alpha_n=\alpha_U\left[\frac{(U-1)P+\left|\rho_k\right|\left((U-2)(U-1)\right)P-(U-1)^2\rho_k^2P+\sigma^2}{UP+U(U-1)\left|\rho_k\right|P+\sigma^2}\right]^{n-U}$$

Moreover, $E\left(\varepsilon_{i,k+1}\varepsilon_{j,k+1}\right)=E\left(\varepsilon_{i,k}\varepsilon_{j,k}\right)-\frac{\left(E\left(Y_{k+1}\varepsilon_k\right)\right)^2 sgn(\rho_k)}{E\left(Y_{k+1}^2\right)}$.

The correlation coefficient ρ_k is $\rho_k=\frac{E\left(\varepsilon_{i,k}\varepsilon_{j,k}\right)}{\alpha_k}$. The updating formula for ρ_k is

$$\rho_{k+1}=\frac{\rho_k\left(UP+P(U-1)U\left|\rho_k\right|+\sigma^2\right)-\left(P+P(U-1)^2\rho_k^2+2P(U-1)\left|\rho_k\right|\right)sgn(\rho_k)}{(U-1)P+P\left|\rho_k\right|(U-2)(U-1)-P(U-1)^2\rho_k^2+\sigma^2}$$

For every P and σ^2, the series $\{\rho_k\}$ converges to $\rho_k=(-1)^k\rho^*$.

theoretical bounds

At the end of the n-code the receiver estimate is $\hat{\vartheta}_i^n=\vartheta_i+\varepsilon_{i,n}$

An error occurs if $\left|\varepsilon_{i,n}\right|>\frac{1}{2(M_i-1)}$. The error probability is

$P_{e,i}\le Pr\left(\left|\varepsilon_{i,n}\right|>\frac{1}{2(M_i-1)}\right)$. Given $M_i=2^{nR_i}$ then

$$P_{e,i}\le 2Q\left[\frac{2^{\left[n\left[\frac{1}{2}\log_2\left(\frac{UP+PU(U-1)\left|\rho^{*2}\right|+\sigma^2}{(U-1)P+\left|\rho_k\right|\left((U-2)(U-1)\right)P-(U-1)^2\rho_k^2P+\sigma^2}\right)-R_i\right]\right]}}{2\sqrt{\alpha_U}\left[\frac{(U-1)P+P\left|\rho^{*2}\right|(U-1)(U-2)-P(U-1)^2\rho^{*2}+\sigma^2}{UP+U(U-1)\left|\rho^{*2}\right|+\sigma^2}\right]^{\frac{U}{2}}}\right]$$

$$\text{If: } R_i\le\frac{1}{2}\log_2\left(\frac{UP+PU(U-1)\left|\rho^*\right|+\sigma^2}{(U-1)P+\left|\rho_k\right|\left((U-2)(U-1)\right)P-(U-1)^2\rho_k^2P+\sigma^2}\right)$$

$P_{e,i}$ can be made as small as desired as n increases.

We have shown that the argument of rigth-hand side of (P_e) may be made arbitrarily large by increasing n and reliable communication is possible at all rates under bound (2). In presence of feedback, the capacity region is:

$$C^{FB}=\begin{Bmatrix} R_i\le\frac{1}{2}\log_2\left(\frac{UP+PU(U-1)\left|\rho^*\right|+\sigma^2}{(U-1)P+\left|\rho_k\right|\left((U-2)(U-1)\right)P-(U-1)^2\rho_k^2P+\sigma^2}\right) & i=1,2,\dots,U \\ \sum_{i=1}^{U}R_i\le\frac{1}{2}\log_2\left(1+\frac{UP\left(1+(U-1)\left|\rho^*\right|\right)}{\sigma^2}\right) & \end{Bmatrix}$$

Simulation of the proposed code with two users

The TDM of 500000 symbols at R=1 was simulated for different code length n and SNR (Fig. 7). Performance is reported in Fig.6 showing that the algorithm N users-equal power includes Ozarow's code with two users-equal power.

Simulation of the proposed code with three users

The algorithm was implemented with three users with power one (thus $R_1=R_2=R_3=R$). Figure 8 shows SER for different code lengths and rates with SNR=10 dB. Rates greater than 0.7 are beyond bound (2) ($R\le 0.7104$). The capacity region is:

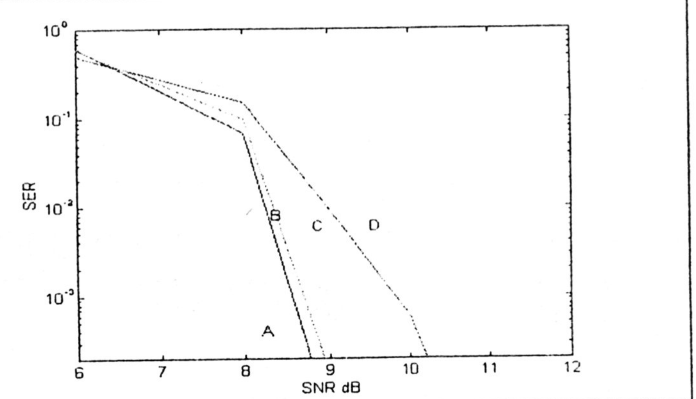

Fig. 7. Simulation results with different SNR, different code lengths n, and R=1.A: n=12,B: n=10,C: n=8,D: n=6

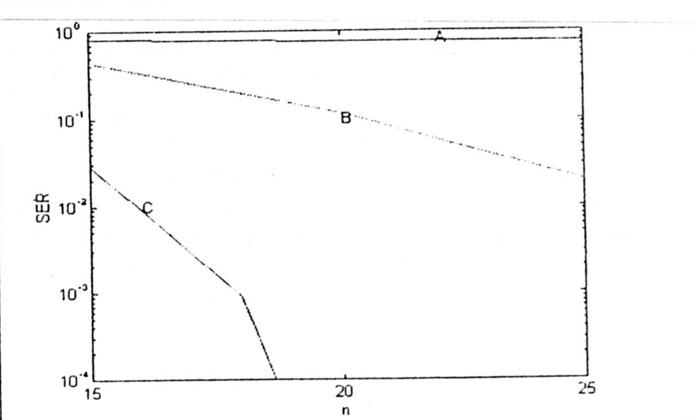

Fig. 8. Simulation results with different (n,R) codes and SNR=10 dB.A: R=0.8,B: R=0.7,C: R=0.6

$$C^{FB}=\begin{Bmatrix}\left(R_1^{FB},R_2^{FB},R_3^{FB}\right): 0\le R_1^{FB}\le 0.7104, 0\le R_2^{FB}\le 0.7104, 0\le R_3^{FB}\le 0.7104 \\ 0\le R_1^{FB}+R_2^{FB}+R_2^{FB}\le 2.9336\end{Bmatrix}$$

Without feedback, the capacity region is:

$$C=\begin{Bmatrix}\left(R_1,R_2,R_3\right): 0\le R_1\le 1.7297, 0\le R_2\le 1.7297, 0\le R_3\le 1.7297 \\ 0\le R_1+R_2+R_3\le 2.4771\end{Bmatrix}$$

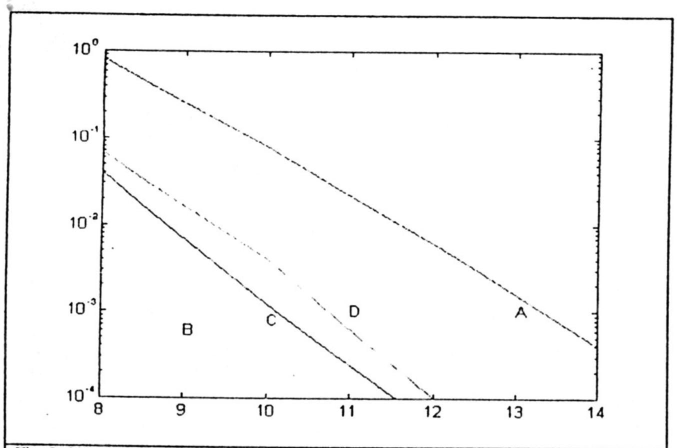

Fig. 9. Simulation with different SNR, different code lengths n and R=0.6. A: uncoded TDM transmission without feedback, B: n=20,C: n=15, D: n=10

The TDM of 500000 symbols at R=0.6 was simulated for different code lengths n and SNR (Fig.9). Curve A shows the results for a channel without feedback and uncoded transmission. At SER=10^{-3}, with n=15 vs. n=10, the code gains 0.5 dB. For n=20 vs. n=15 there is a further gain of 1.5 dB. The gain, for $6 \leq n \leq 12$ and SER=10^{-3}, is between 2.5 and 4.5 dB.

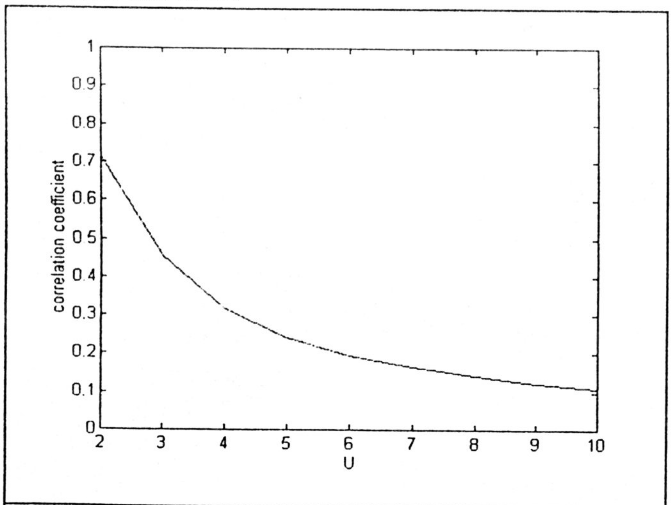

Fig. 10. Absolute value of the correlation coefficient as a funcion of the number of users.

the case of U users

Figure 10 shows the absolute value of the correlation coefficient as a function of the number of users U with P=1 and SNR=10 dB. Note that, when U increases, the correlation between users decreases and tends to zero, making ineffective the use of feedback.

Define R the bound given by (2), that is, the maximum achievable rate, T the maximum achievable sum of rates and C, the channel capacity. Table 1 contains the values computed for different users in the case of P=1 and SNR=10 dB.

Note that, when U increases, the maximum achievable sum of rates decreases, whereas C increases.

4. Conclusions

In this paper, an extension of the Ozarow's code [7], originally developped for the MAC channel with feedback in the case of two users, to the case of several equal-power-users was presented.

Results of computer simulation show that, in the analyzed case of equal-power-users, the use of feedback with the proposed code leads to an achievable rate which is higher than with no feedback, as long as the number of users is small. More specifically, the algorithm is still successful for three users but it is not so for a larger number of users. This result can be interpreted on the basis of a decrease of the correlation between users as the number of users increases. The above consideration leads to the conclusion that the simplicity of the code is strictly related to its inadequacy when the number of users is significant. Future investigation is still needed to understand the structure of a code which would allow to approach the capacity bound in a Multiple Access Channel with several users.

U	R	T	C
2	1.2842	2.5684	2.5693
3	0.7104	2.1312	2.93336
4	0.4744	1.8976	3.1528
5	0.356	1.78	3.3154
6	0.2851	1.7106	3.4506
7	0.2378	1.6646	3.5627
8	0.2041	1.6328	3.6595
9	0.1787	1.6083	3.7448
10	0.1590	1.59	3.8211

Table 1- U: number of users; R: bound given by (2); T: maximum achievable sum of rates; C: channel capacity. Values computed for different users in the case of P=1 and SNR=10 dB.

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