Evaluating BER in Sparse IR UWB Networks under the Pulse Collision Model

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Abstract

In this paper, we provide a novel analytical expression for the average BER in Impulse Radio Ultra Wide Band (IR UWB) networks affected by Multi User Interference (MUI). BER is evaluated based on the observation that interference in IR is provoked by collisions occurring between pulses belonging to different transmissions. The reference scenario consists of multiple asynchronous users transmitting IR-UWB signals using Pulse Position Modulation (PPM) in combination with Time Hopping (TH) coding. The proposed method requires specification of a similar set of system parameters as Gaussian-based approaches, but shows improved accuracy in estimating BER, in particular when sparse network topologies are taken into account.

1. Introduction

Different methods have been proposed in the recent past for evaluating the effect of Multi User Interference (MUI) on the performance of Impulse Radio Ultra Wide Band (IR UWB) networks [1]-[5]. Most of these methods basically extend to the UWB case the models that were conceived for non-impulsive SS-CDMA systems, such as the Standard Gaussian Approximation (SGA) [6], the Characteristic Function (CF) method [7], and the Gaussian Quadrature Rule (GQR) method [8]. Among the above methods, however, only the SGA provides a feasible way for deriving power allocation in IR-UWB networks [9]-[11]. By SGA, in fact, few system parameters and reduced computational cost are required for estimating BER. The SGA, however, is based on the central limit theorem, and provides thus accurate BER estimations only for scenarios with high MUI levels [12]. When sparse network topologies are considered, the gap between theoretical and measured BERs may be as large as several orders of magnitude [13]. In this letter, we propose a novel MUI model for estimating performance of sparse IR UWB networks, which is based on the observation that interference in IR is provoked by collisions occurring between pulses belonging to different transmissions. Modeling BER based on the concept of pulse collisions was firs proposed in [14]. Here, we extend [14] by further refining the pulse collision model and by introducing a complete receiver structure. A novel analytical expression for the average BER is then provided for the case of IR-UWB signals employing Pulse Position Modulation (PPM) in combination with Time Hopping (TH) coding. Propagation over AWGN channels is taken into account. Power control at the reference receiver is not required for BER computation under the proposed approach.

This paper is organized as follows. Section 2 defines the system model. Section 3 evaluates BER under the Pulse Collision model. Section 4 validates the proposed model by simulation of different network scenarios. Finally, Section 5 concludes the paper.

2. System Model

The system model consists of a reference transmitter TX which emits IR-UWB-TH-PPM signals to a reference receiver RX. The binary sequence **b** generated by TX is formed by independent and equally probable binary symbols "0" and "1". The transmitted signal writes:

$$s_{TX}(t)=\sqrt{E_{TX}}\sum_j p_o\left(t-jT_S-\theta_j-\varepsilon b_{\lfloor j/N_S\rfloor}\right) \qquad (1)$$

where $p_0(t)$ is the energy-normalized waveform of the transmitted pulses, E_{TX} is the energy of each pulse, T_S is

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the average pulse repetition period, $0 \le \theta_j < T_S$ is the time shift of the j-th pulse provoked by the TH code, ε is the PPM shift, b_x is the x-th bit of **b**, N_S is the number of pulses transmitted for each bit, and $\lfloor x \rfloor$ is the inferior integer part of x. A general flat AWGN channel model is assumed. The impulse response for the channel between TX and RX is thus given by $h(t)=\alpha\delta(t-\tau)$, where α and τ are the amplitude gain and propagation delay. TX and RX are assumed to be perfectly synchronized, that is, RX has perfect knowledge of τ. The channel output is corrupted by thermal noise and MUI generated by N_i interfering IR-UWB devices. The received signal thus writes:

$$s_{RX}(t) = r_u(t) + r_{mui}(t) + n(t) \quad (2)$$

where $r_u(t)$, $r_{mui}(t)$, and n(t) are the useful signal, MUI, and thermal noise, respectively.

As regards $r_u(t)$, one has:

$$r_u(t) = \sqrt{E_u}\sum_j p_0\left(t - jT_S - \theta_j - \varepsilon b_{\lfloor j/N_S \rfloor} - \tau\right) \quad (3)$$

where $E_u=\alpha^2 E_{TX}$.

As regards $r_{mui}(t)$, we assume that all interfering signals are characterized by same T_S, and thus:

$$r_{mui}(t) = \sum_{n=1}^{N_i}\sqrt{E^{(n)}}\sum_j p_0\left(t - jT_S - \theta_j^{(n)} - \varepsilon b^{(n)}_{\lfloor j/N_S^{(n)} \rfloor} - \tau^{(n)}\right) \quad (4)$$

where $E^{(n)}$ and $\tau^{(n)}$ are received energy per pulse and delay for the n-th interfering user. The relative delay $\Delta\tau^{(n)}=\tau-\tau^{(n)}$ is modelled as a random variable uniformly distributed between 0 and T_S. The terms θ_j, $b_x^{(n)}$ and $N_S^{(n)}$ in (4) are the time shift of the j-th pulse, the x-th bit generated by user n, and the number of pulses per bit for the n-th transmitter, respectively. TH shifts and data bits are randomly generated and independent one of another. In particular, $\theta_j^{(n)}$ are uniformly distributed in the range $[0,T_S)$, and $b_x^{(n)}$ have equal probability to be "0" or "1".

Finally, signal n(t) in (2) is Gaussian noise, with double-sided power spectral density $\mathcal{N}_0/2$.

The optimum single-user receiver for the above system model is composed by a coherent correlator followed by a ML detector [1]. The input of the detector Z(x), for a generic bit b_x, expresses as follows:

$$Z(x) = \int_{xN_ST_S+\tau}^{(x+1)N_ST_S+\tau} s_{RX}(t)\, m_X(t-\tau)\, dt = Z_u + Z_{mui} + Z_n \quad (5)$$

where $m_x(t)$ is the correlation mask for b_x, i.e.:

$$m_x(t) = \sum_{j=xN_S}^{(x+1)N_S}\left(p_0\left(t - jT_S - \theta_j\right) - p_0\left(t - jT_S - \theta_j - \varepsilon\right)\right) \quad (6)$$

According to (5), Z(x) consists of the signal term Z_u, the MUI contribution Z_{mui}, and the noise contribution Z_n, which is Gaussian with zero mean and variance $\sigma_n^2 = N_S\mathcal{N}_0\gamma(\varepsilon)$, where $\gamma(\varepsilon)=1-R_0(\varepsilon)$, and where $R_0(\varepsilon)$ is the autocorrelation function of $p_0(t)$. Bit b_x is estimated by comparing Z(x) with a zero-valued threshold: when Z(x)>0 decision is "0", when Z(x)<0 decision is "1". For independent and equiprobable transmitted bits, the average BER at the output of the detector is thus:

$$BER = Prob\left(Z(x) < 0 \,|\, b_x = 0\right) \quad (7)$$

2. BER Evaluation

By observing that the signal term Z_u in (5) is given by $N_S\sqrt{E_u}\gamma(\varepsilon)$ for $b_x = 0$, BER in (7) rewrites:

$$BER = Prob\left(Z_{mui} < -\left(N_S\sqrt{E_u}\gamma(\varepsilon) + Z_n\right)\right) = Prob\left(Z_{mui} < -y\right) \quad (8)$$

where y is a Gaussian random variable with mean $N_S\sqrt{E_u}\gamma(\varepsilon)$ and variance $N_S\mathcal{N}_0\gamma(\varepsilon)$.

BER in (8) can be evaluated by first computing the conditional BER for a generic y value, and by then averaging over all possible y values, that is:

$$BER = \int_{-\infty}^{+\infty} Prob\left(Z_{mui} < -y \,|\, y\right) p_Y(y)\, dy \quad (9)$$

where $p_Y(y)$ is the Gaussian probability density function of y.

In the proposed approach, conditional probability of error $Prob(Z_{mui}<-y|y)$ takes into account collisions between pulses of different transmissions. In every bit period, the number of possible collisions at the input of the receiver, denoted with N_C, is confined between 0 and N_SN_i, given N_S pulses per bit and N_i interfering users. Under the reasonable assumption that the events of collision are independent of one another, $Prob(Z_{mui}<-y|y)$ rewrites:

$$\Pr ob(Z_{mui} < -y \mid y) = \sum_{N_C=0}^{N_S N_i} \Pr ob(Z_{mui} < -y \mid y, N_C) P_{CP}(N_C) \quad (10)$$

where $P_{CP}(N_C)$ indicates the probability of having N_C pulse collisions within one single bit interval. For independent interferers, $P_{CP}(N_C)$ can be reasonably expressed through the binomial distribution, that is:

$$P_{CP}(N_C) = \binom{N_S N_i}{N_C} P_{C0}^{N_C} (1 - P_{C0})^{N_S N_i - N_C} \quad (11)$$

where P_{C0} is the probability that a single interfering device produces a colliding pulse within T_S. P_{C0} can be computed as the fraction of T_S during which the receiver may be affected by the presence of an interfering pulse, that is:

$$P_{C0} = \frac{\min(2T_M + \varepsilon, 4T_M, T_S)}{T_S} \quad (12)$$

where T_M is the length of the pulse waveform. When substituting (10) into (9), one obtains:

$$BER = \sum_{N_C=0}^{N_S N_i} P_{CP}(N_C) \int_{-\infty}^{+\infty} \text{Prob}(Z_{mui} < -y \mid y, N_C) p_Y(y) dy \quad (13)$$

As shown in [4], the cumulative density function of MUI caused by one single interferer can be reasonably fitted by a linear function. Based on [4], we propose for $\text{Prob}(Z_{mui}<-y|y,N_C)$ a linear model including multiple interferers with different received powers, as shown in Figure 1. $\text{Prob}(Z_{mui}<-y|y,N_C)$ is analytically expressed by:

$$\Pr ob(Z_{mui} < -y \mid y, N_C) = \begin{cases} 1 & \text{for } y \le -Z_{max}(N_C) \\ 1 - \frac{P_{CP}(N_C)}{2}\left(1 + \frac{y}{Z_{max}(N_C)}\right) & \text{for } -Z_{max}(N_C) < y \le 0 \\ \frac{P_{CP}(N_C)}{2}\left(1 - \frac{y}{Z_{max}(N_C)}\right) & \text{for } 0 < y \le Z_{max}(N_C) \\ 0 & \text{for } y > Z_{max}(N_C) \end{cases} \quad (14)$$

where $Z_{max}(N_C)$ is defined as the maximum value for the MUI term Z_{mui}, when N_C collisions have occurred at the reference receiver. For a given N_C, there are as many $Z_{max}(N_C)$ possible values as the number of possible combinations of N_C collisions among the N_i interfering users. For a given N_C, however, different estimates for $Z_{max}(N_C)$ can be obtained. The proposed estimate for $Z_{max}(N_C)$ is:

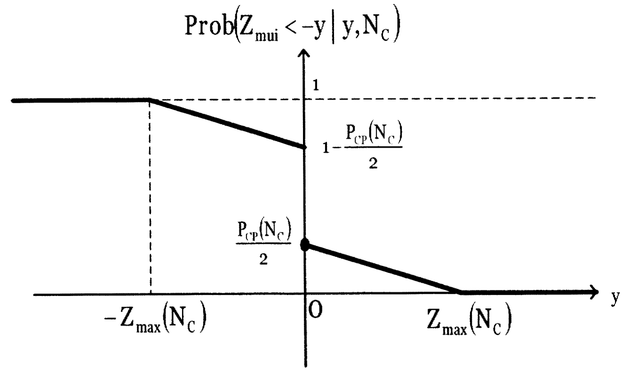

Figure 1 – Linear model for the conditional probability of error $\text{Prob}(Z_{mui}<-y|y,NC)$, given y and given N_C.

$$Z_{max}(N_C) = \sum_{j=1}^{N_i} \left(\left\lceil \frac{N_C - j + 1}{N_i} \right\rceil \sqrt{E_S^{(j)}} \right) \quad (15)$$

where $E_S^{(1)}, E_S^{(2)}, \ldots, E_S^{(Ni)}$ are the interfering energies $E^{(1)}, E^{(2)}, \ldots, E^{(Ni)}$ of (4), but sorted in descending order so that $E_S^{(j)} \ge E_S^{(j+1)}$ for $j \in [1, Ni-1]$. Note that the expression in (15) estimates $Z_{max}(N_C)$ by privileging those users with the highest interfering energies.

Given $Z_{max}(N_C)$, we can finally introduce the conditional probability function of (14) into (13). One has:

$$\begin{aligned} BER = & \frac{1}{2}\text{erfc}\left(\sqrt{\frac{1}{2}\frac{N_S E_u}{\mathcal{N}_0}\gamma(\varepsilon)}\right) + \\ & + \sum_{N_C=0}^{N_S N_i} \frac{P_{CP}(N_C)^2}{2}\left[-\text{erfc}\left(\sqrt{\frac{1}{2}\frac{N_S E_u}{\mathcal{N}_0}\gamma(\varepsilon)}\right) + \right. \\ & + \frac{1}{2}\text{erfc}\left(\sqrt{\frac{1}{2}\frac{N_S E_u}{\mathcal{N}_0}\gamma(\varepsilon)} - \sqrt{\frac{1}{2}\frac{Z_{max}(N_C)^2}{N_S \mathcal{N}_0 \gamma(\varepsilon)}}\right) \\ & + \frac{1}{2}\text{erfc}\left(\sqrt{\frac{1}{2}\frac{N_S E_u}{\mathcal{N}_0}\gamma(\varepsilon)} + \sqrt{\frac{1}{2}\frac{Z_{max}(N_C)^2}{N_S \mathcal{N}_0 \gamma(\varepsilon)}}\right) + \\ & \left. - \int_{-Z_{max}(N_C)}^{+Z_{max}(N_C)} y \, p_Y(y) dy \right] \end{aligned} \quad (16)$$

Since $p_Y(y)$ is positive and symmetrical around its mean value $y_m>0$, it is easy to recognize that the last integral in (16) is always positive. One thus obtains:

$$BER \le BER_{upbond} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{1}{2}\frac{N_S E_u}{\mathcal{N}_0}\gamma(\varepsilon)}\right) + \sum_{N_C=0}^{N_S N_i} \frac{P_{CP}(N_C)^2}{2}\left[-\mathrm{erfc}\left(\sqrt{\frac{1}{2}\frac{N_S E_u}{\mathcal{N}_0}\gamma(\varepsilon)}\right) + \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{1}{2}\frac{N_S E_u}{\mathcal{N}_0}\gamma(\varepsilon)} - \sqrt{\frac{1}{2}\frac{Z_{max}(N_C)^2}{N_S\mathcal{N}_0\gamma(\varepsilon)}}\right) + \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{1}{2}\frac{N_S E_u}{\mathcal{N}_0}\gamma(\varepsilon)} + \sqrt{\frac{1}{2}\frac{Z_{max}(N_C)^2}{N_S\mathcal{N}_0\gamma(\varepsilon)}}\right)\right] \quad (17)$$

If we approximate the BER with its upper bound given in (17), we have:

$$BER \approx \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{1}{2}\frac{N_S E_u}{\mathcal{N}_0}\gamma(\varepsilon)}\right) + \sum_{N_C=0}^{N_i N_S} \frac{P_{CP}(N_C)}{2}\Omega\left(\frac{N_S E_u}{\mathcal{N}_0}\gamma(\varepsilon), \frac{Z_{max}(N_C)^2}{N_S\mathcal{N}_0\gamma(\varepsilon)}\right) \quad (18)$$

where:

$$\Omega(A,B) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{A}{2}} - \sqrt{\frac{B}{2}}\right) + \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{A}{2}} + \sqrt{\frac{B}{2}}\right) - \mathrm{erfc}\left(\sqrt{\frac{A}{2}}\right) \quad (19)$$

The BER expression in (18) includes a first term that only depends on signal to thermal noise ratio at RX input, and a second term accounting for MUI. Note that for computing (18), no additional information with respect to the BER computation with the SGA is requested.

4. Simulation Results

In Figs. 2 and 3, performance of the proposed MUI model is evaluated in two different scenarios. In both cases, transmitted signals have $N_S = 1$ and $T_S = 60$ns, leading to R_b=16.66 Mb/s. In both cases, $p_0(t)$ is the second derivative Gaussian waveform [2], with $T_M = 1$ ns and $\varepsilon = 1$ ns.In the case of Figure 2, the network consists of one reference user with received energy per bit $E_b = E_u$, and 3 interfering users with received energy per pulse $E^{(1)} = E_u$, $E^{(2)} = 4E_u$, and $E^{(3)} = 1/4(E_u)$, respectively. In the case of Figure 3, the network consists of one reference user with received energy per pulse E_u, and 5 interfering users with received energy per pulse $E^{(1)}=E_u$, $E^{(2)}=4E_u$, $E^{(3)}=8E_u$, $E^{(4)}=1/4(E_u)$, and $E^{(5)}=1/8(E_u)$, respectively. In both cases, performance is expressed by BER vs. signal to noise ratio $E_b/\mathcal{N}_0$, and BER estimates based on Pulse Collision are plotted against simulation values and theoretical BER values derived under the SGA. Simulation values have been obtained by averaging the BER measured at the receiver output after the transmission of 10^6 bits. In particular, 20 different runs of the simulation have been performed for each $E_b/\mathcal{N}_0$ value under examination.

Note that Pulse Collision values very well fit simulation data, while SGA underestimates BER. Estimation accuracy provided by the proposed method is thus higher than that provided by SGA. Differently from existing non-Gaussian approaches, however, the proposed approach does neither require knowledge of additional system parameters, as in GQR methods, nor necessitates numerical computation of open-ended integrals, as in the CF method.

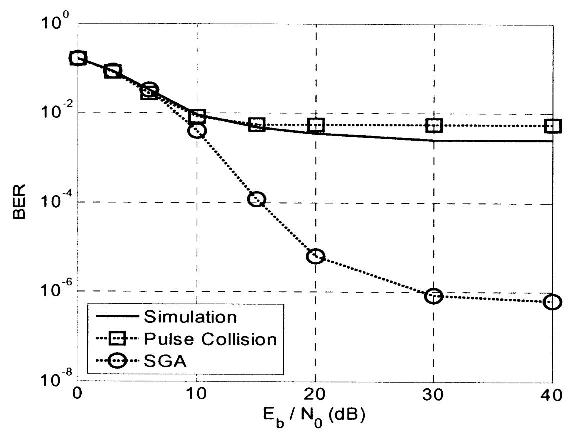

Figure 2 – BER vs. E_b/N_0 with $N_S = 1$, $T_S = 60$ ns, N_i=3, $E^{(1)}=E_u$, $E^{(2)}=4E_u$, $E^{(3)}=(1/4)E_u$.

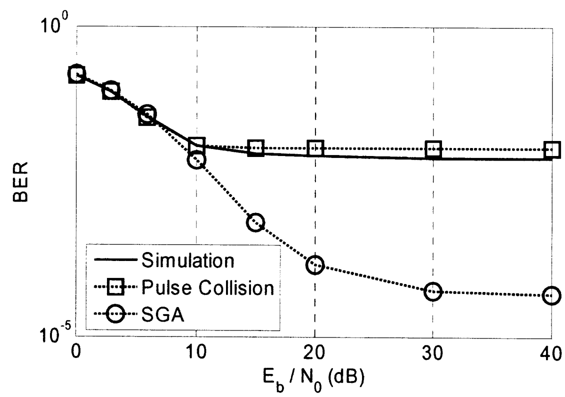

Figure 3 – BER vs. E_b/N_0 with $N_S = 1$, $T_S = 60$ ns, N_i=5, $E^{(1)}=E_u$, $E^{(2)}=4E_u$, $E^{(3)}=8E_u$, $E^{(4)}=1/4(E_u)$, and $E^{(5)}=1/8(E_u)$.

5. Conclusions

In this paper, a novel approach for estimating BER in IR UWB networks affected by MUI was presented. Differently from existing solutions [2]-[5], which basically extend to the UWB case results that are known for SS-CDMA systems, the proposed method analyzes MUI under a novel perspective. In particular, the proposed MUI model explicitly takes into account the peculiar way in which information is structured and conveyed by IR-UWB devices. In IR-UWB, information bits are coded into sequences of short pulses. MUI can thus be re-analyzed by observing that interference at the reference receiver is provoked by collisions occurring between pulses belonging to different transmissions. Based on this observation, a novel analytical expression for the average BER was derived for the reference scenario where IR-UWB-PPM-TH signals propagate over AWGN channels, and terminals adopt single user receivers with soft decision detection.

The proposed approach showed high accuracy in estimating receiver performance by simulation of different network topologies. Estimation accuracy provided by the proposed method results to be much higher than that provided by conventional Gaussian-based approaches, in particular when sparse network topologies are taken into account. Differently from existing non-Gaussian approaches [3]-[5], however, the proposed approach does neither require knowledge of additional system parameters, as in GQR methods, nor necessitates numerical computation of open-ended integrals, as in the CF method.

A natural extension of this work is to include propagation over multipath-affected channels. The Pulse Collision approach presented in this paper is in fact feasible enough to include more advanced receiver structures, such as RAKE, and as such promises to be effective for predicting the behavior UWB networks when channel models more complex than the simple AWGN channel are considered.

Acknowledgments

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