Multiple Access Design for Impulse Radio Communication Systems

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***Abstract*- In this paper, we address the problem of designing time hopping codes for impulse radio multiple access communication systems. Time hopping techniques are first reviewed, including a new code that we proposed [2] for time hopping, and performance in terms of cyclic correlation properties are given. Advantages and disadvantages of different time hopping codes in terms of resource assignment for multiple-access communication systems are analyzed and discussed.**

I. INTRODUCTION

Impulse radio (IR) is a spread spectrum technique which uses very short duration pulses and pulse position modulation for transmitting information.

The transmitted signal is:

$$s(t) = \sum_{i=-\infty}^{\infty} \sum_{j=0}^{N_s-1} g\left(t - jT_f - b_i\delta\right)$$

where $g(t)$ represents the pulse, N_s is the number of pulses per bit and $T_b = N_s \cdot T_f$ is the bit duration. The sequence b_k represents the information bits.

Multiple access is achieved using time hopping codes, thus the transmitted signal is:

$$s(t) = \sum_{i=-\infty}^{\infty} \sum_{j=0}^{N_s-1} g\left(t - jT_f - c_jT_c - b_i\,\delta\right)$$

where an additional shift of c_jT_c is provided by the hopping code, with $0 \le c_j \le N_h$ and $N_h \cdot T_c < T_f$.

When the number of users is N_u and noise $n(t)$ is additive, the received signal is:

$$s_{rec}(t) = \sum_{k=1}^{N_u} \sum_{i=-\infty}^{\infty} \sum_{j=0}^{N_s-1} g\left(t - jT_f - c_j^{(k)}T_c - b_i^{(k)}\delta\right) + n(t) \qquad (1)$$

When the receiver is the pulse correlator, as described in [1], the Signal to Noise Ratio SNR at the receiver output can be written as follows:

$$SNR = \frac{P_{av}}{N_s\sigma_a^2(N_U-1)+\sigma_{rec}^2} = \frac{(N_s m_p)^2}{N_s\sigma_a^2(N_U-1)+\sigma_{rec}^2} \qquad (2)$$

where P_{av} is the average useful power, σ_{rec}^2 is the power of the thermal noise, σ_a^2 is the interference average power provoked by one user in the time interval T_f, and m_p is the signal at the correlator output in that same interval T_f.

In time hopping spread-spectrum systems, multilevel sequences are used to specify which time interval T_c is used for transmission at any given time T_f.

Since several users share the same resource (time), it is important to keep mutual interference as low as possible. This mutual interference is controlled by the cross correlation of the time hopping sequences.

Let $\boldsymbol{X} = \left(x_0, x_1, \ldots x_{N_p-1}\right)$ and $\boldsymbol{Y} = \left(y_0, y_1, \ldots y_{N_p-1}\right)$ indicate two hopping sequences of period N_p with x_i and $y_i \in \{0,1,\ldots N_h-1\}$.

One of the best hopping sequence performance indicator is the Hamming cross-correlation function, defined as:

$$H_{XY}(\tau) = \sum_{i=0}^{L} h(x_i, y_{i+\tau}) \quad 0 \le \tau \le N_p - 1 \quad L = N_p - 1 - \tau$$

where: $$h(a,b) = \begin{cases} 0, & a \ne b \\ 1, & a = b \end{cases} \qquad (3)$$

Equation (3) represents the number of coincidences or hits between the two sequences ***X*** and ***Y***, when ***X*** is delayed by τ. Therefore, the Hamming cross-correlation function defined in (3) represents an *aperiodic* cross-correlation measure.

Properties of codes in terms of aperiodic cross-correlation are reported in [2]; the scenario which justifies the use of the aperiodic cross-correlation function is related to non-continuous transmissions, where a sufficiently large guard interval is introduced in-between two transmissions. However, the adoption of non-continuous transmission may not be desirable, since it does lower the throughput, and can also cause loss of synchronism between transmitters and receivers with long time hopping codes.

It is therefore very important to analyze cyclic cross-correlation properties of time hopping codes by defining the periodic Hamming cross-correlation function. In this case there is no need of guard intervals, the bit rate is exactly $R_b = 1/T_b$, and the only synchronism required is the chip

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synchronism (that is time intervals T_f belonging to transmissions of different users must be synchronized). The hypothesis of chip synchronism is made in the definition of the Hamming cross-correlation function, and therefore for the calculus of the correlation properties of the codes studied in the present paper.

Let $\boldsymbol{Y}_\tau = \left(y_{N_p-\tau}, y_{N_p-(\tau-1)}, \ldots y_0, y_1, \ldots y_{N_p-1-\tau}\right)$ $0 \le \tau \le N_p - 1$, be the cyclic shift of codeword $\boldsymbol{Y}$ by τ positions. The *periodic* Hamming cross-correlation function is:

$$H_{XY}^{p}(\tau) = H_{XY_\tau}^{a}(0) \tag{4}$$

The Signal to Noise Ratio given by (2), under the hypothesis of chip synchronism is:

$$SNR = \frac{\left(N_s m_p\right)^2}{\left\lfloor N_s / N_p \right\rfloor \cdot E\left(H^p\right) \cdot \left(N_U - 1\right) + \sigma_{rec}^2} \tag{5}$$

where $E\left(H^p\right)$ is an average over each pair of codewords for each value of τ, and $\lfloor x \rfloor$ is the integer part of x.

In this paper, we analyze properties of the codes presented in [2] when the cross-correlation function is *periodic*.

The paper is organized as follows. In section 1 we review some time hopping techniques in terms of code construction, and we give the periodic correlation properties. Moreover, a new code proposed in [2] for time hopping multiple access is described and analyzed. Section 2 reports the error probability performance of the codes. In section 3, advantages and disadvantages of different time hopping codes in terms of resource assignment for multiple-access communication systems are analyzed and discussed.

I. Time Hopping Codes: Code Construction and Correlation Properties

A. *Pseudorandom code*

A codeword is obtained as follows: for a given N_h, the N_p code symbols are obtained by casually extracting N_p numbers between 0 and $N_h - 1$.

The cyclic cross-correlation function between two codewords of length N_p is:

$$E\left(H^p(\tau)\right) = \frac{N_p}{N_h}$$

Therefore, the signal to noise ratio given by (5) can be written as follows:

$$SNR = \frac{\left(N_s m_p\right)^2}{N_s / N_h \cdot m_p^2 \cdot \left(N_U - 1\right) + \sigma_{rec}^2}$$

B. *Code construction 1*

The chips c_j^k of the codeword of user k (where index j=1..p indicates time), $\boldsymbol{C^k} = \left(c_0^k, c_1^k, \ldots c_{N_p-1}^k\right)$, are obtained as follows:

$c_j^k = \left[(k + j - 1) \bmod p\right]$ with p prime

and $pT_c < T_f$. This construction generates a family of p codes (p users) of length $N_p = p$ [4].

The periodic cross-correlation function (4) of code construction 1 has the following property: for each pair of different codewords, there exists one (and only one) value of τ for which the two codewords have exactly p hits (full collision), while for all the other values of τ the two codewords are perfectly orthogonal (no hits). In fact, let the first codeword be

$$\boldsymbol{C1} = \left\{c_j^{k1}\right\} = \left[(k1 + j - 1) \bmod \boldsymbol{p}\right]$$

and the other codeword, shifted by τ, be

$$\boldsymbol{C2}(\tau) = \left\{c_j^{k2}(\tau)\right\} = \left[(k2 + j + \tau - 1) \bmod \boldsymbol{p}\right] \quad .$$

The two codewords collide if and only if $k1 = \left[(k2 + \tau) \bmod p\right]$. In this event, the collision must be recovered at the DLC level with the retransmission of the collided packets. For this reason, even if $E\left(H^p\right)$ is equal to one, it is possible to adopt this family of perfectly orthogonal codes, stated that the only case of full collision is recovered at higher levels. In this case $E\left(H^p\right)$=0 and the expression for SNR is:

$$SNR = \frac{\left(N_s m_p\right)^2}{\sigma_{rec}^2}$$

C. *Code construction 2: a new code*

The time hop for a user is given by $c_j^{m,k} = \left(\frac{j}{k} + m\right) \bmod p$, p prime.

Index j=1..p-1 identifies time; indexes k=1..p-1 and m=1..p-1 identify the user. In this way it is possible to generate a family of $(p-1)^2$ codes of length $N_p = p - 1$.

In [2] we proposed this construction, already used for frequency hopping [3], for time hopping multiple access.

The periodic cross-correlation function given by (4) was computed by computer generation of all the different codewords for a fixed value of p. The maximum and average number of hits, for different values of p, is reported in Table 1. As shown, the codewords are not orthogonal, but the average number of hits is very low even in case of high codeword lengths.

TABLE I
PERIODIC CROSS-CORRELATION PROPERTY OF CODE CONSTRUCTION 2

p	Maximum number of hits	Average number of hits $E(H^p)$
3	2	0.75
5	3	0.875
7	4	0.9167
11	4	0.95
19	4	0.9722

II. ERROR PROBABILITY

Bit error rate of the proposed time hopping codes was computed. The parameter values were fixed as follows: pulse amplitude $A = 10^{-9}$; pulse width $\tau_g = 0.8\ ns$; $T_f = 100\ ns$; $N_s = 2000$ and different values of N_h.

The error probability is given by the 2PSK formula: $P_e = 2Q\sqrt{SNR \cdot sin^2\left(\frac{\pi}{2}\right)}$.

Figure 1 shows the increase in error probability of a time-hopping pulse position modulated system as a function of the number of users with access to the same resource. All users are supposed to transmit at a rate equal to 5 kbit/s. Figure 1 shows that the best code is obtained with construction 1. In fact, except one case of full collision, it generates perfectly orthogonal codewords. In the case of full collision, the messages associated with the two collided codewords cannot be received and the two transmitters must retransmit the message with a different delay τ. This must be dealt at the DLC level. In order to do so, a certain amount of signalling must be spent for message recovering, with the consequence of a higher delay. The pseudo-random and construction 2 codes have comparable BER. The cross correlation depends on the value of N_h; the higher is N_h the smaller is the correlation. However, the pseudo-random code is more flexible, since the value N_p can be chosen independently of N_h, something which is not possible with the other codes. Figure 2 shows that for $N_h = 199$ the pseudo-random code has good performance even for a high number of users.

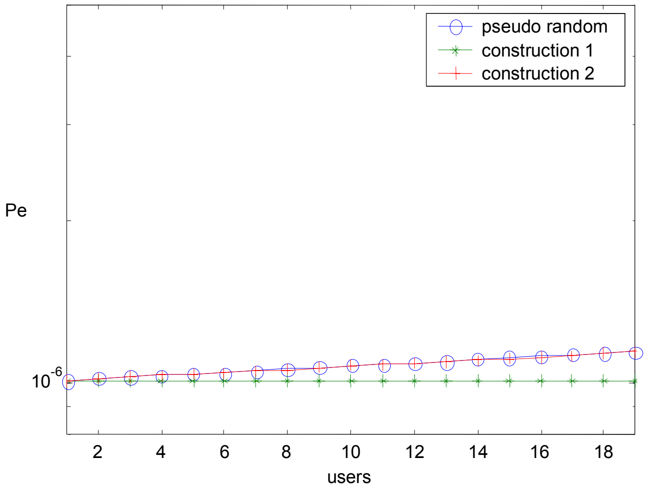

Fig. 1. Error Probability versus total number of users with $p = 19$, $T_f = 100\ ns$; $N_s = 2000$. The pseudo-random code is with $N_h = 19$.

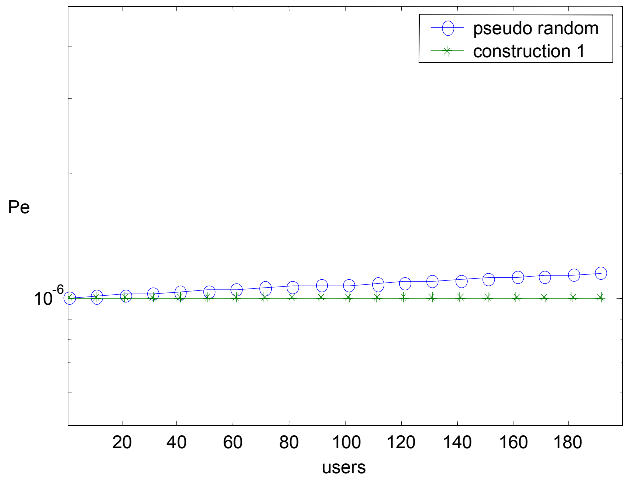

Fig. 2. Error Probability versus total number of users with $N_h = 199$, $T_f = 100\ ns$; $N_s = 2000$.

III. Discussion: multiple access and MAC layer design

From the point of view of interference between users, the best code is the code with construction 1. In fact, unless one case of full collision, it generates perfectly orthogonal codewords, cancelling the phenomenon of mutual interference. The case of full collision must be dealt at the DLC level; some signalling must be spent for the message recover, producing a higher delay. The code generated with construction 2, that is the new code, has very good correlation properties, comparable to the ones of the pseudo-random code. Moreover, it can address a number of users that is much higher than the code with construction 1.

Advantages and disadvantages of adopting the codes of section II in the design of the multiple access module of the system must also address the design of the MAC layer.

The MAC layer has to fulfill the QoS requests of the network layer (including bit rate, delay, Packet Error Rate). The MAC will comply with the requests by adjusting the MAC parameters.

In a time hopping impulse radio system, MAC parameters are several: the time hopping codes, the number of transmitted pulses per bit N_s, the nominal distance between two pulses T_f, the period of the time hopping code $T_p = N_p \cdot T_f$, the pulse shape and its duration. These MAC parameters are involved in link performance.

In particular, time hopping codes must have a few important properties: they must be orthogonal or quasi-orthogonal in order to generate small interference between users; they must be addressed by few parameters in order to simplify the exchange of codes between nodes; they must be numerous in the selected family; they must be defined for different lengths N_p and different N_h ($N_h \cdot T_c \leq T_f$).

The pseudo-random code allows an unlimited number of users with code length N_p, but it is very difficult to address. In fact, it does not have a structure and the exchange of codes between two communicating nodes consists of the transmission of the whole code on a given control channel.

The other codes on the contrary have a construction and can be addressed with a few parameters. In particular, one integer (*m*) is needed to address N_p users for code construction 1, and two integers (*k*,*m*) to address N_p^2 users for code construction 2.

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