Joint communication, ranging, and positioning in low data-rate UWB networks

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Abstract - This work illustrates a possible solution for joint communication, ranging, and positioning for application in UWB sensor networks. The proposed system is based on the adoption of a specifically designed low data-rate (LDR) UWBtailored MAC, named Uncoordinated, Wireless, Baseborn medium access for UWB communication networks $(UWB)^2$, that performs ranging based on the exchange of control packets. The ranging information provided by the MAC is the input to a distributed positioning protocol. Performance was evaluated by simulation and measured as the percentage of positioned terminals as well as the percentage of positioning error, for varying network connectivity and ranging accuracy. Results show that the excellent ranging accuracy provided by UWB is an indispensable feature for applications requiring highly accurate positioning. Results also show that good network connectivity is an additional mandatory condition for designing robust location-aware protocols.

1 Introduction

The ad-hoc networking paradigm offers the possibility of designing flexible self-organizing networks and allows the definition of new network scenarios and applications, which were precluded to traditional infrastructure-based wireless networks [1].

Effective deployment of ad-hoc networks requires, however, to address a whole new set of design issues, ranging from hardware up to the application. Among new requirements one of the most challenging is introducing energy-awareness for extending the life of networks composed of terminals with limited power supply.

Energy-awareness is particularly relevant for sensor networks. The size of such networks, con-

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sisting of hundreds of terminals, makes battery replacement in all network components strongly unlikely. In such a context, knowing the spatial position of terminals can significantly improve energy management, by means, for example, of location-based routing protocols [2]. Position information is also crucial where data provided by sensors depend upon single sensor position, as for example in fire monitoring. The sensors are typically low-cost devices, and, therefore, the cost of equipping all sensors with Global Positioning System (GPS) cannot be afforded. Under these restrictions, sensors can build a map of positions by applying a dedicated positioning protocol, possibly based on a distributed approach. Most of distributed positioning protocols proposed in the literature rely on the availability of distance information between terminals. The system requires, therefore, the presence of a Medium Access Control (MAC) module providing ranging to the positioning module.

Ultra Wide Band (UWB) radio has the appeal, among others, of measuring distances with high precision. Thanks to this feature, UWB has gained popularity world-wide, and has become a top candidate for location-aware ad-hoc and sensor networks [3].

Ranging is not, however, the only concern; network density is another factor to be considered due to its impact on positioning accuracy. This is particularly true for networks where the set of active sensors varies in time due to sensors limited battery autonomy.

In this paper, propose a solution for UWB ranging and positioning based on a distributed MAC protocol, specifically designed to take advantage of the UWB ranging capability, in combination with the distributed Self Positioning Algorithm [4]. We evaluate the performance of the proposed solution in terms of positioning accuracy as a function of ranging accuracy and network density by means of computer simulations. The paper is organized as follows. Section 2 analyzes the ranging capabilities of UWB, taking into account the power limitations imposed by regulation. Section 3 introduces the proposed MAC protocol and evaluates its performance, while Section 4 briefly describes the Self Positioning Algorithm. Section 5 presents the simulation results, while Section 6 draws conclusions.

2 UWB ranging

UWB radio, thanks to its GHz-wide bandwidth, is particularly suited for being adopted as a basis for TOA based ranging, as proposed for example in [5]. The accuracy of the TOA estimation expressed by the variance of the TOA estimation error $\sigma_{\hat{\tau}}^2$ is in fact related to the bandwidth of the signal and SNR at the receiver. According to the general theory of ML estimators, in fact, the lower limit for $\sigma_{\hat{\tau}}^2$ in presence of Additive White Gaussian Noise is given by the Cramer-Rao lower bound [6]:

$$\sigma_{\hat{\tau}}^2 = \frac{N_0}{2\int_{-\infty}^{+\infty} (2\pi f)^2 |G(f)|^2 df}$$
(1)

The maximum theoretical ranging performance made available by UWB signals can be thus evaluated by calculating such lower bound for an UWB pulse g(t) that fully exploits the energy made available by the FCC emission masks for indoor UWB devices, presented in fig. 1.



Figure 1: FCC emission masks for UWB indoor devices.

Let us suppose that the pulse g(t) has a constant bilateral Power Spectral Density $|G(f)|^2$, i.e.:

$$|G(f)|^{2} = \begin{cases} G_{0} \quad for \quad f \in [f_{L}, f_{H}] \cup [-f_{H}, -f_{L}] \\ 0 \quad outside \end{cases}$$
(2)

Eq. (1) can be thus written as follows:

$$\sigma_{\hat{\tau}}^{2} = \frac{N_{0}}{8\pi^{2} \int_{-\infty}^{+\infty} f^{2} |G(f)|^{2} df} = \frac{N_{0}}{8\pi^{2} 2 \int_{f_{L}}^{f_{H}} f^{2} G_{0} df} = \frac{N_{0}}{8\pi^{2} 2G_{0} \left[\frac{f^{3}}{3}\right]_{f_{L}}^{f_{H}}} = \frac{N_{0}}{\frac{8}{3}\pi^{2} 2G_{0} \left(f_{H}^{3} - f_{L}^{3}\right)}$$
(3)

And one can write:

$$\sigma_{\hat{\tau}}^2 = \frac{N_0}{\frac{8}{3}\pi^2 2G_0(f_H - f_L)(f_H^2 + f_H f_L + f_L^2)} = \frac{N_0}{\frac{8}{3}\pi^2 2G_0 B(f_H^2 + f_H f_L + f_L^2)}$$
(4)

Eq. (4) shows that the variance in delay estimation is inversely proportional to the signal monolateral bandwidth occupation B and to a term which depends on the lower and upper frequencies f_H and f_L . It can be easily shown that, for a fixed bandwidth B, this term increases as f_H increases.

Let us consider an IR-UWB signal which fully exploits the frequency band [3.1 - 10.6] GHz with the maximum PSD allowed by the FCC. We obtain the following values: B = 7.5 GHz, $f_H = 10.6$ GHz, $f_L = 3.1$ GHz, $2G_0 = 7.413 \cdot 10^{-14}$ W/Hz and $N_0 \cong 4 \cdot 10^{-21}$ W/Hz for a noise temperature $T_s = FT_0$ with F = 7 dB. The limit given by eq. 4 then writes:

$$\sigma_{\hat{\tau}}^2 = 8.82 \cdot 10^{-39} \tag{5}$$

This corresponds to a lower bound on average distance estimation error equal to $c\sigma_{\hat{\tau}} = 2.82 \cdot 10^{-11}$ m. The above result provides only a theoretical bound for delay estimation error. Receiver hardware limitations, reduced efficiency in the generation of the transmitted signal, and the presence of multipath and multi-user interference lead to a far lower accuracy [7] and UWB ranging accuracy is expected to be in the order of tens of centimeters. It has been proved that with this ranging accuracy one can obtain a positioning accuracy in the order of 50 centimeters [8].

3 The $(UWB)^2$ MAC protocol

In this paper, we assume that information data is organized in packets that are exchanged between nodes in an Aloha fashion, as defined in the specifically designed low data-rate (LDR) UWB-tailored MAC named Uncoordinated, Wireless, Baseborn medium access for UWB communication networks $(UWB)^2$ [9].

 $(UWB)^2$ takes advantage for data transmission of the multiple access capabilities warranted by the TH Codes, and relies for the access to the common channel on the high MUI robustness provided by the processing gain of UWB. The proposed protocol also takes into account synchronization requirements.

 $(UWB)^2$ is a multi-channel MAC protocol. Multi-channel access protocols have been widely investigated in the past, since the adoption of multiple channels may significantly increase the achievable throughput. In multi-channel protocols the overall available resource is partitioned into a finite number of elements. Each element of the resource partition corresponds to a channel. According to the definition of resource, a channel can therefore correspond to:

- 1. A time slot, as in Time Division Multiple Access (TDMA);
- 2. A frequency band, as in Frequency Division Multiple Access (FDMA);
- 3. A code, as in Code Division Multiple Access (CDMA).

The design of an UWB MAC may adopt any of the above solutions. The IEEE 802.15.3a standard for high data rate UWB personal Area Networks, for example, proposes a TDMA MAC for UWB [10]. TH-IR UWB, however, provides a straightforward partition of the resource in channels, each channel being associated to a TH code. The design of a multi-channel CDMA MAC protocol forms therefore the natural basis for the design of a MAC in TH-IR UWB. Multi-channel CDMA MAC algorithms, commonly referred to as multi-code, have been intensively investigated for Direct Sequence (DS) CDMA networks. Among all we cite random CDMA access [11], and, more recently, Multi-Code Spread Slotted Aloha [12]. Note, however, that although in the last years most of the research efforts were focused on DS CDMA, Frequency Hopping (FH) CDMA and TH CDMA also provide viable solutions.

Performance of multi-code MAC protocols are limited by two factors:

- 1. MUI, caused by the contemporary transmission of different packets from different users on different codes;
- 2. Collisions on the code, caused by the selection of the same code by two different transmitters within radio coverage.

Robustness of the system to MUI is determined by the cross correlation properties of the codes; The lower the cross correlation between different codes, the higher the number of possible simultaneous transmissions. The effect of code collisions can be mitigated by adopting appropriate code selection protocols. The task of assigning codes to different transmitters in the same coverage area is a challenging issue in the design of distributed networks. Within this framework, Sousa and Silvester ([13]) provided a thorough overview of possible code assignment solutions:

- 1. Common code: all terminals share the same code, relying on phase shifts between different links for avoiding code collisions.
- 2. Receiver code: each terminal has a unique code for receiving, and the transmitter tunes on the code of the intended receiver for transmitting a packet.
- 3. Transmitter code: each terminal has an unique code for transmitting, and the receiver tunes on the code of the transmitter for receiving a packet.
- 4. Hybrid: a combination of the above schemes.

The Common code scheme is sort of a limit case for a multi-code protocol, since no real multi-code capability is exploited. If phase shifts are too small, this solution collapses into the single Aloha channel. Note however that, in the case of very low data rate UWB networks, even the Common code can be an appealing solution, since the processing gain guaranteed by the low duty cycle of UWB can provide by itself enough protection from MUI to avoid the additional complexity of multi-code management.

The Receiver code scheme has the main advantage of reducing receiver complexity, since a terminal must only listen to its receiving code. On the other hand, multiple transmissions involving the same receiver may likely result in collisions, since the same code is adopted by all transmitters. Oppositely, the Transmitter code scheme avoids collisions at the receiver, since each transmitter uses its own code and thus two transmissions directed to the same receiver use different codes. On the other hand, the adoption of a Transmitter solution requires in principle a receiver to be capable of listening to all possible codes in the network.

Hybrid schemes allow a trade-off between the above conditions. A Hybrid scheme may foresee the use for signaling of either the Receiver or Common code schemes over which the receiver can read the information about the code which will be used for data. A Transmitter code scheme may then be used for data. When the set of codes is limited, however, the Transmitter code scheme may be subject to collisions due to reassignment of the same code. In this case, a code assignment protocol is required for optimizing the use of the limited set of available codes. An example of such a protocol is presented in [14]. The solution proposed in [14] is a distributed assignment protocol for CDMA multihop networks: it guarantees that, if code C is used by terminal T, code C is never selected within a two-hops range from T, avoiding thus the occurrence of collisions.

The $(UWB)^2$ protocol applies the multi-code concept to the specific case of a TH-IR UWB system. $(UWB)^2$ adopts a Hybrid scheme based on the combination of a common control channel, provided by a Common TH code, with dedicated data channels associated to Transmitter TH codes. The adoption of a Hybrid scheme can be motivated as follows:

- 1. It simplifies the receiver structure, since data transmissions (and corresponding TH codes) are first communicated on the control channel;
- 2. It provides a common channel for broadcasting. This is a key property for the operation of higher layers protocols. Broadcast messages are for example required for routing and distributed positioning protocols.

Note that the use of a Common code at the beginning of each transmission also allows an easy transition to the adoption of a Common code solution, whenever the bit rate and the offered traffic are low enough to allow the generated MUI noise to be managed in each receiver with the UWB processing gain alone. As regards code assignment, a unique association between MAC ID and Transmitter Code can be obtained by adopting, for example, the algorithm described in [15] which avoids implementing a distributed code assignment protocol.

 $(UWB)^2$ was designed for distributed networks dedicated to low data rate applications, as such synchronization between transmitter and receiver is not available at the beginning of packet transmission because of clock drifts in each terminal during inactivity periods. A synchronization trailer long enough to guarantee the requested synchronization probability is thus added to each packet. The length of the trailer depends on current network conditions, and is provided to the MAC by the synchronization logic.

3.1 Transmission procedure

Terminal T periodically checks the status of the transmit queue. Detection of one or more MACPDUs triggers the following transmission procedure:

- 1. The ID of the intended receiver R is extracted from the first PDU in the queue;
- 2. T determines the number N_{PDU} of MACPDUs in the queue directed to R;
- 3. T checks if other MACPDUs were sent to R in the last T_{ACTIVE} seconds. If this is the case, T considers R as an Active receiver, and moves to step 5 of the procedure;
- 4. If R is not an Active receiver, T generates a Link Establish (LE) PDU. The LE PDU is composed by the following fields:
 - SyncTrailer Used for synchronization purposes
 - TxNodeID The MAC ID of transmitter T
 - RxNodeID The MAC ID of receiver R
 - TH_{Flag} This flag is set to true if the standard TH code associated to TxNodeID will be adopted for transmission of DATA PDUs. The flag is set to false if a different TH code is going to be adopted.
 - TH Code (optional) If the TH_{Flag} is set to false, the information on the TH-code to be adopted is provided in this field.
 - FEC/CRC Bits for error correction/revelation
- 5. Terminal T sends the LE PDU and waits for a Link Confirm (LC) response PDU from R.

- 6. If the LC PDU is not received within a time T_{LC} , the LE PDU is re-transmitted for a maximum of N_{LC} times, before the transmission of the MACPDU is assumed to be failed.
- 7. After receiving the LC PDU, T switches to the TH code declared in the LE PDU and transmits the DATA PDU. The DATA PDU is composed of the following fields:
 - SyncTrailer Used for synchronization purposes
 - Header, including the fields TxNodeID, RxNodeID, PDU_{Number} and N_{PDU}
 - ACK-flag Used to inform the receiver R if an ACK PDU should be sent in order to inform the transmitter T on the result of the transmission
 - Payload Containing data information
 - FEC/CRC Bits for error correction/revelation
- 8. Once the transmission is completed, T checks again the status of the data queue, and repeats the procedure until all MACPDUs in the transmit queue are served.

When the ACK-flag field is set to 1 in the DATA PDU, the transmitter expects an ACK PDU to be sent by the receiver, in order to schedule a retransmission of a packet if its reception was corrupted by noise or interference, following a predefined backoff scheme. The effect of the selected backoff scheme on performance will be analyzed later on in this section. As regards the transmission of the ACK PDU, two solutions are possible: either the receiver R transmits such PDU on the Common TH-Code, or it transmits the ACK PDU on a Receiver specific TH-Code, at the price of an additional overhead required for communicating such code to the transmitter T in the case such code cannot be derived from the MAC ID of R.

Note furthermore that when the MACSDU is constituted by a broadcast packet (e.g. a routing control packet), the MAC will adopt a simplified transmission procedure, where the DATA PDU that encapsulates the MACSDU is directly transmitted on the Common TH-Code, without performing the LE/LC exchange. The broadcast nature of such PDU would in fact make impossible the reception of a LC PDU by all interested terminals. Furthermore, for this kind of PDUs the ACK-flag will be automatically set to 0 in order to avoid the transmission of several ACK PDUs by each neighbor of T receiving the broadcast PDU. A broadcast ID known to all terminals is set as Receiver ID in these PDUs in order to inform neighbors of the broadcast

nature of the transmission.

Such simplified procedure guarantees of course a lower protection of broadcast PDUs from interference; on the other hand, it makes possible for the upper layers to have a straightforward mean to communicate broadcast information. Furthermore, the potential loss of a control broadcast packet is usually much less critical than the loss of a DATA packet since updated control information is usually retransmitted either on a periodic basis or within a short time.

3.2 Reception procedure

A terminal R in Idle state listens to the Common TH code, indicated as TH-0. When a Sync-Trailer is detected, R performs the following reception procedure.

- 1. R checks the RxNodeID field. If the value in the field is neither the MAC ID of R nor the broadcast ID, the reception is aborted and the reception procedure ends;
- 2. Since in the following we are not considering broadcast packets, let us assume that the RxNodeID contains the MAC ID of R. In this case, since R is assumed in Idle state, MACPDUs directed to this terminal will necessarily be LE PDUs;
- 3. Following the reception of a LE packet, R creates a LC PDU. The LC PDU is structured as follows:
 - SyncTrailer Used for synchronization purposes
 - TxNodeID The MAC ID of T
 - RxNodeID The MAC ID of R
 - FEC/CRC Bits for error correction/revelation
- 4. R sends the LC PDU and moves in the Active state, listening on the TH code indicated in the LE PDU. If no DATA PDU is received within a time T_{DATA} the receiver falls back to Idle state and the procedure ends.
- 5. When a DATA PDU is received, R processes the payload, and extracts N_{PDU} from the header. If the ACK-flag is set to 1, R generates and sends an ACK PDU reporting the status of the transmission. Next, if $N_{PDU} > 0$, R remains in Active state, since at least

 N_{PDU} more DATA PDUs are expected to be received from T. If $N_{PDU} = 0$, R goes back to the Idle state.

It should be noted that the above procedures are related to the setup of a single link. During the reception procedure for example R also keeps on listening to the common code. It is assumed in fact that a terminal can act as a receiver on one or more links while acting as a transmitter on another link.

The performance of the $(UWB)^2$ was analyzed by means of simulations in order to evaluate its behavior in terms of throughput and delay.

The simulation scenario consisted in N terminals, randomly located in an area of 80x80 m^2 size. Each terminal was characterized by a radio transmission range of 120 meters in order to guarantee almost full connectivity between terminals. Each terminal generated MACPDUs to other terminals in the network following a Poisson process characterized by an average inter-arrival time T_{PDU} . The size of each MACPDU was set to L = 2000 bits, and all terminals adopted the same synchronization sequence of length $L_{sync} = 100$ pulses. Performance of the $(UWB)^2$ protocol was evaluated for a number of terminals N varying between 25 and 50, and for T_{PDU} values in the interval [1.25, 0.039063] secs, corresponding to data rates between 1600 bits/s and 51200 bits/s, respectively.

Two performance indicators were considered: throughput, defined as the ratio between received MACPDUs and transmitted MACPDUs, and delay, both evaluated in presence of retransmissions. Two different backoff algorithms were considered:

- Immediate retransmit in this algorithm retransmissions are performed as soon as the information of the transmission error is sent back by means of the ACK packet.
- Binary Exponential Backoff (BEB) in this algorithm retransmissions are performed after a random delay. The average delay before attempting a retransmission for the r-th time is equal to N_r times the transmission time of a DATA PDU; the value of N_r is randomly extracted in the interval [2⁰, 2^{min(r,rmax)}]. In our simulations, we chose $r_{max} = 10$.

The measured values for throughput and delay are presented in figs. 2 and 3, respectively.

Figure 2 shows that measured throughput was higher than 0.985 in all simulation cases. Furthermore, the two backoff schemes considered led to comparable values in all simulations, highlighting the fact that most of PDUs collisions are not destructive thanks to the MUI resiliance guaranteed by UWB.

This conclusion is confirmed by fig. 3, showing that the average delay is only slightly increased as the number of offered packets increase, and is in all cases close to the minimum value given by the transmission time of a MACPDU at the bit rate of 1 Mb/s. Furthermore, the adoption of the Binary Exponential Backoff scheme led to higher delays, since in the rare cases where a destructive collision occurred, transmitters were forced to wait a longer average time before attempting a retransmission.



Figure 2: Throughput as a function of the offered traffic expressed in Packets/s (Empty squares: Binary Exponential Backoff scheme, Filled circles: Immediate retransmit scheme).

In the view of allowing distributed positioning and location-aware routing, $(UWB)^2$ collects distance information between transmitters and receivers during the exchange of control packets, as shown in fig. 4.

The $(UWB)^2$ MAC can provide the input required by a distributed positioning protocol, i.e. distance information between each pair of terminals in physical connectivity.



Figure 3: Delay as a function of the offered traffic expressed in Packets/s - The dashed line shows the delay due to transmission time of a MACPDU at the bit rate of 1 Mb/s (Empty squares: Binary Exponential Backoff scheme, Filled circles: Immediate retransmit scheme).

The most general solution for a distributed positioning protocol is to be anchor-free, that is no position information is available at network start, as in the Self Positioning Algorithm (SPA) [4]; in this paper we thus combined $(UWB)^2$ with the SPA. The SPA is briefly described in the next section.

4 Distributed positioning

The Self-Positioning Algorithm (SPA) [4] leads to a relative network topology, and uses a TOA technique in order to obtain range measurements. The algorithm builds a node-centered coordinate system (called Local Coordinate system) for each node in the network, and forces all nodes to converge to a unique relative coordinate system (called Network Coordinate system).

In order to build its own Local Coordinate system, each node *i* performs the following actions:

- 1. Detect its set of one-hop neighbors K_i ; This phase is accomplished by using beacons, in order to maintain an up-to-date map of one-hop neighbors
- 2. Evaluate the set of distances from its neighbors K_i ; The distance measurement from each one-hop neighbor is obtained by means of TOA estimation



Figure 4: Procedure for bidirectional distance estimation in $(UWB)^2$.

3. Send D_i and K_i to its one-hop neighbors.

In this way each node i will know directly its distances from all its one-hop neighbors, the IDs of its two-hop neighbors, and a subset of the distances from its one-hop neighbors to its two-hop neighbors, as shown in fig. 5.



Figure 5: Example of network knowledge available at terminal i - red circle represents terminal i, blue circles represent one-hop neighbors of i, white circle represent a two-hop neighbor of i. Lines between nodes represent known distances.

The determination of the local coordinate system in a 2D scenario requires the selection of two additional terminals p, q in the K_i set. p and q must not lie on the same line with i. Furthermore, since all reciprocal distances between i, p and q must be known, an additional constraint in the choice of p and q is set: $p \in K_q$ (or, equivalently, $q \in K_p$). The local coordinate system is defined as follows: p lies on the positive x axis, while q has a positive y component. As a consequence, coordinates of i, p and q are:

$$\begin{cases} X_i = 0, \ Y_i = 0 \\ X_p = d_{ip}, \ Y_p = 0 \\ X_q = d_{iq} \cdot \cos \gamma, \ Y_q = d_{iq} \cdot \sin \gamma \end{cases}$$
(6)

where γ is the angle (p, i, q), defined as:

$$\gamma = \arccos \frac{d_{iq}^2 + d_{ip}^2 - d_{pq}^2}{2 \cdot d_{iq} \cdot d_{ip}} \tag{7}$$

Given the terminals i, p and q the position of a fourth terminal j can be evaluated by trilateration if the distances d_{ij} , d_{pj} and d_{qj} are known. In fact, we obtain:

$$j_x = d_{ij} \cos \alpha_j$$

$$j_y = \begin{cases} d_{ij} \sin \alpha_j & if \ \beta_j = |\alpha_j - \gamma| \\ -d_{ij} \sin \alpha_j & if \ \beta_j = \alpha_j + \gamma \end{cases}$$
(8)

where γ is the angle (p, i, j) and β_j is the angle (j, i, q), given by:

$$\begin{aligned}
\alpha_j &= \arccos \frac{d_{ij}^2 + d_{ip}^2 - d_{pj}^2}{2 \cdot d_{ij} \cdot d_{ip}} \\
\beta_j &= \arccos \frac{d_{iq}^2 + d_{ij}^2 - d_{qj}^2}{2 \cdot d_{ij} \cdot d_{iq}}
\end{aligned} \tag{9}$$

Depending on the selected couple p, q, i will be able of evaluating the position of only a subset of its one-hop neighbors. This subset is called Local View Set LVS_i , and it is a subset of K_i : the couple p, q should be chosen in order to maximize the size of LVS_i . Note that a terminal can be included in the LVS_i either based on its distances from i, p and q, or based on its distances from i and two other terminals already included in the LVS_i . If we define $C_i = \{(p,q) \in K_i \text{ such that } p \in K_q\}$ as the set of all possible couples ps and qs, the best couple will be:

$$(p,q) = \arg \max_{(p_k,q_k) \in C_i} |LVS_i(p_k,q_k)|$$
(10)

Following the above strategy, each node will define a coordinates system in which it occupies the position (0,0): in order to define a global network topology, all node-centered systems of coordinates must be linearly transformed in order to have a unique orientation (i.e. the same direction for x and y axes of all nodes) and thus obtain the Network Coordinate system. If we consider two terminals i and k the linear transformation can be performed if the two following conditions are met:

- $i \in LVS_k$ and $k \in LVS_i$
- $\exists j \neq i, k$ such that: $j \in LVS_i$ and $j \in LVS_k$;

Note that if there is no a common one-hop neighbor j, the linear transformation cannot be performed. Figure 6 presents an example of terminal j which satisfies the condition.



Figure 6: Example of common neighbor j satisfying the condition for linear transformation of reference systems of nodes i and k.

Assuming that k must modify its system to get the same orientation of system centered on i, two possible scenarios can be foreseen, depending on local coordinate systems of both k and i:

- 1. k must rotate its local system of a quantity called correction angle;
- 2. k must rotate its local system of a quantity called correction angle and must mirror the system over x or y axis.

The node j is used to understand if the system has simply to be rotated or it has also to be mirrored, by evaluating the following conditions:

1.

$$if \{ [(\alpha_j - \alpha_k) < \pi \text{ and } (\beta_j - \beta_i) < \pi] \}$$

or $\{ [(\alpha_j - \alpha_k) > \pi \text{ and } (\beta_j - \beta_i) > \pi] \}$ (11)

2.

$$if \{ [(\alpha_j - \alpha_k) > \pi \text{ and } (\beta_j - \beta_i) < \pi] \}$$

or $\{ [(\alpha_j - \alpha_k) < \pi \text{ and } (\beta_j - \beta_i) > \pi] \}$ (12)

When condition (11) is met, mirroring is required, and the correction angle is $\beta_i + \alpha_k$. Oppositely, when condition (12) is met, no mirroring is required, and the correction angle is $\beta_i - \alpha_k + \pi$.

An example of the two scenarios is represented in figs. 7 and 8, respectively.



Figure 7: Example of coordinate system conversion in which mirroring is requested.

Once that k obtains the same orientation as i, all nodes in LVS_k can do the same, because they know their position in the system centered on k and also the position of node k in the system of node i. Furthermore, the two systems have the same orientation, and this means that for each node $l \in LVS_k$ the position in the new system is simply given by the sum of two vectors.

Terminals belonging to LVS_l will use the same approach, as the coordinate system adopted by l will now have the same orientation of the coordinate system adopted by k, and the network coordinate system will propagate throughout the network. Terminals that were not able to build their own local coordinate system can still obtain their position if they are able to communicate with three terminals which already received the network coordinate system. In the solution proposed in this paper the SPA receives as inputs the distance estimations provided by the



Figure 8: Example of coordinate system conversion in which mirroring is not requested.

 $(UWB)^2$ MAC. Performance of this solution is evaluated in the next section.

5 Performance analysis

The performance of a distributed positioning algorithm can in general be defined in terms of its effectiveness in building a global coordinate system and providing each terminal with its position in the network. Performance is mainly determined by:

- network connectivity degree, that is the average number of neighbors in physical reach within one hop;
- 2. ranging accuracy.

The network connectivity degree depends on the power available at each terminal, that is the transmission range of a terminal, and on the density of nodes in the network.

The ranging accuracy, on the other hand, derives from the adopted transmission technique. In order to evaluate the effect of both network connectivity and ranging accuracy on positioning accuracy, we simulated a network of 20 mobile terminals distributed in an area of $80x80m^2$ adopting the $(UWB)^2$ MAC and the SPA positioning algorithm.

A first set of simulation investigated the effect of network connectivity as a function of transmission range. Simulations were performed with varying transmission ranges, from 20 meters (low network connectivity) up to 60 meters (high network connectivity), and assuming perfect ranging, i.e. no error in distance estimation.

Results of simulations are shown in Fig. 9, which reports the percentage of terminals that were able to evaluate their own position, as a function of the transmission range.

Figure 9 shows that for medium to high network connectivity degrees (i.e. transmission range above 40 meters) a large number of terminals are able to position themselves, and use thus this information to optimize resource management and routing.

We then analyzed the effect of the density of nodes. The density was varied by varying the number of terminals while keeping constant the network area size. Figure 10 shows the percentage of positioned terminals as a function of the number of terminals, i.e. density, for three different values of the transmission range (40, 50, and 60 m). We can predict the performance of the



Figure 9: Percentage of terminals sharing the same coordinate system as a function of the transmission range.

positioning protocol as terminals run out of energy by reading the data of fig. 10 from right to left.



Figure 10: Percentage terminals sharing the same coordinate system as a function of the number of terminals for three values of transmission range (circles: range = 40 m, diamonds: range = 50 m, squares: range = 60 m).

Finally we analyzed the effect of the accuracy in ranging in the specific case of a highly connected network (transmission range = 60 m). Three different cases were taken into account, characterized by a ranging error with uniform distribution in the intervals [-0.1m, 0.1m], [-1m, 1m], [-10m, 10m].

Figure 11 shows the percentage of positioning error as a function of ranging error.

Figure 11 shows in fact that ranging errors in the order of 0.1 m (case 1) and 1 m (case 2) lead to satisfactory performance (errors in positioning below 5%) while the third case (ranging errors



Figure 11: Percentage positioning error as a function of ranging error.

in the order of 10 meters) is not compatible with applications requiring accurate positioning. The ranging accuracy provided by UWB appears thus as an indispensable feature when high accurate positioning is required by the application layer.

6 Conclusions

In this paper, a solution for joint communication, distributed ranging and positioning for application in UWB sensor networks was proposed. The proposed solution is based on the adoption of the $(UWB)^2$ MAC protocol that performs ranging based on the exchange of control packets. The ranging information provided by the MAC is the input to the distributed positioning protocol (SPA). Performance was evaluated by simulation and measured as the percentage of positioned terminals as well as the percentage of positioning error, for varying network connectivity and ranging accuracy. Results show that the excellent ranging accuracy provided by UWB is an indispensable feature for applications requiring highly accurate positioning. Results also show that good network connectivity is an additional mandatory condition for designing robust location-aware protocols.

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