An Application of MMSE Predistortion to OFDM Systems

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) particularly suffers from the presence of nonlinearities since the signal amplitude is Rayleigh distributed. The degradation introduced by the nonlinear amplifier in the transmitter can be significantly reduced by using an analog cubic predistorter minimum mean square error (MMSE) predistorter proposed by the authors in [1].**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is based on the transmission of a given set of signals on several subcarriers. Each subcarrier is quadrature amplitude modulation (QAM) or phase shift keying (PSK) modulated. While in frequency division multiplexing (FDM) systems the signals conveyed on the subcarriers do not overlap in frequency, in OFDM systems they can overlap; however, they are, in frequency, orthogonal to each other, and thus, they do not interfere, and can be demodulated by using a correlation receiver. With respect to single-carrier (SC) systems, multicarrier systems present the advantage of being more robust in applications involving channels affected by severe multipath propagation, such as in a mobile system [2]. With respect to FDM systems, the OFDM architecture is more efficient in terms of bandwidth and the transceiver structure is simply based on discrete Fourier transform (DFT) circuits [3].

However, in radio systems it is important to consider the distortion introduced by the high power amplifier, in this case a traveling wave tube (TWT), which is worsened in the OFDM systems by the long-tailed distribution (Rayleigh) of the signal amplitude. For example, a degradation of 7.2 dB is expected in OFDM/4QAM systems when nonlinearities are taken into account [3], [4].

One way of limiting the negative effects of such a distortion is to introduce a cubic analog predistorter before the TWT. The nonlinearities introduced by the TWT in the case of 256/512-QAM modulated systems have been analyzed in a previous paper [1], in which the use of a cubic predistorter [called minimum mean square error (MMSE)] significantly improving the system performance has been proposed.

In this paper, the effect of the presence of a strong nonlinearity in the transmitter, when OFDM modulation is adopted, will be analyzed by use of simulation trials. The improvement of the performance obtained with the MMSE predistorter will be shown. Results will also be given in terms of dependency upon the specific data sequence.

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II. Basic Principles of OFDM

In what follows, potentially complex parameters and functions will be indicated in bold letters. An OFDM modulated signal consists of the sum of several signals which are modulated at different carrier frequencies equally spaced by $\Delta f = 1/T_0$, and orthogonal in frequency to each other.

The binary sequence which forms the input to the OFDM modulator, is subdivided into groups of K bits used to generate blocks of N symbols $\{\alpha_o, \cdots, \alpha_m, \cdots, \alpha_{N-1}\}$, where α_j is one of L possible different elements and $K = N \log_2 L$. Finally, each symbol QAM-modulates a different carrier.

All QAM-modulators use the same rectangular pulse shape $g_T(t)$, of finite duration T and such that

$$g_T(t) = \begin{cases} 1/\sqrt{T_0} & \text{for: } -T_G = T_0 - T \le t \le T_0 \\ 0 & \text{elsewhere} \end{cases} \tag{1}$$

where T_G indicates the guard interval which is introduced in order to contrast the effects of multipath. The total bit-rate is $f_b = 1/T \cdot N \log_2 L$. If $\mathbf{c}_m = a_m + jb_m$ indicates the point in the constellation associated to α_m, the OFDM signal corresponding to the block of symbols considered is

$$\begin{aligned} x(t) = a \cdot g_T(t) \sum_{m=0}^{N-1} \{ & a_m \cos[2\pi(f_p + f_m) \cdot t + \phi] \\ & - b_m \sin[2\pi(f_p + f_m) \cdot t + \phi]\} \\ f_m \equiv (m - N/2) & \cdot \Delta f, \quad -T_G \le t \le T_0 \end{aligned} \tag{2}$$

while the corresponding complex envelope is

$$\begin{aligned} \underline{x}(t) &= a \cdot g_T(t) \sum_{m=0}^{N-1} \mathbf{c}_m \cdot e^{j2\pi f_m \cdot t} \\ &\equiv \sum_{m=0}^{N-1} \mathbf{c}_m \varphi_m(t) \equiv a \cdot g_T(t) \cdot S(t) \end{aligned} \tag{3}$$

in which $\varphi_m(t) \equiv a \cdot g_T(t) \cdot \exp\{j2\pi f_m t\}$ and $S(t)$ is a periodic function of period T_0.

Under the hypothesis of ideal propagation, the transmitted symbols can be recovered from the received signal if a set of functions $\psi_n(t)$ orthonormal to $\varphi_m(t)$ exists. In traditional OFDM systems, one has $\psi_m(t) \equiv a \cdot g_R(t) \cdot \exp\{-j2\pi f_m t\}$ with

$$g_R(t) = \begin{cases} 1/a\sqrt{T_0} & \text{for } 0 \le t \le T_0 \\ 0 & \text{elsewhere.} \end{cases} \tag{4}$$

Under the above conditions, the OFDM signal can be demodulated using a correlation receiver.[1] When additive white noise

[1]However, when the propagation is not ideal, and the channel can be modeled by a linear and shift-invariant system with impulse response $h(t)$ or transfer function $H(f)$, the received signals carried at different frequencies

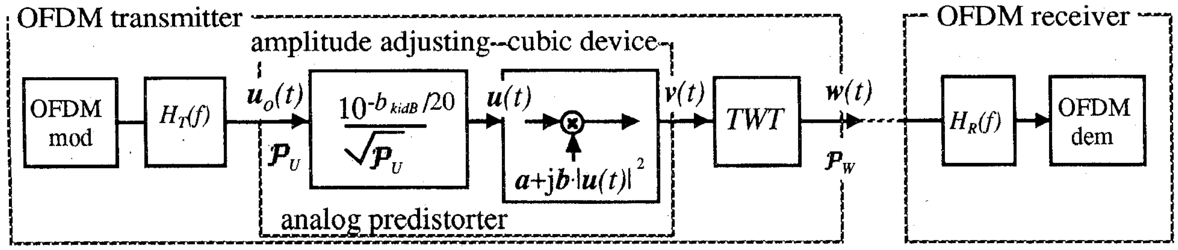


Fig. 1. General scheme of an OFDM system which includes an analog cubic predistorter before the TWT.

is also present, the output of the correlation receiver is given by $\mathbf{r}_k = \mathbf{c}_k + \mathbf{n}_k$, in which $\mathbf{n}_k$ is a zero mean, complex-valued Gaussian variable with variance σ^2. The probability of symbol error for each subcarrier is $P_e \cong 2(1 - 1/\sqrt{L}) \cdot \operatorname{erfc}\{1/[\sigma(\sqrt{L} - 1)]\}$ [5].

A useful expression for σ^2 is derived in Appendix A.

III. The MMSE Predistorter

Fig. 1 shows the scheme of an OFDM system. The TWT input back-off, expressed in dB, is indicated by $b_{ki\mathrm{dB}}$. The complex envelope of the predistorter input, $\mathbf{u}_o(t) = u_o(t) \cdot \exp\{j\Theta_o(t)\}$, is normalized to its average power $\mathcal{P}_U$ and reduced by $b_{ki\mathrm{dB}}$. The analog predistorter is a cubic device characterized by two complex parameters $\mathbf{a}$ and $\mathbf{b}$ and precedes a TWT, which introduces a nonlinear instantaneous distortion described by the input-output relation [6]

$$\mathbf{w}(t) = \mathbf{v}(t)\frac{2}{1+v^2(t)}e^{-j\Theta_o\frac{v^2(t)}{1+v^2(t)}}. \tag{5}$$

The maximum instantaneous emitted power is thus one (0 dB).

The distortion due to the TWT + predistorter produces a nonlinear relation between the output power $\mathcal{P}_{W\mathrm{dB}}$ and $b_{ki\mathrm{dB}}$, with saturation at 0 dB, and intermodulation noise. Intermodulation and thermal noise contribute to the overall system degradation. Let $\mathcal{P}_{R\mathrm{dB}}$ be the requested power at the receiver input, for an error probability P_e, when only thermal noise is present. When intermodulation noise is also present, the received power must be increased by a value $\Delta\mathcal{P}_{R\mathrm{dB}}(b_{ki\mathrm{dB}})$ which depends upon $b_{ki\mathrm{dB}}$; let $\mathcal{P}_{R\mathrm{dB}}(b_{ki\mathrm{dB}}) = \mathcal{P}_{R\mathrm{dB}} + \Delta\mathcal{P}_{R\mathrm{dB}}(b_{ki\mathrm{dB}})$ be the received power in presence of thermal and intermodulation noise. As $b_{ki\mathrm{dB}}$ varies, the system margin is modified by

$$\text{Margin Loss } = \Delta M_{\mathrm{dB}} = \mathcal{P}_{W\mathrm{dB}}(b_{ki\mathrm{dB}}) - \Delta\mathcal{P}_{R\mathrm{dB}}(b_{ki\mathrm{dB}}). \tag{6}$$

The "margin loss" is the difference in margin between the actual amplifier and no-backoff, linear amplifier. Therefore, there exists an optimal $b_{ki\mathrm{dB}}$ which depends upon the predistorter structure. In the present analysis, the use of a cubic predistorter $\mathbf{v}(t) = \mathbf{u}(t) \cdot (\mathbf{a} + j\mathbf{b} \cdot u^2)$ was adopted; $\mathbf{a}$ and $\mathbf{b}$ minimize the error between the actual predistorter output $\mathbf{v}(t)$ and the output of a reference predistorter characterized by the following relation between $\mathbf{u}(t)$ and $\mathbf{w}'(t)$ (hard-limiter)

$$\mathbf{w}' = \begin{cases} 2\mathbf{u} & \text{for } u \le 1/2 \\ \mathbf{u}/u & \text{for } u > 1/2. \end{cases} \tag{7}$$

This cubic predistorter (optimized as described above) is called MMSE and is extensively described in [1]. Table I shows the values for $\mathbf{a}$ and $\mathbf{b}$ as a function of $b_{ki\mathrm{dB}}$ [1].

TABLE I
$\mathbf{a}$ AND $\mathbf{b}$ VALUES (REAL AND IMAGINARY PARTS) AS A FUNCTION OF $b_{ki\mathrm{dB}}$ [1, EQ. (13)]

b_{kidB}	$Re\{a\}$	$Imm\{a\}$	$Re\{b\}$	$Imm\{b\}$
0	1.07	-0.59	-0.16	0.09
2	1.26	-0.66	-0.29	0.13
4	1.41	-0.67	-0.44	0.14
6	1.46	-0.57	-0.50	-0.03
8	1.35	-0.33	-0.21	-0.64
10	1.14	-0.05	0.64	-1.76
12	0.98	0.09	1.58	-2.60
14	0.97	0.06	1.73	-2.30
16	0.99	0.02	1.38	-1.62
18	1.00	0.00	1.18	-1.32
20	1.00	0.00	1.10	-1.20

IV. Results of Simulation Trials

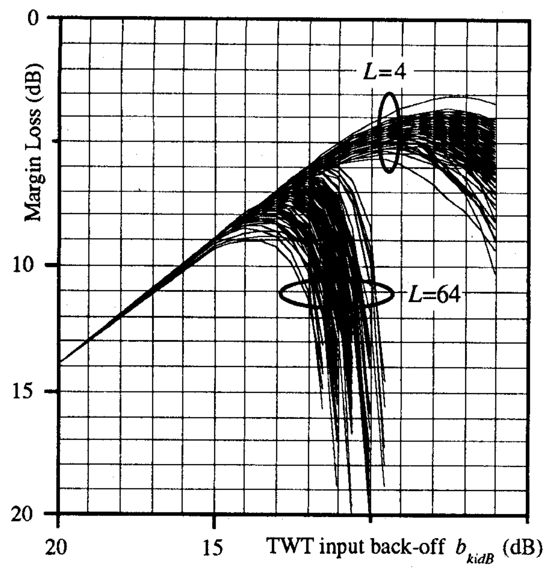
Each simulation trial considered the transmission of $N\log_2 L$ randomly generated bits. The curve corresponding to (2) was derived for $b_{ki\mathrm{dB}}$ between -15 and -6 dB. The complex signal envelope was obtained by using a bandwidth eight times the nominal band $B = N\Delta F$.

Since the OFDM signal has an amplitude distribution which extends much further than in the SC case, and since the distortion depends essentially on the frequency of emission of high amplitude values, we expect: i) a strong dependence of the margin loss on the specific emitted sequence, which increases with L and decreases with N and ii) a higher $b_{ki\mathrm{dB}}$ than in SC cases.

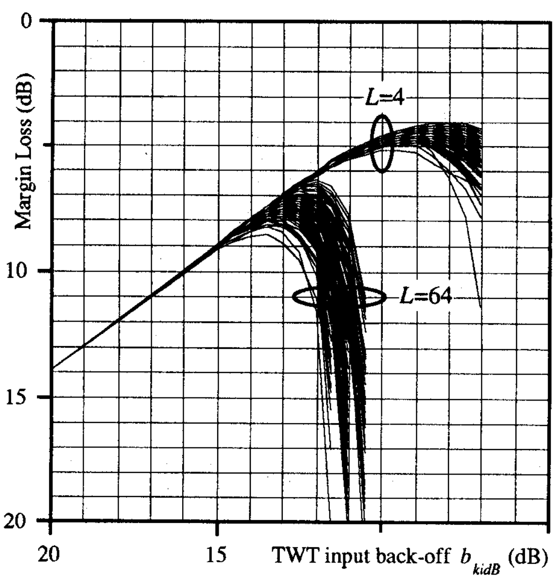
Fig. 2 represents the margin loss versus $b_{ki\mathrm{dB}}$ for the reference predistorter and 128 different input sequences, and for $N = 256$ and 1024. Confirming our first expectation, both the optimal value of $b_{ki\mathrm{dB}}$ and the margin loss vary with the input sequence by as much as 3 dB.

Fig. 3 represents the margin loss versus $b_{ki\mathrm{dB}}$ averaged over 128 input sequences. In agreement with [3], high $b_{ki\mathrm{dB}}$ values

are orthogonal only if $h(t)$ is limited in duration between 0 and T_G. In this case, the output of the correlation receiver is multiplied by a factor $H(f_k)$, which must be estimated in order to derive $\mathbf{c}_k$.



(a)



(b)

Fig. 2. Margin loss as a function of $b_{k i \mathrm{dB}}$, for 128 different data sequences, and for two values of N (# subcarriers) and of L (# levels/subcarrier). Fig. 2(a) and (b) show the results for $N = 256; L = 4$ or 64 levels/subcarrier, and $N = 1024; L = 4$ or 64 levels/subcarrier, respectively.

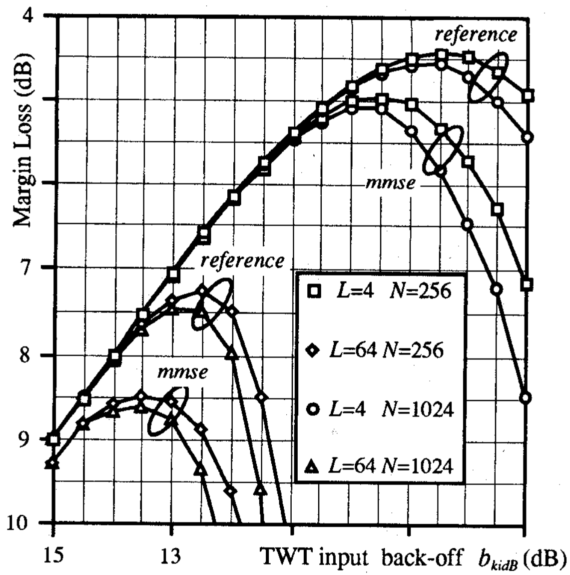


Fig. 3. Margin loss as a function of $b_{k i \mathrm{dB}}$, for $N = 256$ or 1024, and $L = 4$ or 64 levels/subcarrier, in the case of the reference predistorter versus the MMSE predistorter.

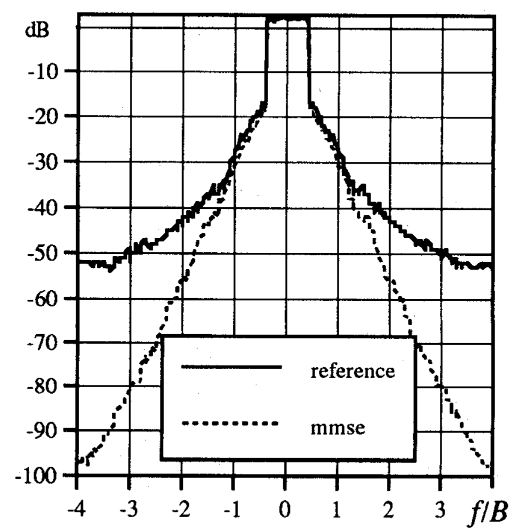


Fig. 4. PSD's at the TWT output for the reference and the MMSE predistorter.

are needed (9.5 to 12.5 dB for $L = 4$ to 64 levels/carrier) and a margin loss as high as -4.5 to -7.5 dB is observed. Most important, Fig. 3 shows that the results for the MMSE predistorter approximate tightly those of the reference case; The relative margin reduction is always less than ≈ 1 dB. In addition, Fig. 4 shows that the use of the MMSE reduces the off-band terms of the power spectral density (PSD) after the TWT.

V. Conclusion

The results reported in the present paper show that the MMSE predistorter is an effective countermeasure against TWT nonlinear distortions in multicarrier systems. The presence of the MMSE predistorter produces a margin loss very similar to the reference case (difference always less than ≈ 1 dB).

With respect to SC systems, the OFDM system shows a margin loss variability versus the transmitted sequence between 1 and 3 dB, suggesting the use of appropriate line coders for minimizing peak-to-average output power [7].

Appendix A

The variance σ^2 of the additive noise $\mathbf{n}_k$ can be derived under the hypotheses of independent and uniformly distributed symbols, and thermal noise with PSD $\mathcal{N}_o$. Let $\lambda^2 \equiv \mathbf{E}\{|\mathbf{c}_k|^2\} = 2\cdot(\sqrt{L}+1)/3\cdot(\sqrt{L}-1)$ be the constellation power, the signal power in (2) is

$$\begin{aligned}\mathcal{P}_X &= \frac{1}{2}\mathcal{P}_{\underline{X}} \\ &= \frac{a^2}{2}\cdot\mathcal{E}\left\{\frac{1}{T}\int_0^T g_T^2(t)\sum_{k=0}^{N-1}\sum_{n=0}^{N-1}\mathbf{c}_k\mathbf{c}_n^* \right.\\ &\qquad \left.\cdot e^{-j2\pi(n-m)\cdot\Delta t\cdot t}\cdot dt\right\} = N\frac{a^2}{2T_0}\lambda^2 \qquad (8)\end{aligned}$$

and the corresponding energy/symbol is given by: $\mathcal{E}_L = a^2\lambda^2 T/2T_0$. In such conditions, since $\mathbf{n}_k = \int_0^{T_0}\mathbf{n}(t)\cdot g_R(t)\cdot e^{-j2\pi f_k t}dt$, the variance σ^2 is usefully expressed by

$$\begin{aligned}\sigma^2 &= \mathcal{E}\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{n}(t)g_R(t)\mathbf{n}^*(\theta)g_R(\theta)e^{-j2\pi f_k(t-\theta)}dtd\theta\right\} \\ &= \mathcal{N}_o\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}u_o(t-\theta)g_R(t)g_R(\theta)e^{-j2\pi f_k(t-\theta)}dtd\theta \\ &= \mathcal{N}_o\int_0^{T_0} g_R^2(t)\cdot dt \\ &= \mathcal{N}_o/a^2 = (\lambda^2/2)\cdot(1+T_G/T_0)2\mathcal{N}_o/\mathcal{E}_L.\end{aligned}$$

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