Practice #5 - October 15, 2021

Ultra Wide Band Radio Fundamentals

Spectral Characteristics of UWB Signals

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Outline

- PPM analog modulator
  - Deterministic modulation
  - Random modulation
- PPM-TH-UWB

Outline

PPM-TH similarity with analog theory

1. **PPM-TH-UWB**: if θ_k is the dither process:

$$x(t) = \sum_{k \in \mathbb{Z}} p(t - kT_s - \theta_k)$$

2. **PPM-analog modulator**: modulation with $m(t)$

$$x(t) = \sum_{k \in \mathbb{Z}} p(t - kT_s - m(kT_s))$$

If max $|m(t)| \leq T_s/2$, then the pulses are strictly non-overlapping.

1. **Deterministic** modulation.
2. **Random** modulation.

Deterministic modulation

CHECKPOINT 3–1

PPM-analog modulator:

$$x(t) = \sum_{k \in \mathbb{Z}} p(t - kT_s - m(kT_s))$$

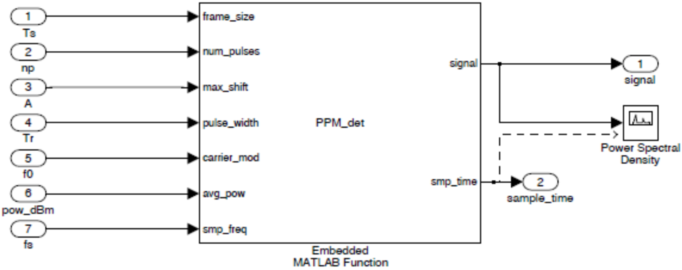
with modulating signal:

$$m(t) = A \cos(2\pi f_c t).$$

For the sake of simplicity, consider rect-pulse:

$$p(t) = \mathrm{rect}_{Tr}(t - T_r/2).$$

We shall generate the previous signal and evaluate its PSD.



The function declaration could be:

```
[signal,smp_time] = PPM_det(Ts,np,A,f0,Tr,pow,fs)
```

Parameters:

- $\mathbf{T_s}$ (mean pulse rep time): 2 [ns],
- **np** (number of pulses): 10^4,
- $\mathbf{T_r}$ (rect pulse-width): 0.5 [ns],
- $\mathbf{f_s}$ (sampling freq): 100 [GHz],
- **pow** (avg signal power): -30 [dBm].

Two cases:

1. **PPM-shift absence**:
   - **A** (max PPM-shift): 0 [s],
   - $\mathbf{f_0}$ (carrier freq): 0 [Hz].
2. **PPM-shift presence**:
   - **A** (max PPM-shift): 1 [ns],
   - $\mathbf{f_0}$ (carrier freq): 50 [MHz].

What to expect?

PSD explicit expression:

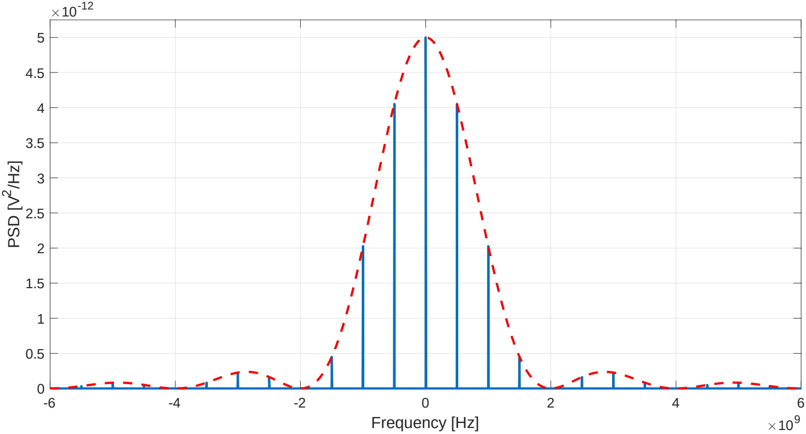
$$S_{XX}(f) = \frac{1}{T_s^2} \sum_{m\in\mathbb{Z}} \sum_{n\in\mathbb{Z}} \left| J_n(2\pi A f_{m,n}) \right|^2 \left| P(f_{m,n}) \right|^2 \delta(f - f_{m,n})$$

with $f_{m,n} = m/Ts + nf_0$.

Note that: $J_n(x) \simeq 0$ if $|n| > |x|$, so $|n| > 2\pi A \frac{m}{T_s}$

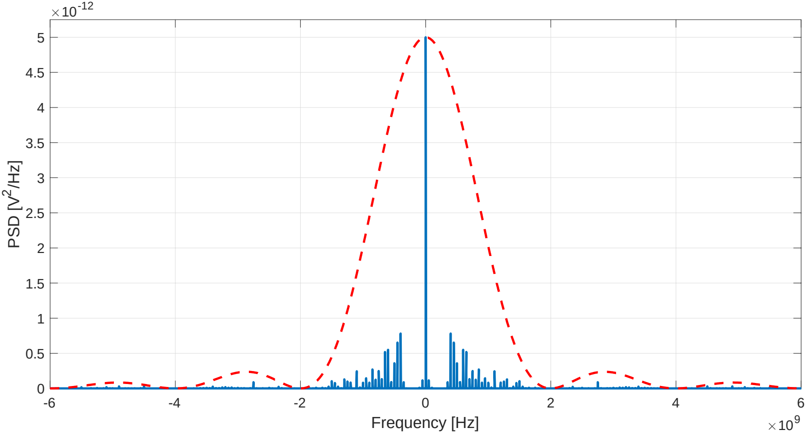
(so they are negligible in the summation)

1. **PPM-shift absence**:
   - **spectral-peaks**: lines due to the absence of modulation,
   - **PSD-envelope**: “sinc-like” (due to the pulse shape).



2. PPM-shift presence:

- **spectral-peaks**: more lines, located @ $f = f_{m,n}$ and organized in **clusters** centered @ m/T_s ,
- **PSD-envelope**: distortion of the “sinc-like” shape.



What is the effect of the Bessel functions of first kind?

From **PSD** expression:

$$P_{x_{PPM}}(f) = \frac{1}{T_S^2} \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \left| J_n\left(2\pi A\left(\frac{m}{T_S} + nf_0\right)\right)\right|^2 \left|P\left(\frac{m}{T_S} + nf_0\right)\right|^2 \delta\left(f - \left(\frac{m}{T_S} + nf_0\right)\right)$$

Consider:

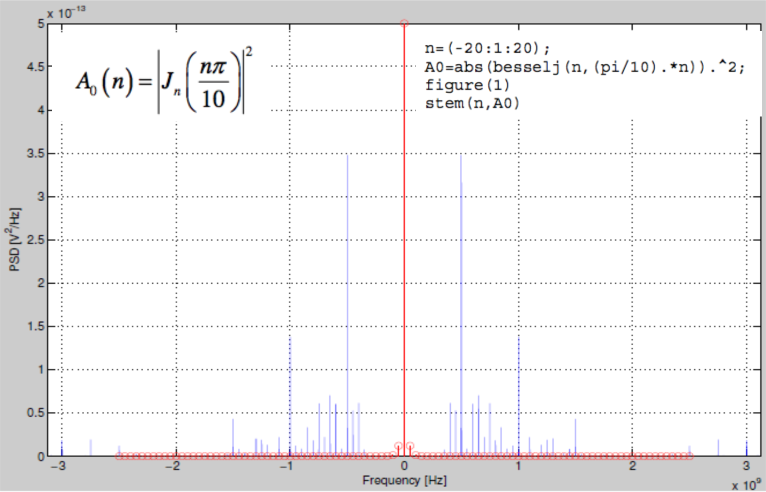
$$A = T_s/2, f_0 = 1/(10T_s)$$

$$J_n\left(2\pi \frac{T_S}{2}\left(\frac{m}{T_S} + \frac{n}{10T_S}\right)\right) = J_n\left(m\pi + \frac{n\pi}{10}\right)$$

$$A_0(n) = \left|J_n\left(\frac{n\pi}{10}\right)\right|^2 \qquad A_{+1}(n) = \left|J_n\left(\pi + \frac{n\pi}{10}\right)\right|^2 \qquad A_{-1}(n) = \left|J_n\left(-\pi + \frac{n\pi}{10}\right)\right|^2$$

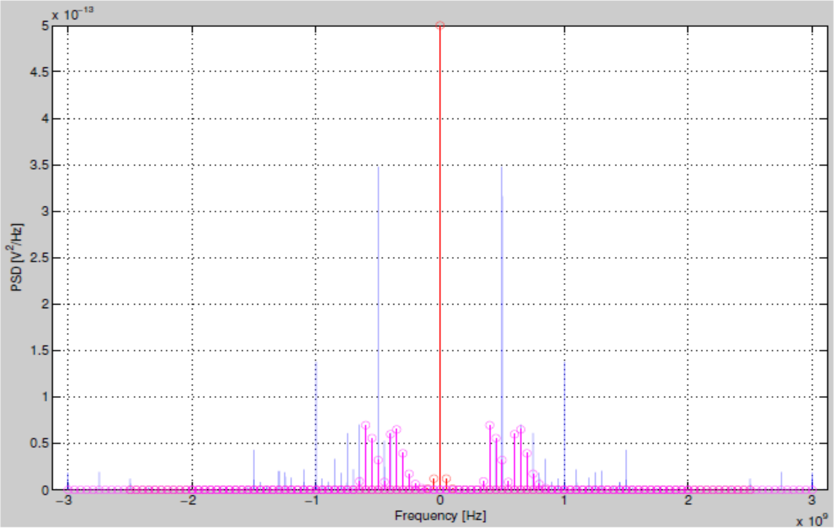
Bessel order-**zero** cluster

(J_n contribution for $m = 0$)



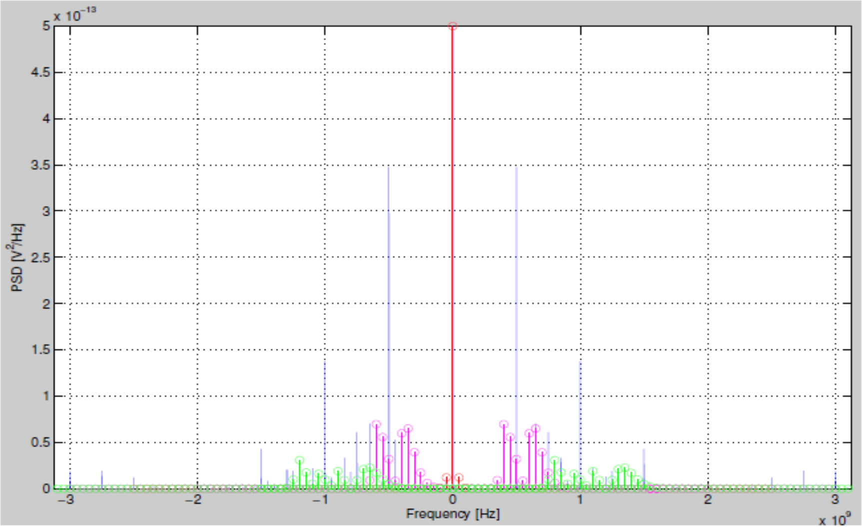
Bessel **first**-order cluster

(J_n contribution for $m = \pm 1$)



Bessel **second**-order cluster

(J_n contribution for $m = \pm 2$)



Random modulation

CHECKPOINT 3–3

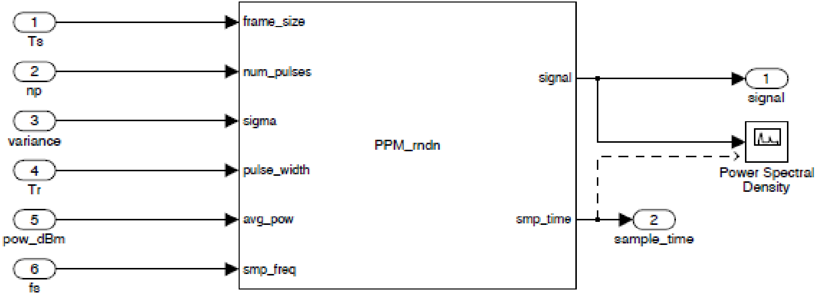
PPM-analog modulator:

$$x(t) = \sum_{k \in \mathbb{Z}} p(t - kT_s - m(kT_s))$$

where $m[k] := m(kT_s)$ are IID samples of the SSS continuous stochastic process $m(t)$.

We will choose $\boldsymbol{m[k] \sim N(0, \sigma^2)}$ and $\boldsymbol{p(t) = rect_{T_r}(t - T_r/2)}$.

We shall generate the previous signal and evaluate its PSD.



Embedded MATLAB Function

The function declaration could be:

```
[signal,smp_time] = PPM_rndn(Ts,np,sigma,Tr,pow,fs).
```

Parameters:

- $\mathbf{T_s}$ (mean pulse rep time): 2 [ns],
- **np** (number of pulses): 10^4,
- $\mathbf{T_r}$ (rect pulse-width): 0.5 [ns],
- $\mathbf{f_s}$ (sampling freq): 100 [GHz],
- **pow** (avg signal power): -30 [dBm].

We will choose: *sigma* (std dev of the gaussian) = 0.1 [ns].

What to expect?

PSD explicit expression:

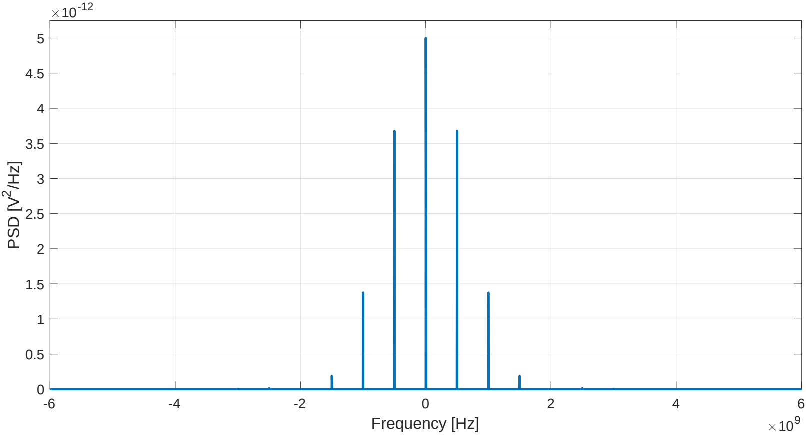
$$S_{XX}(f) = \frac{|P(f)|^2}{T_s}\left[1 - |W(f)|^2 + \frac{|W(f)|^2}{T_s}\,\mathrm{comb}_{\frac{1}{T_s}}(f)\right]$$

with $\mathrm{comb}_T(t) := \sum_{n\in\mathbb{Z}} \delta(t - nT)$.

Note the presence of both a continuous and a discrete component.

spectral-peaks: **discrete term** predominant, due to a lack of asynchrony (σ too small with respect to pulse rep time),

PSD-envelope: "**gaussian-like**" (no longer "sinc-like"), due to $|W(f)|^2$ (it determines the BW jointly with $P(f)$).



Outline

- PPM analog modulator
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- PPM-TH-UWB

PPM-TH-UWB

CHECKPOINT 3–4

PPM-TH-UWB:

$$x(t) = \sum_{k \in \mathbb{Z}} p(t - kT_s - \theta_k)$$

with dither process:

$$\theta_k = c_k T_c + a_k \varepsilon.$$

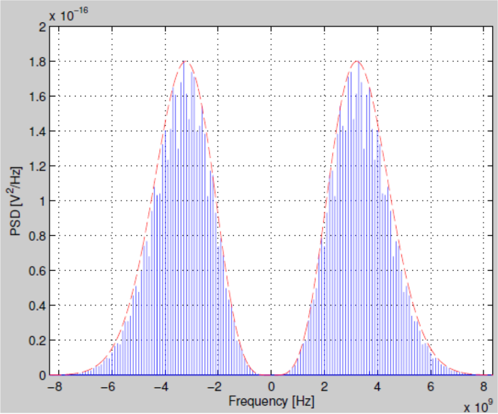
Depending on the values of ε and N_p (= code length), we can have three interesting cases:

1. **no PPM-shift, no TH-code**,
2. **no PPM-shift, TH-code**,
3. **PPM, TH-code**.

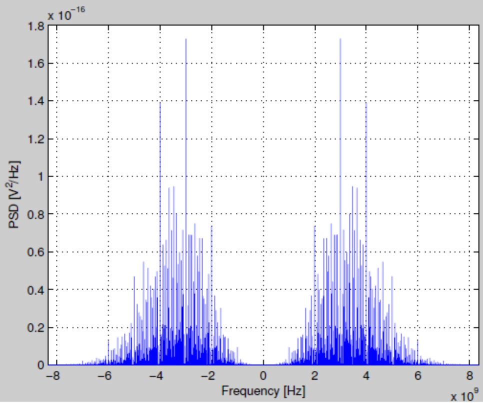
Parameters:

```
nbits  = 10;             % bitstream length
Ns = 5;                  % channel coder repetition factor
Np = 5;                  % Time Hopping code length
Nh = 5;                  % code max value
Tc = 1e-9;               % Chip time
Tf = 5e-9;               % frame time
dPPM =  0.25e-9;         % PPM shift
Tm = 0.9e-9;             % Pulse Duration [s]
powdBm = -30;            % avg Tx Power [dBm]
fs = 30e9;               % sampling frequency [GHz]
```

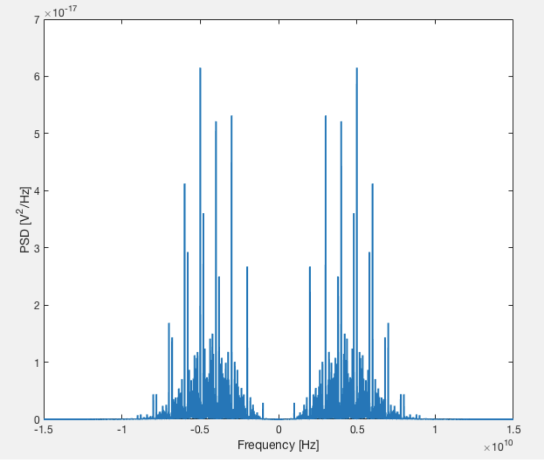
1. **no PPM-shift** ($\varepsilon = 0$), **no TH-code** ($N_p = 1$):
   periodic signal, so we expect spectral peaks @ m/T_s.
   - **spectral-peaks**: lines located @ $f = m/T_s$,
   - **PSD-envelope**: FT of 2nd derivative of a gaussian.



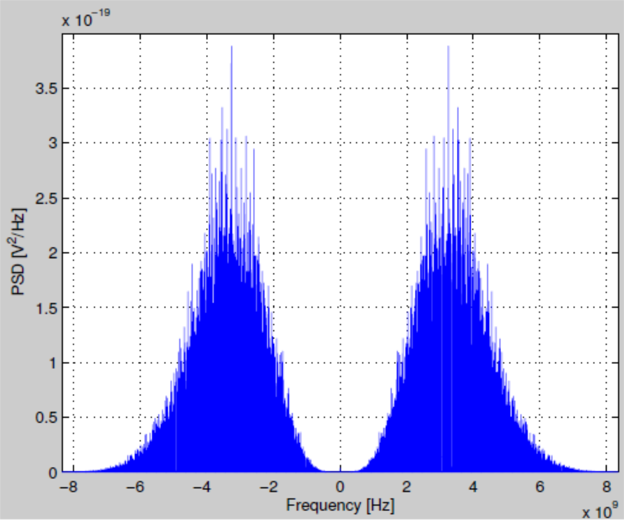
2. **no PPM-shift** ($\varepsilon = 0$), **TH-code** ($N_p = N_s = 5$):
   spectral peaks @ $m/(N_s T_s) = m/T_b$ due to the ($N_p = N_s$)–induced periodicity, thus much closer spectral lines (*whitening* effect of the code)
   - **spectral-peaks**: lines located @ $f = m/T_b$,
   - **PSD-envelope**: FT of 2[nd] derivative of a gaussian.



Even if the TH code reduces the number of peaks with high power contribution, it turns out that **it is not possible to remove all the peaks by only increasing the periodicity of the TH code.**
Check the case : Ns = 5, Np = 5e3.



However, we can remove the strong peaks in the PSD by increasing N_h (max TH code value).



3. dPPM=0.25e-9 s, Np=Nh=5