Design and performance of a code for the Multiple Access Channel with feedback

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***Abstract* - In a previous paper [2], an extension of the deterministic feedback code proposed by Ozarow for the two-user Gaussian Multiple Access Channel (MAC) [1] was presented; the Ozarow's code was extented to the case of U-users (U ≥ 2) in the specific case in which all users have equal power. In the present paper, a performance analysis of the proposed code compared to BCH codes is reported. It is shown that a large gain over BCH codes with same trasmission rate and same redundancy is obtained with the new code, if redundancy exceeds a critical value wich was determined by computer simulation.**

I. INTRODUCTION

The present paper presents the results of a further investigation on the extension of the Ozarow's code to the case of three users characterized by equal power as proposed in [2]. In particular, the Ozarow's code and its extension to three users are compared to BCH codes.

In section 2, the Ozarow's code capacity region is reported. In section 3, the proposed extension of the above code to several users [2] is briefly presented in terms of error probability and capacity region. In section 4, the extension of Ozarow's code to the case of three users, vs. BCH codes is reported. Computer simulations of the above codes with same transmission rate and same redundancy are presented.

II. THE CAPACITY REGION OF OZAROW'S CODE

Ozarow's code [1] refers to a situation in which two users transmit messages to a central node through an ideal channel. Ozarow's code is designed for channels with feedback and leads to an achievable capacity region, C^{FB}, given by:

$$C^{FB} = \bigcup_{0 \le \rho \le 1} \left\{ \begin{array}{l} \left(R_1^{FB}, R_2^{FB}\right): 0 \le R_1^{FB} \le 1/2 \cdot \log_2\left(1 + \frac{P_1}{\sigma^2}\left(1-\rho^2\right)\right), \\ 0 \le R_2^{FB} \le 1/2 \cdot \log_2\left(1 + P_2\left(1-\rho^2\right)/\sigma^2\right) \\ 0 \le R_1^{FB} + R_2^{FB} \le \frac{1}{2}\log_2\left(1 + \frac{P_1 + P_2 + 2\sqrt{P_1 P_2}\,\rho}{\sigma^2}\right) \end{array} \right\}$$

where ρ is the correlation coefficient of the two transmitted variables, P_i is the transmitter power, σ^2 is the variance of the Gaussian noise.
The above region includes the capacity region of the Gaussian MAC without feedback.

III. EXTENSION OF OZAROW'S CODE

We proposed [2] to extend Owarow's code to U users ($U \ge 2$) with equal power P.
The error probability formula for each user is:

$$P_{e,i} \le 2Q\left[\frac{2^{\,n\left[\frac{1}{2}\log_2\left(\frac{UP + PU(U-1)\left|\rho^{*2}\right| + \sigma^2}{(U-1)P + \left|\rho^*\right|\left((U-2)(U-1)\right)P - (U-1)^2\rho^{*2}P + \sigma^2}\right) - R_i\right]}}{2\sqrt{\alpha_U}\left[\frac{(U-1)P + P\left|\rho^{*2}\right|(U-1)(U-2) - P(U-1)^2\rho^{*2} + \sigma^2}{UP + U(U-1)\left|\rho^{*2}\right| + \sigma^2}\right]^{-\frac{U}{2}}}\right]$$

where ρ^* is the correlation coefficient between any of two transmitted variables. We have shown that the argument of the rigth-hand side of (P_e) may be made arbitrarily large by increasing n. In presence of feedback, the capacity region is:

$$C^{FB} = \left\{ \begin{array}{l} R_i \le \frac{1}{2}\log_2\left(\frac{UP + PU(U-1)\left|\rho^*\right| + \sigma^2}{(U-1)P + \left|\rho^*\right|\left((U-2)(U-1)\right)P - (U-1)^2\rho^{*2}P + \sigma^2}\right), i = 1,2,\ldots,U \\ \sum_{i=1}^{U} R_i \le \frac{1}{2}\log_2\left(1 + \frac{UP\left(1 + (U-1)\left|\rho^*\right|\right)}{\sigma^2}\right) \end{array} \right\}$$

IV. COMPARISON OF FEEDBACK CODES VS. BCH

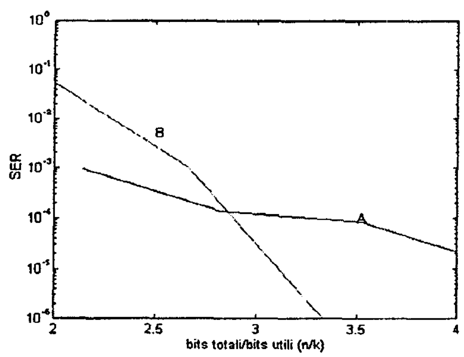

Fig.1. Symbol Error Rate vs. redundancy
curve A: BCH codes with SNR=12 dB and R=1.5 bits/sec/Hertz;
curve B: extension of Ozarow' s code with three users, SNR=12 dB and R1=R2=R3=0.5 bits/sec/Hertz

Figure 1 shows the comparison results between different BCH codes (characterized by different redundancy values) and the new code with three users. All codes have SNR=12 dB and R=1.5 bit/sec/Hertz. The value of n/k at which the two curves cross, indicates the minimum redundancy value over which the extension of Ozarow's code performs better than BCH codes.
The above results can be motivated by the fact that, in the case of three users, ρ^* reaches a lower value than Ozarow's algorithm (two users). This means that the maximum achievable rate in the capacity region is lower than Ozarow's code, and the error probability formula tells that only augmenting the code length n a lower bit error rate can be reached.

REFERENCES

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[2] M. S. Iacobucci and M. G. Di Benedetto, "A feedback code for the Multiple Access Channel (MAC): a case study," published in the proceedings of *IEEE Global Communication Conference* (GLOBECOM), *Communication Theory Mini Conference* (CTMC), Phoenix, Arizona, November '97.

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