Practice #8 - November 05, 2021

Ultra Wide Band Radio Fundamentals

Pulse Shaping

DIET Department

SAPIENZA
UNIVERSITÀ DI ROMA

Outline

Gaussian envelope: properties

Meeting the emission mask
- via random selection
- via LSE minimization

Outline

Gaussian envelope: properties

Meeting the emission mask
- via random selection
- via LSE minimization

Gaussian envelope

The PSD envelope is deeply affected by the pulse shape.

There are three ways for spectral shaping:

- pulse **width** variations,
- pulse **differentiation**,
- combination of **base functions**.

Due to the extremely shortness of pulses, **no modulation** is allowed.

The easiest and cheapest pulse is a bell-shaped pulse combined with its derivatives.

Gaussian pulse shape: time domain

The classical gaussian shape is:

$$g(t) := \mathcal{N}(0, \sigma^2)(t) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{t^2}{2\sigma^2}\right).$$

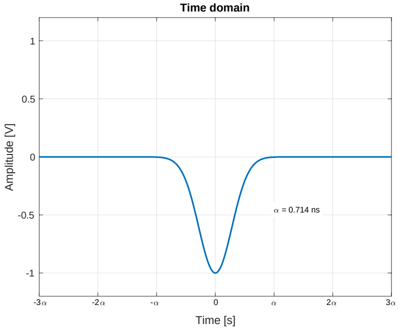
It can be shown by means of mathematical induction principle that:

$$\frac{d^n}{dt^n} g(t) = \frac{(-1)^n}{\sigma^n} \mathrm{H}_n\left(\frac{t}{\sigma}\right) g(t), \quad n \in \mathbb{N}$$

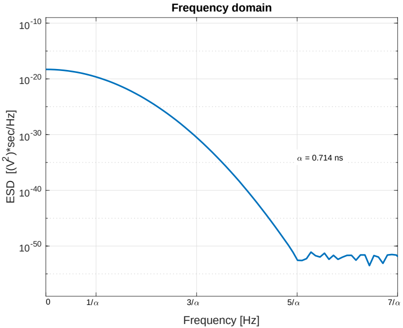
Let be $\sigma^2 = \alpha^2/4\pi$, with *shape factor* α : then,

- the **monocycle** is g'(t);
- the **doublet** is g''(t).

Gaussian pulse waveform



Energy Spectral Density



Analytical expression of a Gaussian pulse

$$p(t) = \pm \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{t^2}{2\sigma^2}} = \pm \frac{\sqrt{2}}{\alpha} e^{-\frac{2\pi t^2}{\alpha^2}}$$

$\alpha^2 = 4\pi\sigma^2$ is the shape factor

Exercise 1.1: Check the effect of shape factor on pulse waveform and corresponding ESD

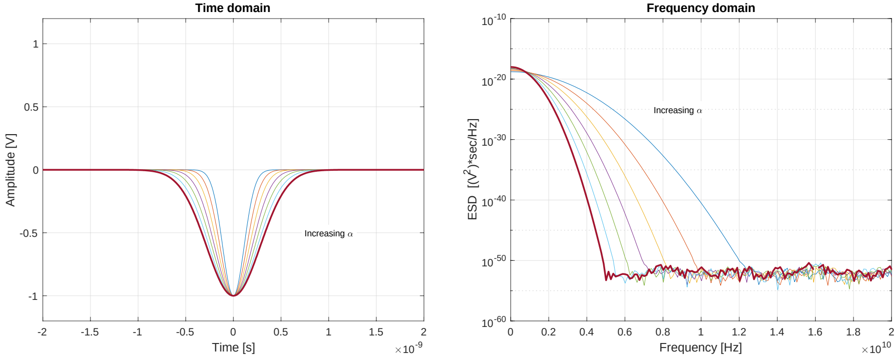
Write the function:

```
shape_factor_variation(alphamin, alphamax, N_alphavalues)
```

Settings

- `alphamin =  0.4e-9`
- `alphamax = 1e-9`
- `N_alphavalues = 7`
- `A = 1 %pulse amplitude`
- `smp = 1024 %number of samples`
- `Tmin = -4e-9 %lower time interval limit`
- `Tmax = 4e-9 %upper time interval limit`

Exercise 1.1: results



Exercise 1.2: Check the effect of differentiation:

- Represent waveforms and ESDs for the 15 first derivatives
  - alpha = 0.714e-9

- Plot f_{peak} as a function of alpha, for 15 derivatives. Verify that the maximum of the spectrum is reached at:

$$f_{peak,k} = \sqrt{k}\frac{1}{\alpha\sqrt{\pi}}$$

- Verify that the bandwidth @ -10 dB is only slightly dependent on derivative's order by plotting BW_{-10dB} [Hz] as a function of alpha, for 15 derivatives.

Exercise 1.2: Check the effect of differentiation

Hints

- ❖ In time and frequency with 15 first derivatives

HINT: create the functions:

```
Gaussian_derivatives(alpha)
Gaussian_derivatives_ESD(alpha)
```

- ❖ max of the spectrum is reached at $f_{peak,k} = \sqrt{k}\frac{1}{\alpha\sqrt{\pi}}$

HINT: create the function:

```
[peakfrequency]=Gaussian_derivatives_peak_frequency(alphamin,
alphamax, N_alphavalues)
```

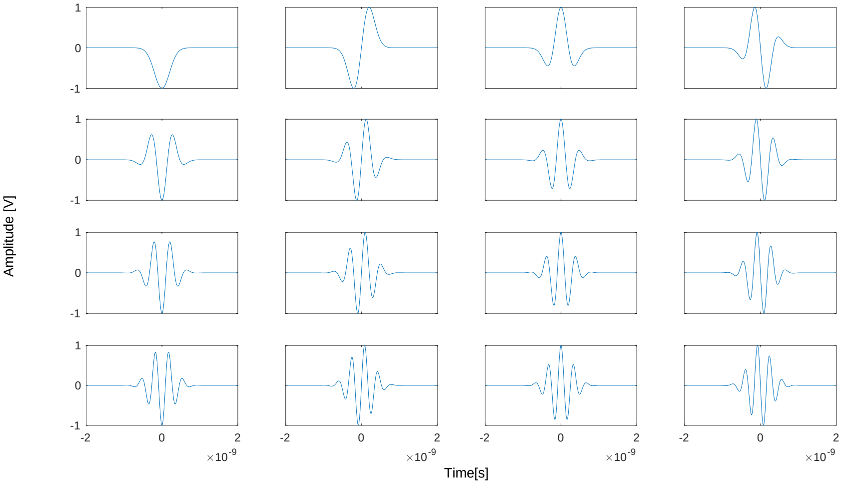
- ❖ Plot the bandwidth @ -10 dB as a function of alpha:

HINTS: - create a function:

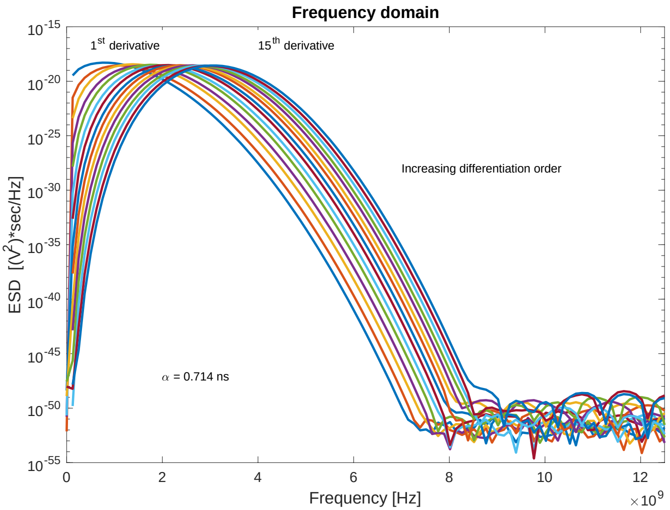
```
Gaussian_derivatives_10dB_bandwidth(alphamin, alphamax,
N_alphavalues)
```

- use the routine `bandwidth mod`.

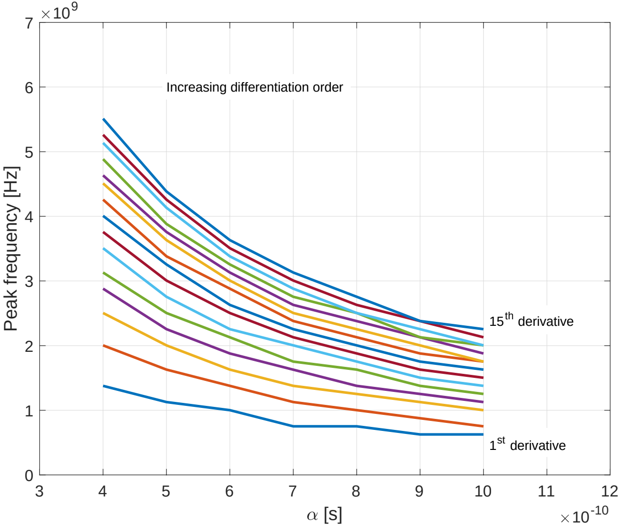
Exercise 1.2: results



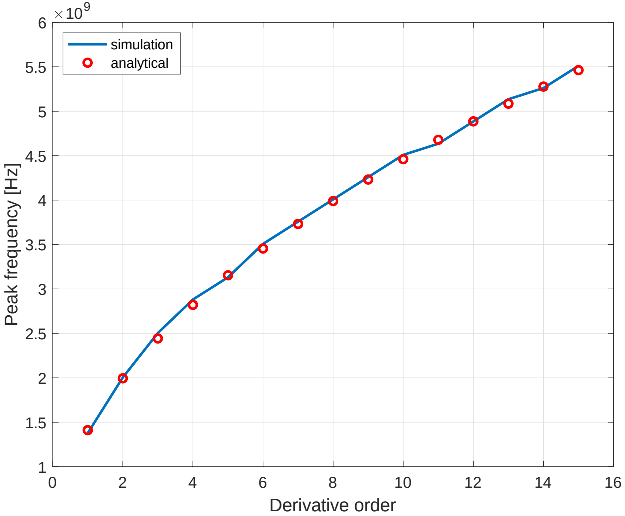
Exercise 1.2: results on ESDs



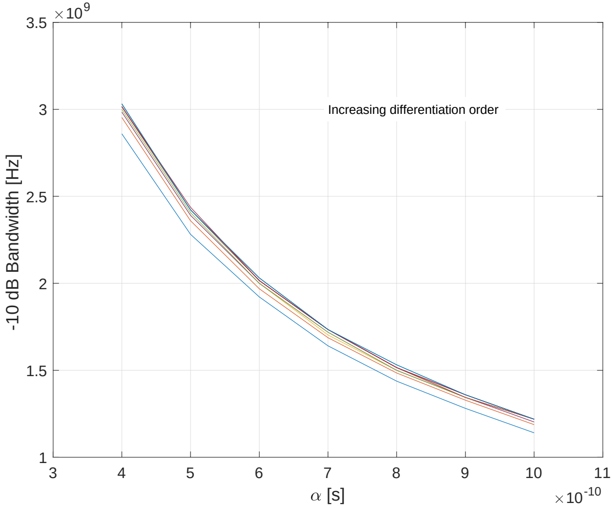
Exercise 1.2: results on peak frequency



Exercise 1.2: results on peak frequency as a function of the derivation order



Exercise 1.2: results on -10 dB bandwidth



Outline

Gaussian envelope: properties

Meeting the emission mask
- via random selection
- via LSE minimization

Meeting the emission mask

Remind: Combining pulse width variation and differentiation

- Pulse width variation and differentiation allow to modify the PSD of the emitted signal
- A single waveform $p(t)$ does not allow to achieve efficient power use at all frequencies
- A set of different waveforms $p_k(t)$ (each corresponding to a different derivative with a different shape factor a_k) can be used to increase efficiency

Meeting the emission mask

Problem: choose $\{a_k\}_{k=0}^{M-1}$ such that the **ESD** of tx pulse

$$p(t) = \sum_{k=0}^{M-1} a_k p_k(t)$$

is as close as possible to the mask.

Possible approaches:

- via random selection
- via LSE

Meeting the emission mask

VIA RANDOM SELECTION

Exercise 2.1: Write a script/ function

```
[c, singlederiv, analyticalderiv, validresult, df] =
random_pulse_combination(i,Ts,attempts)
```

Purpose: The function yields the best coefficient set within the sets found during the 'attempts' iterations and the best coefficient for the solutions based on each single derivative

Returns:

1) the best coefficient set '`c`'
2) the coefficients for the set formed by each single derivative '`singlederiv`'
3) the set of analytical derivatives in time ‘`analyticalderiv`'
4) a flag on the validity of the returned vectors '`validresult`'
5) the fundamental frequency `df`

Meeting the emission mask

VIA RANDOM SELECTION

Exercise 2.1: Write a script/ function

```
[c, singlederiv, analyticalderiv, validresult, df] =
random_pulse_combination(i,Ts,attempts)
```

Settings:

1) the index 'i' indicating which setting must be adopted for the shape factors α of the derivatives
   - `i=1`: *α = 0.714ns for all derivatives*
   - `i=2`: *α = 1.5ns for 1st derivative and α =0.314 for 2nd-15th derivatives*

2) the pulse repetition period (`Ts=1e-7`)

3) the number of attempts in the random selection of the coefficients '`attempts`' (attempts=100)

Meeting the emission mask

VIA RANDOM SELECTION

1. pick $\{a_k\}_{k=0}^{M-1}$ randomly,
2. evaluate the **ESD and its PSD**
3. select the PSD if it is below the emission mask
4. repeat 1. to 3. as many times as necessary
5. choose the sequence $\{a_k^*\}_{k=0}^{M-1}$ which leads to the highest PSD

Meeting the emission mask

VIA RANDOM SELECTION

To select the coefficients, write the function

```
[c, result]= random_coefficients(attempts, basefunction, dt, smp,
Ts,freqsmoothfactor, emissionmask, lowerbasefunction,higherbasefunction)
```

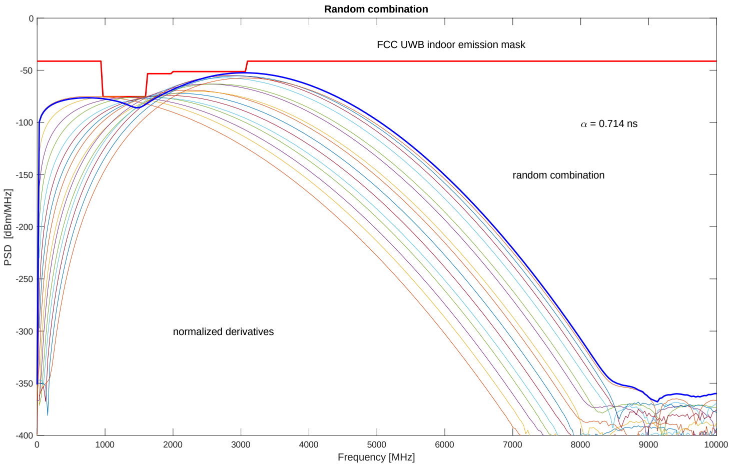
Inputs

1) the number of attempts in the random selection of the coefficients 'attempts' =100
2) the set of BFs 'basefunction' (15 normalized derivative function in time)
3) the sampling period 'dt', given Tmin = -4e-9 (Lower time interval limit), Tmax = 4e-9 (Upper time interval limit)
4) the number of samples in the time domain 'smp', = 1024
5) the pulse repetition period 'Ts'
6) the frequency smoothing factor 'freqsmoothfactor' =8 (FFTsize = freqsmoothfactor*smp)
7) the target emissionmask
8) and 9) the range of BFs to be used in the mask fitting, given by the values 'lowerbasefunction' and 'higherbasefunction'

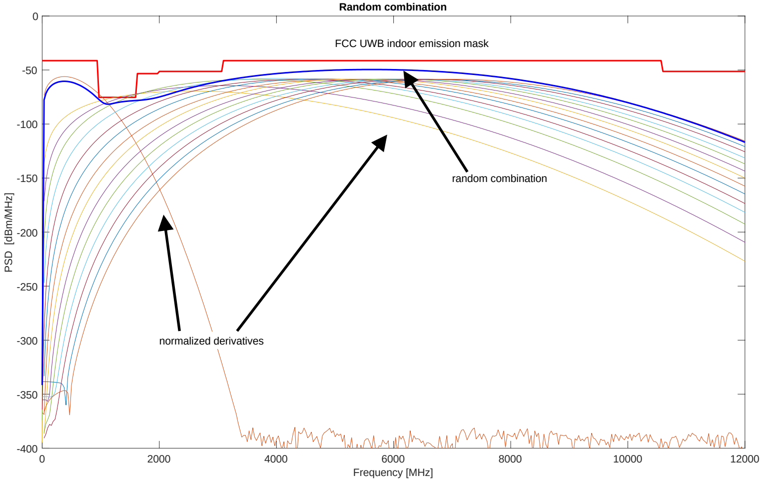
Outputs

1) the best coefficient set 'c'
2) a flag on the validity of the returned set 'result'

Example 1: all functions have same α



Example 2: first derivative has larger α



Meeting the emission mask

VIA LSE

We might minimize the LSE in:

- in time-domain,

$$\min \int_{-\infty}^{\infty} \left| m(t) - \sum_{k=0}^{M-1} a_k \psi_k(t) \right|^2 dt \ ;$$

- in frequency-domain,

$$\min \int_{-\infty}^{\infty} \left| S_{mm}(f) - \sum_{k=0}^{M-1} \alpha_k \Psi_k(f) \right|^2 df \ .$$

Meeting the emission mask

VIA LSE

Exercise 2.2: Write the function

`LSE_pulse_comb(Ts,Tmin,Tmax,smp, frequencysmoothingfactor)`

to implement the LSE selection algorithm for the determination of a combination of the first 15 Gaussian derivatives fitting the FCC indoor emission mask.

Settings:

```
Ts = 1e-7; Tmin = -4e-9;
Tmax = 4e-9; smp = 2^12;
frequencysmoothingfactor = 4;
```

Hints:

- Write the function `[timeemissionmask]=time_mask(Tmin,Tmax,smp)` to define the signal in the time domain corresponding to the FCC indoor emission mask in the frequency domain.
- Use the command `lsqlin`

Meeting the emission mask

VIA LSE