Time Reversal Beamforming in MISO-UWB Channels

G.C. Ferrante*†, J. Fiorina† and M.-G. Di Benedetto*

* Sapienza University of Rome, DIET Department, Rome, ITALY,
†SUPELEC, TELECOM Department, Gif-sur-Yvette, FRANCE.

***Abstract*—Multiple-Input Single-Output (MISO) Ultra Wide Band (UWB) communications over a multipath channel is investigated. After introducing a tractable discrete-time multipath MISO channel model accounting for correlation between multiple channels, analytical results for single-user transmissions are drawn in the following two cases: no precoding at transmitter with All-Rake receiver vs. time reversal (TR) at transmitter with One-finger Rake receiver. Channel state information at both transmitter (CSIT) and receiver (CSIR) is assumed. Optimality of TR conditioned on One-finger Rake receiver structure is proved. Robustness brought by TR combined with One-finger Rake with respect to lack of correlation between channels vs. no precoding is shown. Simulations with realistic reference channels show that combining time reversal with multiple transmit antennas amplifies the performance improvement due to each technique when used alone.**

I. Introduction

Time reversal is a technique first proposed for acoustic systems [4] based on a physical property of waves propagating in reciprocal media: the wave radiated by a source located in a point O within a volume V with surface ∂V can be computed by replacing the original source with equivalent virtual sources placed on surface ∂V; the wave radiated in V by the virtual sources is same as the original field reversed in time.

Applications are based on the idea of recording the wavefront on the closed surface ∂V containing the source by ideally an infinite number of sensors and then transmitting the recorded signals reverted in time simultaneously: waves radiated by all sensors interfere constructively when reaching O. In practice, a finite number of sensors can be placed on ∂V, that implies an approximate focusing.

Time reversal is a signaling scheme focusing a signal in both time and space. When transmitted signals have impulsive nature, as in Impulse Radio Ultra Wide Band (IR-UWB), time reversal further increases impulsiveness thanks to the many resolvable paths of UWB channels even in the single-antenna setting [6], [13].

When multiple antennas at transmitter side are used, time reversal is a beamformer focusing the wideband signal at receiver. Early experimental results for MISO and MIMO time reversal are collected, respectively, in [1] and in [10], [12], [14].

MISO beamforming with N_t antennas in flat-fading channels provides SNR gain of N_t [9].

On the other hand, in SISO multipath channels, a prefilter, that is a filter at transmitter side, matched to the realization of the channel impulse response (CIR), which corresponds to time reversal, can focus in time the signal received: a transceiver using time reversal and One-finger Rake achieves same performance in terms of SNR as that obtained without time reversal and with All-Rake [3]. This is useful when transmitter cost and complexity are not crucial as opposed to receiver simplicity. However, time reversal can also provide SNR gain when more complex receiver is used, that is when more than one finger is employed in Rake.

In this paper we aim at providing theoretical results on MISO time reversal and extending known analyses proposed for SISO time reversal [3], [5], [6], [11] to MISO time reversal; in particular, we study the SNR gain due to time reversal beamforming and investigate the impact of the presence of more fingers in the Rake receiver when multiple antennas at transmitter are used.

The paper is organized as follows. In Section II it is described the continuous-time system with realistic channel model and it is proposed a simplified tractable discrete-time channel model. In Section III, analytical and simulation results are derived. Finally, conclusions are drawn in Section IV.

II. System model

The basic equation of a binary PAM-UWB signal is the following:

$$s(t) = \sum_{m \in \mathbb{Z}} \mathsf{b}[m] p(t - mT_{\mathsf{s}} - c_m T_c) \tag{1}$$

where $s(t)$ is the signal sent by the transmitter, c_m is the pseudorandom user-specific time hopping code, $\mathsf{b}[m] \in \{\pm \mathsf{A}\}$, where $\mathsf{A} > 0$, is the m-th symbol sent at time interval T_{s}, and T_c is called chip period [2]. We have used here Pulse Amplitude Modulation (PAM) but also Pulse Position Modulation [2] can be adopted. When no time reversal is used, $p(t)$ is a transmitted pulse of very short duration denoted in this paper by $\psi(t)$, typically a Scholtz pulse or a bandlimited pulse. Conversely, when time reversal is used, $p(t)$ is the output of a linear time-invariant filter called *time reversal precoder* or *prefilter* with input waveform $\psi(t)$.

Single-user MISO communication with N_t antennas at transmitter side is considered. We focus on the reception of one symbol only, where each symbol is transmitted by amplitude modulating $p(t)$. Under this assumption, ISI is negligible since

978-1-4799-0969-8/13/$31.00 ©2013 IEEE

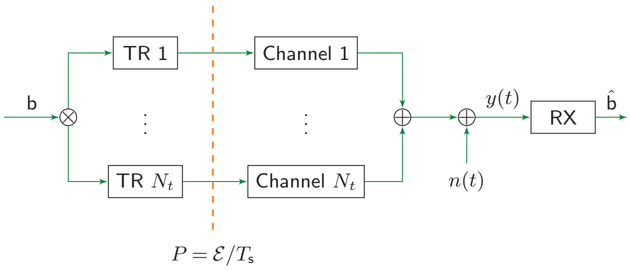

FIG. 1: System model: MISO-TR. N_t antennas are fed by N_t time reversal prefilters, each of which is matched to the corresponding multipath channel. A correlation receiver is adopted in order to estimate the transmitted symbol. Partial TR and Rake can be used in order to reduce complexity of the transceiver.

symbol period T_s is typically longer than channel delay spread and time-hopping code can ensure a time guard.

Received signal $y(t)$ can be written as follows, where we omitted the time-hopping code because of the one-symbol assumption:

$$y(t) = \mathsf{b} \int_{\mathbb{R}} \boldsymbol{h}^\mathsf{T}(t-\tau)\, \boldsymbol{p}(\tau)\, d\tau + n(t), \tag{2}$$

where $\boldsymbol{h}(t)$ and $\boldsymbol{p}(t)$ are the $N_t \times 1$ channel impulse response (CIR) and transmitted signal real-valued vectors, $n(t)$ is an Additive White Gaussian Noise (AWGN) with power spectral density σ_N^2, and b is the transmitted symbol. Note that all antennas transmit the same symbol during a symbol period. The i-th component of $\boldsymbol{h}(t)$ and $\boldsymbol{p}(t)$ is denoted by $h_i(t)$ and $p_i(t)$. Power constraint is intended as follows:

$$\frac{1}{T_\mathsf{s}} \int_{\mathbb{R}} \mathsf{A}^2 \|\boldsymbol{p}(t)\|^2\, dt = \frac{1}{T_\mathsf{s}} \sum_{i=1}^{N_t} \int_{\mathbb{R}} \mathsf{A}^2 p_i^2(t)\, dt \le P,$$

where T_s is the symbol period and P is the power spent by the N_t antennas, and, therefore, by the transmitter.

Given the large bandwidth of UWB systems, the propagation channel is resolved into multiple paths. In general, knowledge of CIR realizations at transmitter side can be exploited for prefilter design; MISO-TR design adopts a bank of N_t prefilters, where each antenna element is fed by the output of a prefilter modulated by b and matched to the CIR between that antenna element and the receiver, hence:

$$p_n(t) = \int_{\mathbb{R}} \psi(\tau) h_n(\tau - t)\, d\tau = (\psi * h_n^\mathsf{r})(t), \tag{3}$$

where $h_n^\mathsf{r}(t) = h_n(-t)$; denoting by $\mathcal{E}$ the maximum energy spent by the transmitter during a symbol period, power constraint reads as:

$$\mathsf{A}^2 \sum_{n=1}^{N_t} \int_{\mathbb{R}} \left|\psi * h_n^\mathsf{r}(t)\right|^2 dt \le P T_\mathsf{s} \triangleq \mathcal{E}.$$

Since the multipath channel $h_i(t)$ can be written as:

$$h_i(t) = \sum_{k \ge 0} a_k^i\, \delta(t - \tau_k^i),$$

partial time reversal prefilters can be also designed, where just a subset of paths is considered [5].

Finally, receiver is assumed to be a matched filter that projects $y(t)$ onto the following template:

$$\int_{\mathbb{R}} \boldsymbol{h}^\mathsf{T}(t-\tau)\, \boldsymbol{p}(\tau)\, d\tau \equiv \sum_{k \ge 0} \tilde{a}_k \psi(t - t_k).$$

where $\{\tilde{a}_k, t_k \colon k \ge 0\}$ represents the set of amplitudes and delays of fingers in the Rake. However, we can also consider a partial Rake receiver where only a subset of $\{\tilde{a}_k, t_k \colon k \ge 0\}$ is known and used: when the strongest paths only are considered, the receiver is known as *Selective-Rake*, whereas when all paths are considered, it is known as *All-Rake*.

A. Channel Model: Continuous-Time

We consider for simulation results the channel proposed in [8] that combines the standard SISO channel IEEE 802.15.3a [7] with the Kronecker channel correlation model for MIMO. We briefly summarize below this model in the special case of MISO and uniform linear array.

Channel impulse response between the i-th transmit antenna and the receiver is as follows:

$$h_i(t) = \sum_{\ell, k \ge 0} a_{k,\ell}^i \delta(t - T_\ell - \tau_{k,\ell} - \Delta^i)\delta(\phi^\mathsf{t} - \phi_{k,\ell}^\mathsf{t})\delta(\phi^\mathsf{r} - \phi_{k,\ell}^\mathsf{r}),$$

where $a_{k,\ell}^i$ is the signed amplitude of ray k in cluster ℓ, T_ℓ is cluster ℓ delay, $\tau_{k,\ell}$ is the delay of ray k in cluster ℓ with respect to T_ℓ, $\Delta^i = (i-1)d/c$ is the incremental delay due to antenna i position where d is the distance between two antennas and c is the speed of light, and $\phi_{k,\ell}^\mathsf{t}$ and $\phi_{k,\ell}^\mathsf{r}$ are the angle-of-departure and angle-of-arrival of ray k in cluster ℓ, respectively. Amplitude of ray k in cluster ℓ for CIR i has the form: $a_{k,\ell}^i = z_{k,\ell} \xi_\ell \beta_{k,\ell}^i$, where $z_{k,\ell}$ is the equiprobable ± 1 amplitude sign due to random reflections, and ξ_ℓ and $\beta_{k,\ell}^i$ are lognormal distributed cluster and ray fading, respectively. Amplitudes and delays statistics are in accordance with the standard IEEE 802.15.3a SISO channel. Multiple transmit antennas correlation is taken into account by assigning a correlation structure to $\boldsymbol{a}_{k,\ell}^\mathsf{T} = (a_{k,\ell}^1, \ldots, a_{k,\ell}^{N_t})$, that is by requiring that $\mathsf{E}[\boldsymbol{a}_{k,\ell} \boldsymbol{a}_{k,\ell}^\mathsf{T}] = \boldsymbol{\Sigma}_{k,\ell}^{\boldsymbol{a}}$. Since only $\beta_{k,\ell}^i = 10^{(\mu_{k,\ell} + \nu_{k,\ell}^i)/20}$ in $a_{k,\ell}^i$ depends on the antenna, we just need to assign a correlation structure to $(\beta_{k,\ell}^i)_{i=1}^{N_t}$, and, in turn, to $\boldsymbol{\nu}_{k,\ell} = (\nu_{k,\ell}^1, \ldots, \nu_{k,\ell}^{N_t})^\mathsf{T}$. Since $\nu_{k,\ell}^i$ is Gaussian, then $\boldsymbol{\nu}_{k,\ell}$ is a zero mean multivariate normal distribution with covariance matrix $\boldsymbol{\Sigma}_{k,\ell}^{\boldsymbol{\nu}}$ that can be derived once fixed $\boldsymbol{\Sigma}_{k,\ell}^{\boldsymbol{a}}$. For further details refer to [8].

B. Simplified Discrete-Time Channel Model

A more tractable channel model for analytical derivations is described below under the assumption of unit-energy, ideal baseband W-bandlimited pulse $\psi(t)$ with band $[-\mathsf{W}/2, \mathsf{W}/2]$.

1) SISO model: Single-Input Single-Output frequency-selective continuous-time AWGN channels can be described by the relation:

$$y(t) = \sum_{n=0}^{N-1} h(t) * \mathsf{b}[n]\psi(t - nT_\mathsf{s}) + n(t),$$

where $\{\mathsf{b}[n]\colon 0 \le n \le N-1\}$ is the set of transmitted symbols, $\psi(t)$ is the bandlimited waveform, T_s is the symbol period, $h(t)$ is the channel impulse response, and $n(t)$ is a realization of the continuous-time AWGN process with power spectral density σ_N^2.

In impulse-radio ultra-wideband communications, adjacent symbols are separated in time by an interval much longer than $1/\mathsf{W}$: in fact, $1/\mathsf{W}$ is shorter than the chip period and each symbol period counts usually tens of chips. Therefore, called $M = T_\mathsf{s}\mathsf{W}$, previous relation can be written as follows:

$$y(t) = \sum_{m=0}^{NM-1} h(t) * \tilde{\mathsf{b}}[m]\psi(t - m/\mathsf{W}) + n(t),$$

where $\tilde{\mathsf{b}}[nM] = \mathsf{b}[n]$ for $0 \le n \le N-1$, and $\tilde{\mathsf{b}}[i] = 0$ when i is not a multiple of M. Since one symbol only is considered, the channel model is as follows:

$$y(t) = \sum_{m=0}^{M-1} h(t) * \tilde{\mathsf{b}}[m]\psi(t - m/\mathsf{W}) + n(t).$$

At receiver, projecting this signal onto $\{\psi(t-m/\mathsf{W})\colon m \in \mathbb{Z}\}$ yields the following discrete-time channel:

$$y_k = \sum_{m\in\mathbb{Z}} h_{k-m}x_m + n_k,$$

where $x_m \coloneqq \tilde{\mathsf{b}}[m]$, $\{n_k\colon k \in \mathbb{Z}\}$ is a set of i.i.d. Gaussian random variables with variance σ_N^2, and

$$h_\ell \coloneqq \frac{1}{\sqrt{\mathsf{W}}} \int_{\mathbb{R}} h(t)\psi(t - \ell/\mathsf{W})\,dt.$$

Typically, most of the energy is included in the first paths, say $0 \le \ell \le L$. Without loss of generality, $L \ll M$ since M can be made as large as needed. Since $x_m = 0$ for $m < 0$ and $m > M-1$, then

$$y_k \approx \sum_{m=0}^{M-1} h_{k-m}x_m + n_k,$$

and this relation can be written in vector form as follows:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n},$$

where $\boldsymbol{H}$ is a $(M+L) \times M$ Toeplitz matrix with first column equal to $(h_0, h_1, \dots, h_L, 0, \dots, 0)^\mathsf{T}$, $\boldsymbol{x} = (x_i)_{i=0}^{M-1}$ and $\boldsymbol{n} = (n_i)_{i=0}^{M+L}$. One-shot communication implies that $x_m = \mathsf{b}[0]\delta_{m,0}$, therefore $\boldsymbol{x} = \mathsf{b}[0]\boldsymbol{e}_1$, where $\boldsymbol{e}_i$ is the $M \times 1$ vector with all zero entries but i-th element equal to 1. Note that $\mathsf{b}[0]$ is equal to b of eq. (2), therefore hereinafter we will denote it with b. Power constraint is expressed as $\mathsf{E}\big[\,\|\boldsymbol{x}\|^2\,\big] = \mathsf{A}^2 \le \mathcal{E}$.

Note that we can consider $\boldsymbol{y}$ as the concatenation of two vectors, $\overline{\boldsymbol{y}}$ and $\underline{\boldsymbol{y}}$, such that $\boldsymbol{y} = [\overline{\boldsymbol{y}}^\mathsf{T}\,\underline{\boldsymbol{y}}^\mathsf{T}]^\mathsf{T}$, where $\overline{\boldsymbol{y}}$ is $M \times 1$ and $\underline{\boldsymbol{y}}$ is $L \times 1$. Similarly, $\boldsymbol{H} = [\overline{\boldsymbol{H}}^\mathsf{T}\underline{\boldsymbol{H}}^\mathsf{T}]^\mathsf{T}$ and $\boldsymbol{n} = [\overline{\boldsymbol{n}}^\mathsf{T}\,\underline{\boldsymbol{n}}^\mathsf{T}]^\mathsf{T}$, where $\overline{\boldsymbol{H}}$ is $M \times M$ and $\underline{\boldsymbol{H}}$ is $L \times M$. Since $M \gg L$, we can neglect $\underline{\boldsymbol{y}}$. Prefiltering can be accounted by a $M \times M$ matrix $\boldsymbol{P}$ as follows: $\overline{\boldsymbol{y}} = \overline{\boldsymbol{H}}\boldsymbol{P}\boldsymbol{x} + \overline{\boldsymbol{n}}$. With prefiltering, transmitted vector is $\boldsymbol{P}\boldsymbol{x}$, therefore power constraint is $\mathsf{E}\big[\,\|\boldsymbol{P}\boldsymbol{x}\|^2\,\big] = \mathsf{A}^2\|\boldsymbol{p}\|^2 \le \mathcal{E}$, being $\boldsymbol{p}$ the first column of $\boldsymbol{P}$. Since the prefilter is a linear time-invariant system, $\boldsymbol{P}$ is Toeplitz and $\boldsymbol{p}$ is the vector containing projections of the prefilter impulse response $p(t)$ onto $\{\psi(t - m/\mathsf{W})\colon 0 \le m \le M-1\}$.

In order to provide a tractable model for the MISO channel, we propose first a simplified channel model for the SISO channel, which helps in illustrating the behavior of the system with and without TR. For SISO NLOS channels, it seems reasonable to consider $\{h_\ell\colon 0 \le \ell \le L\}$ independent, zero-mean Gaussian random variables with variance profile $\{v_\ell\colon 0 \le \ell \le L\}$, *i.e.*, $\mathsf{Var}\,[\,h_\ell\,] = v_\ell$. Therefore, the simplest model we propose is: $h_\ell = \sqrt{v_\ell}h_w(\ell)$, where $\{h_w(\ell)\colon 0 \le \ell \le L\}$ are i.i.d. unit variance Gaussian random variables. Furthermore, when the channel has also a LOS component, we propose a generalization that reads as follows:

$$h_\ell = \sqrt{v_\ell}\left\{\sqrt{\frac{\kappa}{1+\kappa\rho}}\delta_{\ell,0} + \sqrt{\frac{1}{1+\kappa\rho}}h_w(\ell)\right\},$$

where κ, similarly to the “Ricean factor” in flat-fading channels, accounts for the fraction of energy in the LOS component with respect to the NLOS component *in the first path*, and ρ accounts for the fraction of energy of the first path with respect to the sum of the energy of all paths, namely $\rho = v_0/\mathsf{G}$, where $\mathsf{G} = \sum_{\ell\ge 0} v_\ell$ is the total channel gain.

2) MISO model: The discrete-time model is as follows:

$$y(k) = \sum_{m=0}^{M-1} \boldsymbol{h}^\mathsf{T}(k-m)\boldsymbol{x}(m) + n(k),$$

where $\boldsymbol{h}(m)$ is a $N_t \times 1$ vector where the i-th element is the channel impulse response between the i-th transmit antenna and the receiver projected onto $\psi(t-m/\mathsf{W})/\sqrt{\mathsf{W}}$, and $\boldsymbol{x}(m) = \mathsf{b}\mathbf{1}\delta_{m,0}$, where $\mathbf{1}$ is a $N_t \times 1$ vector of ‘1’. Stacking $\{y(k)\colon 0 \le k \le M-1\}$ in a vector, $\overline{\boldsymbol{y}}$, the previous relation can be written as:

$$\overline{\boldsymbol{y}} = \overline{\boldsymbol{H}}\boldsymbol{x} + \overline{\boldsymbol{n}}, \quad \text{with} \quad \boldsymbol{x} = \boldsymbol{e}_1 \otimes \mathbf{1}\mathsf{b}, \tag{4}$$

where $\boldsymbol{x}$ is a $MN_t \times 1$ vector of symbols and $\overline{\boldsymbol{H}}$ is a $M \times MN_t$ block-Toeplitz channel matrix structured as follows:

$$\overline{\boldsymbol{H}} = \sum_{m=0}^{M-1} \boldsymbol{J}_m \otimes \boldsymbol{h}^\mathsf{T}(m) = \sum_{\ell=0}^{L} \boldsymbol{J}_\ell \otimes \boldsymbol{h}^\mathsf{T}(\ell), \tag{5}$$

where $\boldsymbol{J}_m$ is a $M \times M$ matrix with ones on the m-th subdiagonal. Power constraint is: $\mathsf{E}\big[\,\|\boldsymbol{x}\|^2\,\big] = N_t\mathsf{A}^2 \le \mathcal{E}$. Prefiltering can be accounted by a $MN_t \times MN_t$ block-Toeplitz matrix $\boldsymbol{P}$ structured as follows:

$$\boldsymbol{P} = \sum_{m=0}^{M-1} \boldsymbol{J}_m \otimes \boldsymbol{P}(m). \tag{6}$$

Thus, the channel assumes the following form: $\overline{\boldsymbol{y}} = \overline{\boldsymbol{H}}\boldsymbol{P}\boldsymbol{x}+\overline{\boldsymbol{n}}$; from eq. (4), we have:

$$\overline{\boldsymbol{y}} = \overline{\boldsymbol{H}}\boldsymbol{p}\mathsf{b} + \overline{\boldsymbol{n}}. \tag{7}$$

where:

$$\boldsymbol{p} \coloneqq \boldsymbol{P}(\boldsymbol{e}_1 \otimes \mathbf{1}) = \sum_{m=0}^{M-1} \boldsymbol{J}_m \boldsymbol{e}_1 \otimes \boldsymbol{P}(m)\mathbf{1} \equiv \sum_{m=1}^{M} \boldsymbol{e}_m \otimes \boldsymbol{p}(m),$$

having denoted $\boldsymbol{p}(m+1) \coloneqq \boldsymbol{P}(m)\mathbf{1}$. In general, $\boldsymbol{P}(m)$ jointly precodes symbols for all antennas; in MISO-TR prefilters are decoupled, that is, each antenna is fed by the output of a prefilter knowing the channel impulse response between that antenna and the receiver only. This structure implies that $\boldsymbol{P}(m)$ is diagonal, therefore also uniquely defined by a vector that we call $\boldsymbol{p}(m+1)$, that is consistent with previous notation:

$$\boldsymbol{P}(m) = \mathsf{diag}(\boldsymbol{p}(m+1)).$$

Power constraint reads:

$$\mathsf{E}\left[\,\|\boldsymbol{P}\boldsymbol{x}\|^2\,\right] = \mathsf{A}^2\|\boldsymbol{p}\|^2 \leq \mathcal{E}. \tag{8}$$

In order to derive analytical results, we propose the following tractable simplified channel model for MISO channel, that similarly to SISO channels is defined as:

$$\boldsymbol{h}^{\mathsf{T}}(\ell) = \sqrt{v_\ell}\left\{\sqrt{\frac{\kappa}{1+\kappa\rho}}\mathbf{h}_0^{\mathsf{T}}\delta_{\ell,0} + \sqrt{\frac{1}{1+\kappa\rho}}\boldsymbol{h}_w^{\mathsf{T}}(\ell)\boldsymbol{\Theta}_t^{1/2}\right\},$$

where matrix $\boldsymbol{\Theta}_t$ is introduced as in the Kronecker model in order to account correlation between channels of different antennas, $\mathbf{h}_0$ is a deterministic component that account for the line-of-sight path, and $\boldsymbol{h}_w(\ell) \sim \mathcal{N}(\mathbf{0}, \boldsymbol{I})$. We want that $\mathsf{E}\left[\,\|\boldsymbol{h}(\ell)\|^2\,\right] = N_t v_\ell$, therefore we set $\mathsf{Tr}\,\boldsymbol{\Theta}_t = N_t$ and $\|\mathbf{h}_0\|^2 = N_t$. In this way, correlation matrix of $\boldsymbol{h}_w(\ell)$ is as follows:

$$\mathsf{E}\Big[\boldsymbol{h}(\ell)\boldsymbol{h}^{\mathsf{T}}(\ell)\Big] = v_\ell \cdot \left\{\frac{\kappa}{1+\kappa\rho}\mathbf{h}_0\mathbf{h}_0^{\mathsf{T}}\delta_{\ell,0} + \frac{1}{1+\kappa\rho}\boldsymbol{\Theta}_t\right\}. \tag{9}$$

A generalization to MIMO would be straightforward, taking into account also correlations at receiver antennas with a matrix $\boldsymbol{\Theta}_r$. However, since in this paper we investigate MISO channels, we denote hereinafter $\boldsymbol{\Theta}_t$ by $\boldsymbol{\Theta}$, being the subscript redundant.

C. Performance measure

The key performance measure that we are interested in is SNR. The problem is to estimate b from eq. (7) through a linear receiver, that is a $M \times 1$ vector $\boldsymbol{w}$ onto which $\overline{\boldsymbol{y}}$ is projected. We want to compute the SNR of the decision variable $z = \boldsymbol{w}^{\mathsf{T}}\overline{\boldsymbol{y}}$, that is:

$$z = \boldsymbol{w}^{\mathsf{T}}\overline{\boldsymbol{H}}\boldsymbol{p}\mathsf{b} + \boldsymbol{w}^{\mathsf{T}}\overline{\boldsymbol{n}} \eqqcolon \zeta + \nu,$$

where $\zeta \coloneqq \boldsymbol{w}^{\mathsf{T}}\overline{\boldsymbol{H}}\boldsymbol{p}\mathsf{b}$ and $\nu \coloneqq \boldsymbol{w}^{\mathsf{T}}\overline{\boldsymbol{n}}$. Therefore, SNR of z is as follows:

$$\mathsf{SNR} \coloneqq \frac{\mathsf{E}[\,\zeta\,]}{\mathsf{E}[\,\nu\,]} = \frac{\mathsf{A}^2}{\sigma_N^2} \cdot \frac{(\boldsymbol{w}^{\mathsf{T}}\overline{\boldsymbol{H}}\boldsymbol{p})^2}{\|\boldsymbol{w}\|^2},$$

where expectations are taken conditioned on the channel realization. This quantity is, therefore, still a random variable, since the channel is a random process. We are primary interested in its mean value, $\mathsf{E}[\,\mathsf{SNR}\,]$.

III. Results

A. Analytical Derivations

We derive expressions of SNR in terms of channel correlation, receiver number of fingers and transmitter number of antennas. In particular, we compute the two opposed cases of strongly correlated and independent channels, characterized by $\boldsymbol{\Theta} = \mathbf{1}\mathbf{1}^{\mathsf{T}}$ and $\boldsymbol{\Theta} = \boldsymbol{I}$, respectively.

1) All-Rake ($\boldsymbol{w} = \overline{\boldsymbol{H}}\boldsymbol{p}$), General Precoder:

$$\begin{aligned}\mathsf{SNR} &= \frac{\mathsf{A}^2}{\sigma_N^2} \cdot \|\overline{\boldsymbol{H}}\boldsymbol{p}\|^2 \\ &= \frac{\mathsf{A}^2}{\sigma_N^2} \cdot \left\|\sum_{\ell=0}^{M-1}\sum_{m=1}^{M} \boldsymbol{e}_{m+\ell} \otimes \boldsymbol{h}^{\mathsf{T}}(\ell)\boldsymbol{p}(m)\right\|^2,\end{aligned}$$

intending that the sum is extended over $1 \leq m + \ell \leq M$.

2) All-Rake ($\boldsymbol{w} = \overline{\boldsymbol{H}}\boldsymbol{p}$), No Precoding: With no precoding, $\boldsymbol{P} \propto \boldsymbol{I}$, therefore $\boldsymbol{P}(m) \propto \boldsymbol{I}_{N_t}\delta_{m,0}$. According to the power constraint, it is sufficient to specialize the general relation for SNR to:

$$\boldsymbol{p}(m) = \mathbf{1}\sqrt{\frac{\mathcal{E}}{N_t\mathsf{A}^2}}\delta_{m,1},$$

that yields:

$$\mathsf{SNR} = \frac{\mathcal{E}}{N_t\sigma_N^2}\sum_{\ell=0}^{L}\left(\boldsymbol{h}^{\mathsf{T}}(\ell)\mathbf{1}\right)^2. \tag{10}$$

Note that, in the SISO case, this reduces to the well-known result $\mathsf{SNR} = (\mathcal{E}/\sigma_N^2)\sum_\ell h(\ell)^2$.

Using the tractable proposed channel model, the average SNR is:

$$\mathsf{E}[\,\mathsf{SNR}\,] = \frac{\mathcal{E}}{N_t\sigma_N^2}\sum_{\ell=0}^{L}\mathbf{1}^{\mathsf{T}}\,\mathsf{E}\left[\,\boldsymbol{h}(\ell)\boldsymbol{h}(\ell)^{\mathsf{T}}\,\right]\mathbf{1},$$

that, using eq. (9), is:

$$\begin{aligned}\mathsf{E}[\,\mathsf{SNR}\,] &= \frac{\mathcal{E}}{N_t\sigma_N^2}\sum_{\ell=0}^{L} v_\ell\left\{\frac{\kappa}{1+\kappa\rho}(\mathbf{h}_0^{\mathsf{T}}\mathbf{1})^2\delta_{\ell 0} + \frac{1}{1+\kappa\rho}\mathbf{1}^{\mathsf{T}}\boldsymbol{\Theta}\mathbf{1}\right\} \\ &= \frac{\mathcal{E}\,\mathsf{G}}{N_t\sigma_N^2}\left\{\frac{\kappa\rho}{1+\kappa\rho}\cdot\mathsf{H} + \frac{1}{1+\kappa\rho}\cdot\theta\right\}.\end{aligned} \tag{11}$$

having set $\mathsf{H} = (\mathbf{h}_0^{\mathsf{T}}\mathbf{1})^2$, $\theta = \mathbf{1}^{\mathsf{T}}\boldsymbol{\Theta}\mathbf{1}$ and $\mathsf{G} = \sum_{\ell=0}^{L} v_\ell$.

Let analyze particular cases. Assume deterministic LOS component equal for all antennas, that is $\mathbf{h}_0 = \mathbf{1}$, then $\mathsf{H} = N_t^2$. For strongly correlated channels, namely for $\boldsymbol{\Theta} = \mathbf{1}\mathbf{1}^{\mathsf{T}}$, we have $\theta = N_t^2$. In this case:

$$\mathsf{E}[\,\mathsf{SNR}\,] = \frac{\mathcal{E}\mathsf{G}}{\sigma_N^2}N_t.$$

For independent channels, that is $\boldsymbol{\Theta} = \boldsymbol{I}$, we have $\theta = N_t$, and therefore:

$$\mathsf{E}[\,\mathsf{SNR}\,] = \frac{\mathcal{E}\,\mathsf{G}}{\sigma_N^2}\left\{\frac{\kappa\rho}{1+\kappa\rho}\cdot N_t + \frac{1}{1+\kappa\rho}\right\}.$$

3) 1-Rake, time reversal: A 1-Rake is $\boldsymbol{w} = \boldsymbol{e}_i$, where in the case of time reversal $i = L+1$. Time reversal is the use of the precoder:

$$\boldsymbol{p}^{\star} = \frac{\sqrt{\mathcal{E}}}{\mathsf{A}} \cdot \frac{1}{\|\boldsymbol{h}^{\mathsf{r}}\|} \boldsymbol{h}^{\mathsf{r}},$$

where $\boldsymbol{h}^{\mathsf{r}} = \boldsymbol{c}^{L+1}$ and $\boldsymbol{c}^{L+1}$ is the $(L+1)$-th row of $\overline{\boldsymbol{H}}$. We can prove that this precoder maximizes the SNR when One-finger Rake is used.

In fact, consider One-finger Rake $\boldsymbol{w} = \boldsymbol{e}_i$, $1 \leq i \leq M$. Since $\boldsymbol{e}_i^{\mathsf{T}} \overline{\boldsymbol{H}} \boldsymbol{p} = \boldsymbol{c}^{i\mathsf{T}} \boldsymbol{p}$, where $\boldsymbol{c}^i$ is the i-th row of $\overline{\boldsymbol{H}}$, the problem is that of finding the precoding vector:

$$\boldsymbol{p}^{\star} = \arg \max_{\boldsymbol{p}} \mathsf{SNR} = \arg \max_{\boldsymbol{p}} (\boldsymbol{c}^{i\mathsf{T}} \boldsymbol{p})^2,$$

subject to power constraint of eq. (8): $\mathsf{A}^2 \|\boldsymbol{p}\|^2 \leq \mathcal{E}$. This problem is solved, using for instance the Cauchy-Schwarz inequality, by $\boldsymbol{p}_i^{\star} \propto \boldsymbol{c}^i$, that is time reversal. Each $i \geq L+1$ is sufficient for taking into account the whole delay spread of channels, therefore exploiting the whole gain offered. For the sake of simplicity, we choose $i = L+1$.

With the time reversal precoder, achieved SNR is:

$$\mathsf{SNR} = \frac{\mathcal{E}}{\sigma_N^2} \|\boldsymbol{h}^{\mathsf{r}}\|^2 = \frac{\mathcal{E}}{\sigma_N^2} \sum_{\ell=0}^{L} \|\boldsymbol{h}(\ell)\|^2. \tag{12}$$

A well-known result in SISO channels is that this SNR is equal to that obtained with All-Rake receiver and no precoding. This is no longer true with MISO, since eq. (12) is not equivalent to eq. (10).

Using the tractable proposed channel model, the average value of (12) is:

$$\mathsf{E}[\,\mathsf{SNR}\,] = \frac{\mathcal{E}\mathsf{G}}{\sigma_N^2} N_t. \tag{13}$$

The meaning of this relation is that time reversal is insensitive, on average and in terms of SNR, to many parameters characterizing the channel as in (11). In particular, time reversal is robust to the lack of correlation as opposed to All-Rake without precoding: in fact, One-finger Rake, time reversal systems average SNR is larger than that achieved by systems with All-Rake receiver and without precoding, beiung equal only when channels between different transmit antennas and receiver are identical.

B. Simulation Results

In addition to analytical derivations and comparison of performance (in terms of SNR) of systems using time reversal with One-finger Rake vs. All-Rake without prefiltering, we investigate through simulations performance dependence on number of antennas, type of receiver (One-finger Rake vs. All-Rake) and number of taps in the time reversal prefilter. In fact, transmitter may also select a subset of channel paths to form TR: when one tap only is employed, we have no prefiltering; when the whole channel impulse response is taken into account, we have full TR; between this two extrema, when the number of taps in the precoder is limited, we have in general partial TR, and paths are selected with decreasing amplitude.

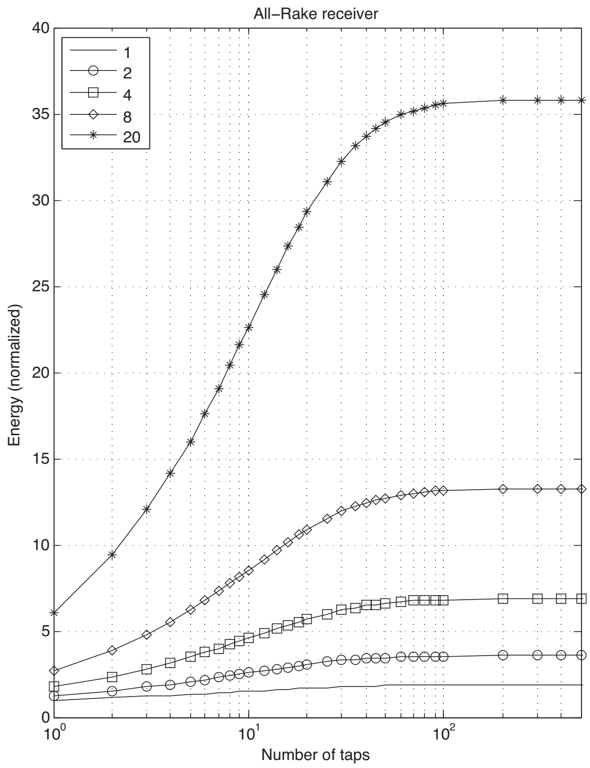

FIG. 2: Energy collected by an All-Rake normalized to the average energy collected without time reversal with single antenna as function of the number of taps of precoder.

TABLE I: Parameters used in simulations.

Channel Model	IEEE 802.15.3a-CH1
Pulse	Scholtz, $\tau = 200$ ps
Antenna distance (d)	0.1 m
Angle spread (Ω)	38°

Simulations are performed with the realistic channel described in Subsection II-A. Parameters used in simulations are reported in Table I.

In Figs. 2 and 3 it is shown the energy collected by an All-Rake and by a One-finger Rake, respectively, normalized to the average energy collected without time reversal with single antenna, as function of the number of paths considered in time reversal. In both cases, time reversal can be used for reducing the number of antennas while maintaining fixed the energy collected by the receiver. For instance, in Fig. 2, a system with 20 antennas without time reversal collects the same energy of a system with 4 antennas employing time reversal with 25 taps. Note that, moreover, energy gain due to time reversal remains roughly constant as function of the number of antennas, therefore the total gain due to multiple antennas and time reversal is roughly the product of each gain. A relative increase of focusing, although of small entity, is observed in Figs. 4 showing the percentage of collected energy as function of the number of fingers in the Rake receiver with time reversal. We confirmed by simulations the

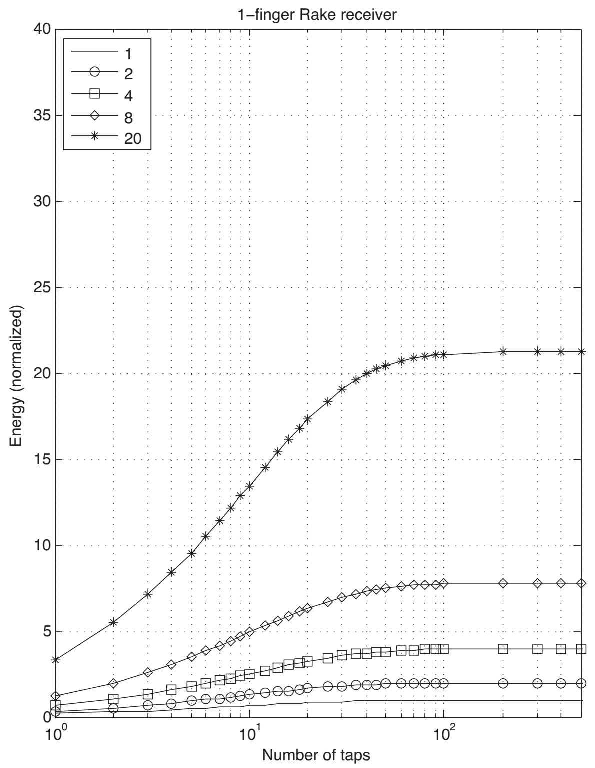

FIG. 3: Energy collected by a 1-Rake normalized to the average energy collected without time reversal with single antenna as function of the number of taps of precoder.

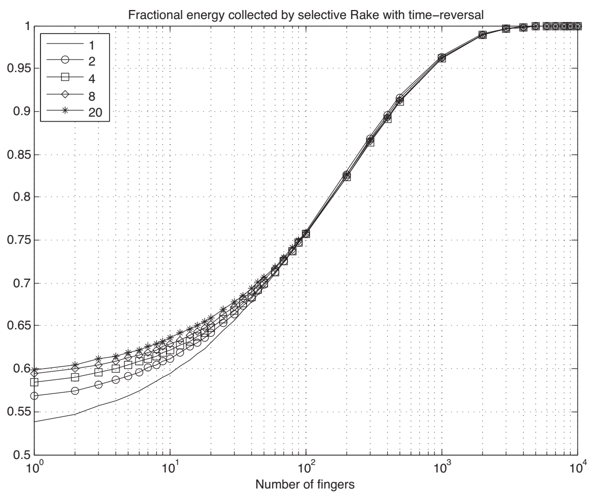

FIG. 4: Fractional energy: time reversal

better performance of time reversal with One-finger Rake with respect to no time reversal with All-Rake, as expected from the analyical derivation.

IV. CONCLUSIONS

In this paper we studied combination of time reversal and multiple transmit antennas providing theoretical arguments for robustness of communications using TR and simulation results emphasizing focusing properties of TR and possible trade-offs in system design. In general, thanks to its focusing properties, time reversal allows to use a One-finger Rake receiver in place of receivers requiring multiple fingers in systems without prefiltering. We proved that MISO-TR is optimum when a One-finger Rake receiver is used and that systems with no time reversal and All-Rake receiver are less robust to channels correlation with respect to systems with time reversal and One-finger Rake receiver, which performance in terms of average SNR is better. Simulation results with realistic channel model and analytical derivations with a more tractable channel model we proposed were presented. We observed that combination of MISO and TR amplify performance of both techniques used independently as well as increased focusing of MISO-TR with respect to SISO-TR; we might expect higher gains with a channel model considering the propagation property of waves on which time reversal design is based.

REFERENCES

[1] Akogun A.E., Qiu R.C., Guo N., "Demonstrating Time Reversal in Ultra-wideband Communications Using Time Domain Measurements," *51st International Instrumentation Symposium*, Knoxville, Tennessee, 8-12 May 2005.

[2] Di Benedetto M.-G., Giancola G., "Understanding Ultra Wide Band, Radio Fundamentals," *Prentice Hall Pearson Education*, 2004.

[3] De Nardis L., Fiorina J., Panaitopol D., Di Benedetto M.-G., "Combining UWB with Time Reversal for improved communication and positioning," *Telecommunication System Journal (Springer), Special Issue on Recent Advances In UWB Systems: Theory and Applications*, 2011, pp. 1-14.

[4] Derode A., Roux P., Fink M.,"Robust acoustic time reversal with high-order multiple scattering", *Phys. Rev. Letters*, 1995, pp. 4206-4209, vol.75

[5] Ferrante G. C., Fiorina J., Di Benedetto M.-G., "Complexity reduction by combining time reversal and IR-UWB," *2012 IEEE Wireless Communications and Networking Conference (WCNC)*, pp.28,31, 1-4 April 2012

[6] Fiorina, J., Capodanno G., Di Benedetto M.-G., "Impact of Time Reversal on multi-user interference in IR-UWB," *2011 IEEE International Conference on Ultra-Wideband (ICUWB)*, vol., no., pp.415,419, 14-16 Sept. 2011

[7] Foerster J., *Channel Modeling Sub-Committee report final*, IEEE Doc. P802.15-02/490r1-SG3a, 2003.

[8] Hong X., Wang C.-X., Allen B., Malik W.Q., "Correlation-based double-directional stochastic channel model for multiple-antenna ultra-wideband systems," *IET Antenna Systems and Propagation for Future Wireless Communications*, pp. 1182-1191, 2007.

[9] Larsson E. G., Stoica P., "Space-Time Block Coding for Wireless Communications," *Cambridge University Press*, 2003.

[10] Nguyen H. T., Andersen J. B., Pedersen G. F., Kyritsi P., Eggers P. C. F., "Time Reversal in Wireless Communications: A Measurement-Based Investigation," *IEEE Transactions on Wireless Communications*, vol. 5, no. 8, pp. 2242-2252, Aug 2006.

[11] Panaitopol D., Fiorina J., Di Benedetto M.-G. "Trade-off Between the Number of Fingers in the Prefilter and in the Rake Receiver in Time Reversal IR UWB," *IEEE International Conference on Ultra-Wideband, ICUWB '09*, Vancouver, Canada, 2009.

[12] Qiu R., Zhou C., Guo N., Zhang J. Q., "Time Reversal With MISO for Ultra-wideband Communications: Experimental Results," *IEEE Antennas and Wireless Propagation Letters,* vol. 5, pp. 269-273, 2006.

[13] Saghir H., Heddebaut M., Elbahhar F., Rivenq A., Rouvaen J.M., "Time-Reversal UWB Wireless Communication-Based Train Control in Tunnel," *Journal of Communications Academy Publishers*, vol.4, no.4, pp. 248-256, 2009.

[14] Zhou C., Guo N., Qiu R. C., "Time-Reversed Ultra-wideband (UWB) Multiple Input Multiple Output (MIMO) Based on Measured Spatial Channels," *IEEE Transactions on Vehicular Technology,* vol. 58, no. 6, pp. 2884-2898, 2009.