Ultra Wide Band Communications

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Lecture 3

Spectral characteristics of UWB radio signals

Outline

- The Power Spectral Density of PPM analog waveforms with reference cases:
  - sinusoidal signal
  - generic periodic signal
  - random signal
- The Power Spectral Density of TH-UWB
- The Power Spectral Density of DS-UWB
- The Power Spectral Density of MB-OFDM

The PSD of PPM analog waveform (1/13)

- The derivation of the Power Spectral Density (PSD) for TH-UWB signals using PPM can follow the same approach of the analog PPM of the old days.
- The analytical expression of a 2PPM-TH-UWB signal has in fact strong similarities with the output of a PPM modulator in its analog form.

$$s_{PPM}(t) = \sum_{j=-\infty}^{+\infty} p\left(t - jT_S - c_j T_C - a_j \varepsilon\right)$$

Analytical expression of a 2PPM-TH-UWB signal

$$x_{PPM}(t) = \sum_{j=-\infty}^{+\infty} p\left(t - jT_S - m(jT_S)\right)$$

Analytical expression of a PPM analog waveform

The PSD of PPM analog waveform (2/13)

$$x_{PPM}(t) = \sum_{j=-\infty}^{+\infty} p\left(t - jT_S - m(jT_S)\right)$$

Analytical expression of a PPM analog waveform

- The PSD of a PPM signal is difficult to evaluate due to the non-linear nature of PPM modulation.
- Results can be provided for three reference cases:
- Case 1: *m(t)* is a sinusoid, $m(t) = A\, cos(2\pi f_0 t)$
- Case 2: *m(t)* is a generic periodic signal, $m(t) = \sum_{-\infty}^{+\infty} m_n e^{jn2\pi t/T_p}$
- Case 3: *m(t)* is a random signal

The PSD of PPM analog waveform (3/13)

Case 1 PPM signals with a sinusoidal modulating signal

$$x_{PPM}(t) = \sum_{j=-\infty}^{+\infty} p\left(t - jT_S - A\cos\left(2\pi f_0 jT_S\right)\right)$$

An expansion of x_{PPM} into sinusoidal components as shown by [Rowe, 1965]

$$x_{PPM}(t) = p(t) * \sum_{j=-\infty}^{+\infty} \delta\left(t - jT_s - A\cos\left(2\pi f_0 jT_s\right)\right)$$

$$= \frac{1}{T_s} \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} (-j)^n J_n\left(2\pi A\left(m\frac{1}{T_s} + nf_0\right)\right) \cdot$$

$$\cdot P\left(m\frac{1}{T_s} + nf_0\right) e^{j2\pi(m\frac{1}{T_s} + nf_0)t}$$

where $P(f)$: FT of $p(t)$

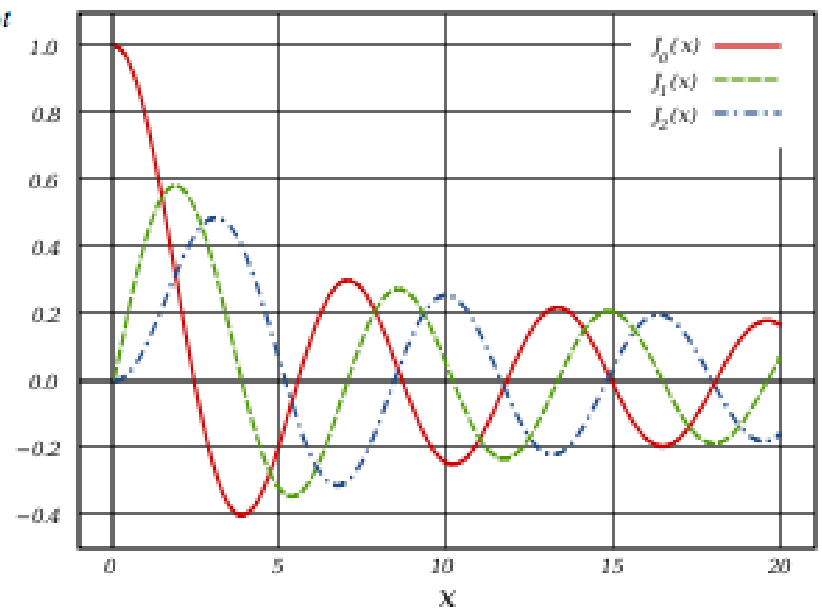
$J_n(.)$: Bessel functions of the first kind

$$J_n(x) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} e^{jx\sin\psi} e^{-jn\psi} d\psi$$

$$J_{-n}(x) = (-1)^n J_n(x), \quad J_n(-x) = (-1)^n J_n(x)$$

and

$$J_n(x) \cong 0 \quad for \quad |n| > |x|$$



The PSD of PPM analog waveform (4/13)

Case 1

PSD of PPM signals with a sinusoidal modulating signal

$$x_{PPM}(t) = p(t) * \sum_{j=-\infty}^{+\infty} \delta\left(t - jT_s - A\cos\left(2\pi f_0 jT_s\right)\right)$$

$$= \frac{1}{T_s} \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} (-j)^n J_n\left(2\pi A\left(m\frac{1}{T_s} + nf_0\right)\right) \cdot$$

$$\cdot P\left(m\frac{1}{T_s} + nf_0\right) e^{j2\pi\left(m\frac{1}{T_s} + nf_0\right)t}$$

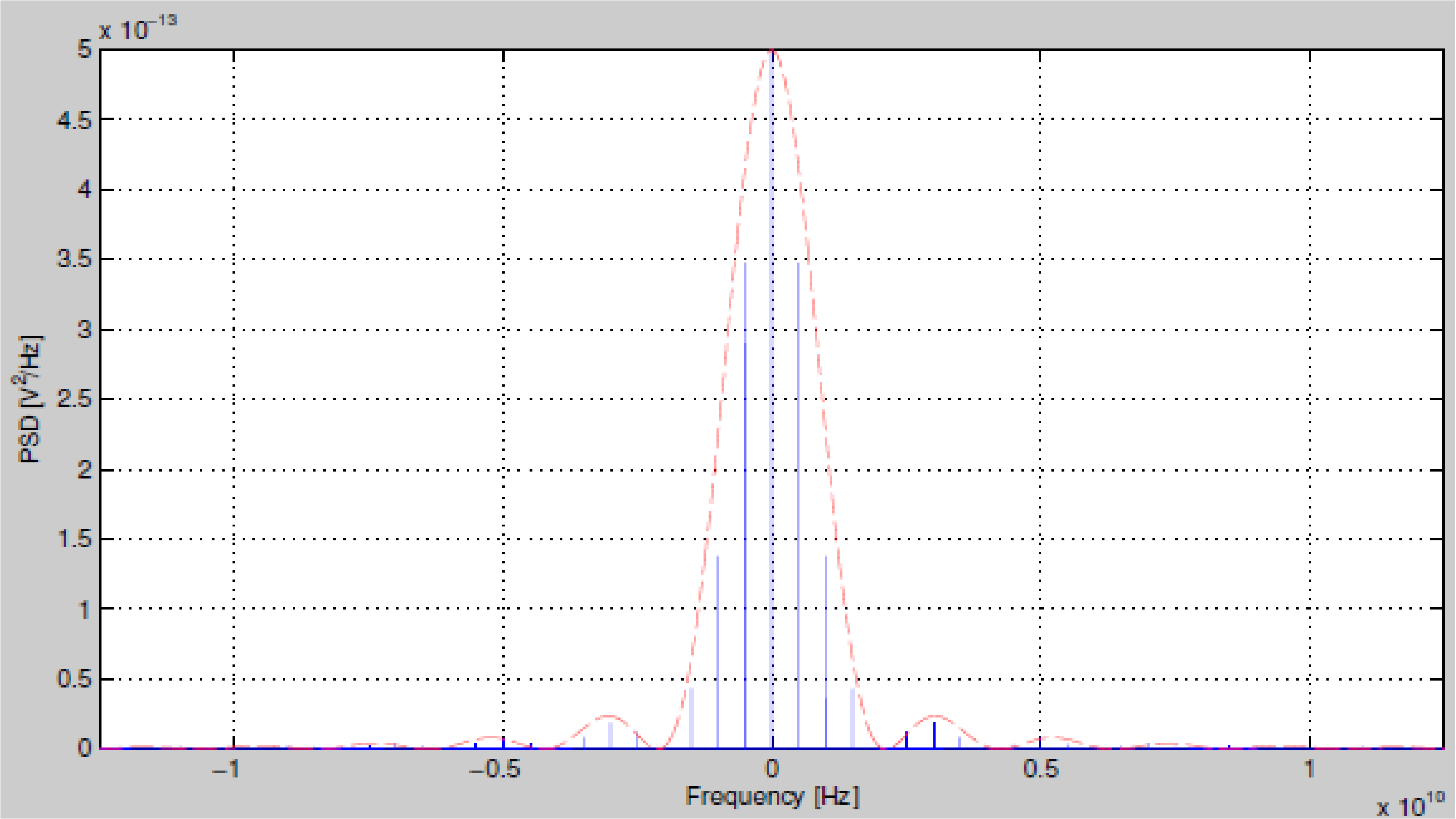
$$P_{x_{PPM}}(f) = \frac{1}{T_S^2} \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \left| J_n\left(2\pi A\left(\frac{m}{T_S} + nf_0\right)\right)\right|^2 \left| P\left(\frac{m}{T_S} + nf_0\right)\right|^2 \delta\left(f - \left(\frac{m}{T_S} + nf_0\right)\right)$$

- The PSD is composed by spectral lines located at all combinations of f_0 and $1/T_S$
- The amplitude of each spectral line is governed by $J_n(x)$ (Bessel function of first kind) and $P(f)$ (FT of p(t))
- If $P(f)$ has limited bandwidth, the bandwidth of the PPM signal is limited as well

The PSD of PPM analog waveform (5/13)

Case 1 PSD of PPM signals with a sinusoidal modulating signal: $f_0 = 0$

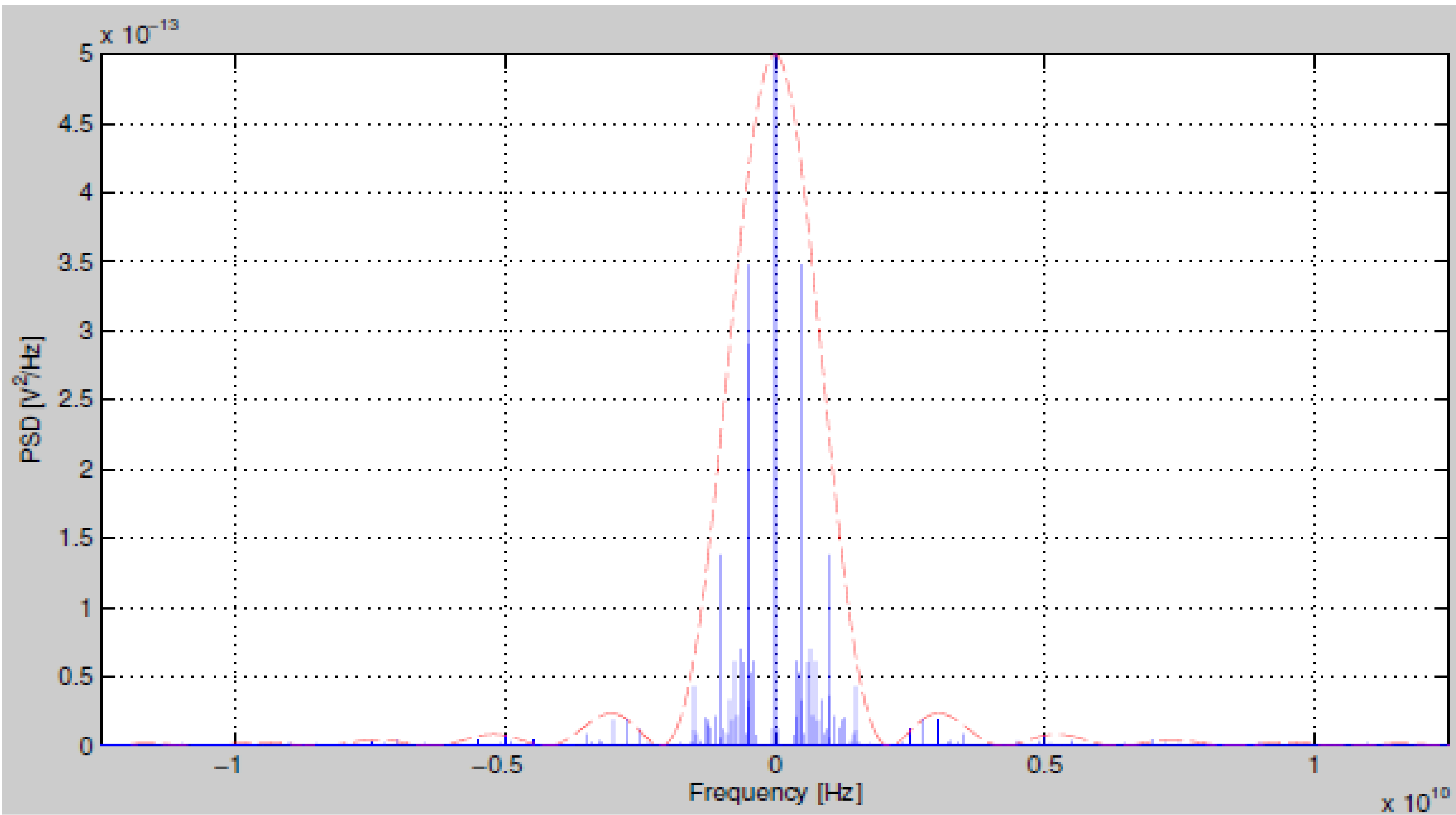
$$P_{x_{PPM}}(f)=\frac{1}{T_S^2}\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\left|J_n\left(2\pi A\left(\frac{m}{T_S}+nf_0\right)\right)\right|^2\left|P\left(\frac{m}{T_S}+nf_0\right)\right|^2\delta\left(f-\left(\frac{m}{T_S}+nf_0\right)\right)$$



The PSD of PPM analog waveform (6/13)

Case 1 PSD of PPM signals with a sinusoidal modulating signal: $f_0 = 50\text{MHz}$

$$P_{x_{PPM}}(f) = \frac{1}{T_S^2} \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \left| J_n\left(2\pi A\left(\frac{m}{T_S} + nf_0\right)\right)\right|^2 \left|P\left(\frac{m}{T_S} + nf_0\right)\right|^2 \delta\left(f - \left(\frac{m}{T_S} + nf_0\right)\right)$$



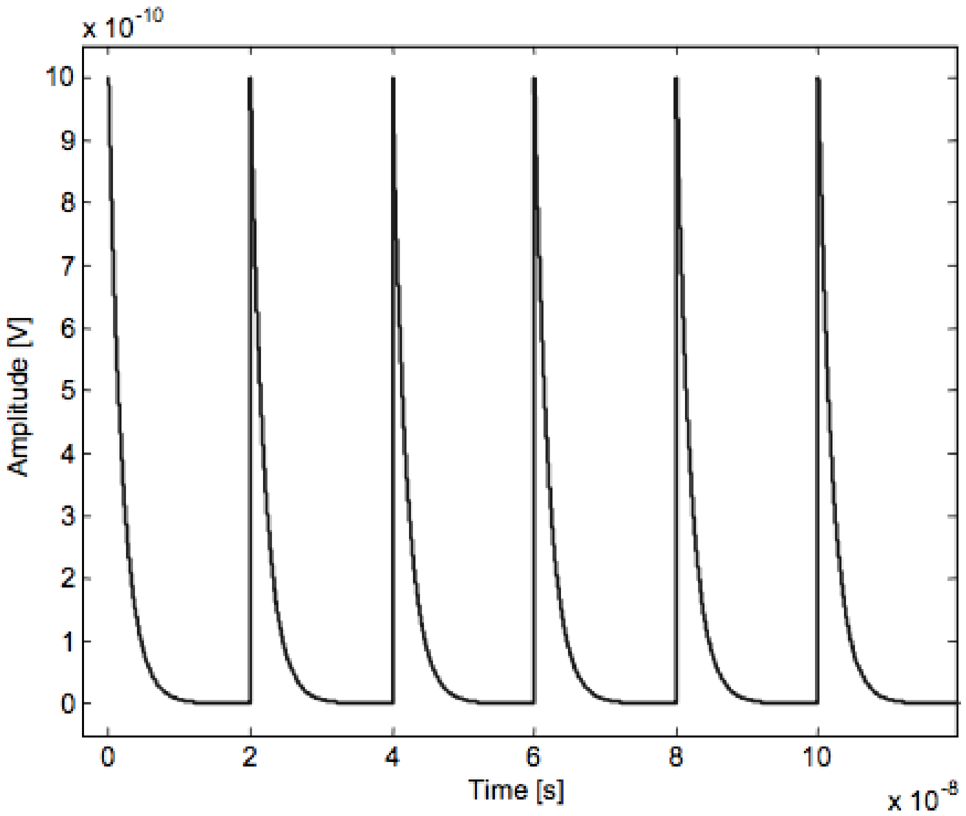
The PSD of PPM analog waveform (7/13)

Case 2

PPM signals with a generic periodic modulating signal

$$x_{PPM}(t) = \sum_{j=-\infty}^{+\infty} p\left(t - jT_S - m(jT_S)\right)$$

$$m(t) = \sum_{n=-\infty}^{+\infty} m_n e^{jn2\pi t/T_P} \quad , T_p \text{ is the period of } m(t)$$



Case 2

PPM signals with a generic periodic modulating signal

$$x_{PPM}(t) = \sum_{j=-\infty}^{+\infty} p\left(t - jT_S - m(jT_S)\right)$$

$$m(t) = \sum_{n=-\infty}^{+\infty} m_n e^{jn2\pi t/T_P}$$, T_p is the period of $m(t)$

where m_n is the n-th Fourier coefficient given by

$$m_n = \frac{1}{T_p}\int_{\alpha}^{\alpha+T_p} m(t) e^{-jn2\pi t/T_p} dt$$

Put $$M = \sum_{n=-\infty}^{+\infty} m_n$$

An expansion of x_{PPM} by applying the multiple Fourier series method is [Rowe, 1965]

$$x_{PPM}(t) = \frac{1}{T_s}\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\sum_{l=-\infty}^{+\infty}(-j)^n J_n\left(2\pi M\left(m\frac{1}{T_s}+nl\frac{1}{T_p}\right)\right)\cdot$$

$$\cdot P\left(m\frac{1}{T_s}+nl\frac{1}{T_p}\right)e^{j2\pi\left(m\frac{1}{T_s}+nl\frac{1}{T_p}\right)t}$$

The PSD of PPM analog waveform (9/13)

Case 2 PSD of PPM signals with a generic periodic modulating signal

$$x_{PPM}(t) = \frac{1}{T_s} \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} (-j)^n J_n\left(2\pi M\left(m\frac{1}{T_s} + nl\frac{1}{T_p}\right)\right) \cdot$$

$$\cdot P\left(m\frac{1}{T_s} + nl\frac{1}{T_p}\right) e^{j2\pi\left(m\frac{1}{T_s} + nl\frac{1}{T_p}\right)t}$$

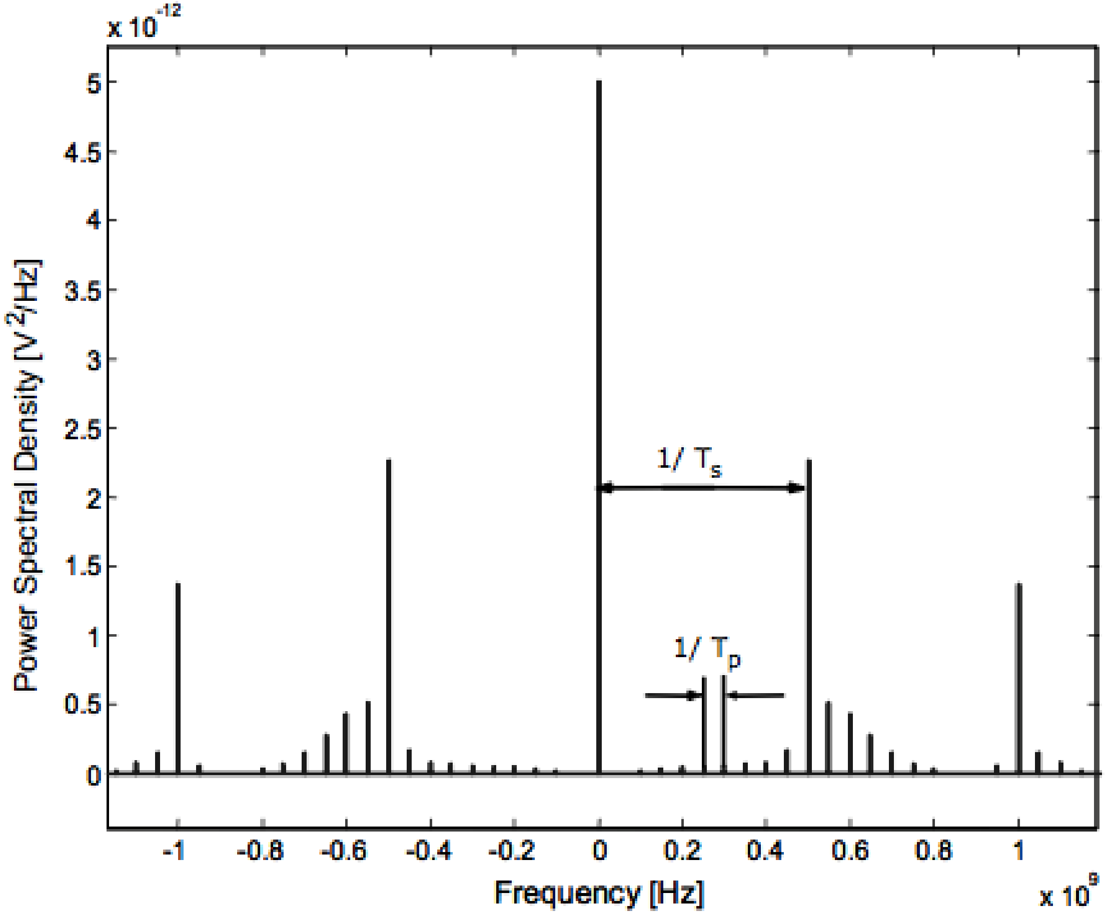
$$P_{x_{PPM}}(f) = \frac{1}{T_S^2} \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} \left| J_n\left(2\pi\left(\sum_{k=-\infty}^{+\infty} m_k\right)\left(\frac{m}{T_S} + \frac{nl}{T_P}\right)\right)\right|^2 \left| P\left(\frac{m}{T_S} + \frac{nl}{T_P}\right)\right|^2 \delta\left(f - \left(\frac{m}{T_S} + \frac{nl}{T_P}\right)\right)$$

- The PSD is composed by spectral lines located at all combinations of $1/T_P$ and $1/T_S$
- Similarly to Case 1, the amplitude of each spectral line is governed by $J_n(x)$ and $P(f)$
- Similarly to Case 1, the bandwidth of the PPM signal is governed by $P(f)$

The PSD of PPM analog waveform (10/13)

Case 2 PSD of PPM signals with a generic periodic modulating signal

$$P_{x_{PPM}}(f)=\frac{1}{T_S^2}\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty}\sum_{l=-\infty}^{+\infty}\left|J_n\left(2\pi\left(\sum_{k=-\infty}^{+\infty}m_k\right)\left(\frac{m}{T_S}+\frac{nl}{T_P}\right)\right)\right|^2\left|P\left(\frac{m}{T_S}+\frac{nl}{T_P}\right)\right|^2\delta\left(f-\left(\frac{m}{T_S}+\frac{nl}{T_P}\right)\right)$$



Case 3

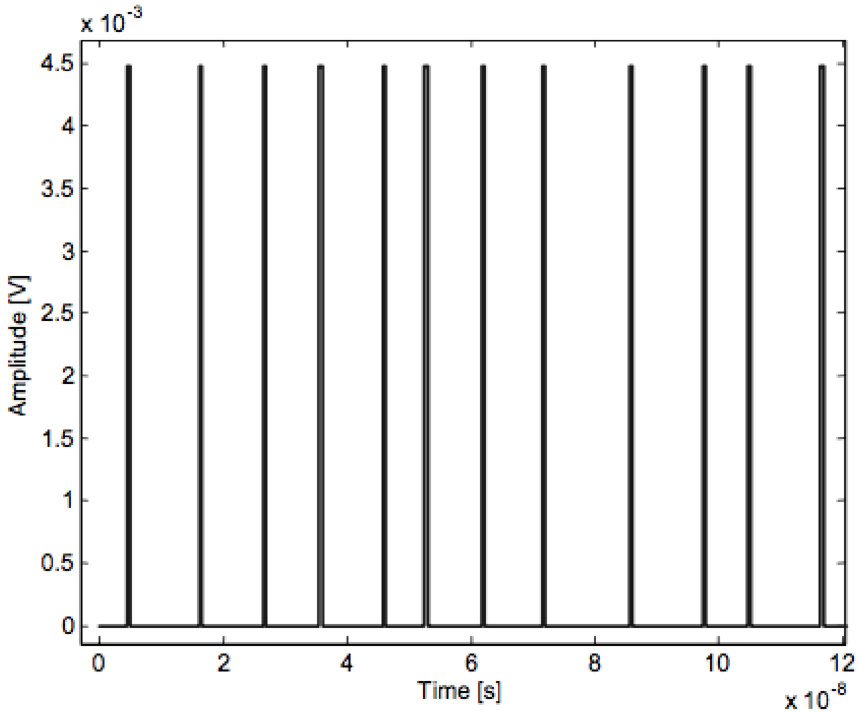
PSD of PPM signals with a random modulating signal

$$x_{PPM}(t) = \sum_{j=-\infty}^{+\infty} p\left(t - jT_S - m(jT_S)\right)$$

$m(t)$ is a strict sense stationary (SSS) random process

$m(jTs)$ are statistically independent

$w(m(jTs))$ probability density function of the samples of $m(t)$



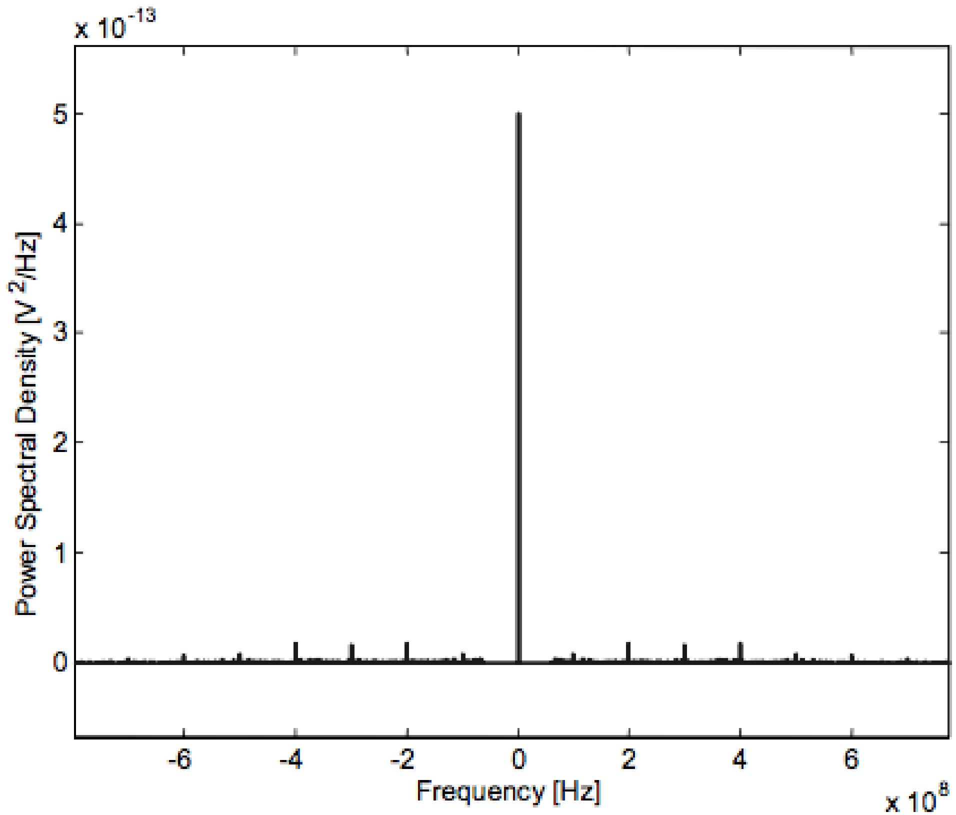
Case 3

PSD of PPM signals with a random modulating signal

$$P_{x_{PPM}}(f) = \frac{|P(f)|^2}{T_S}\left[1 - |W(f)|^2 + \frac{|W(f)|^2}{T_S}\sum_{n=-\infty}^{+\infty}\delta\left(f - \frac{n}{T_S}\right)\right]$$

$W(f)$ is the Fourier transform of the probability density function w

- The PSD is composed of a continuous part controlled by the term $1-|W(f)|^2$, and of a discrete part formed by spectral lines located at multiples of $1/T_S$



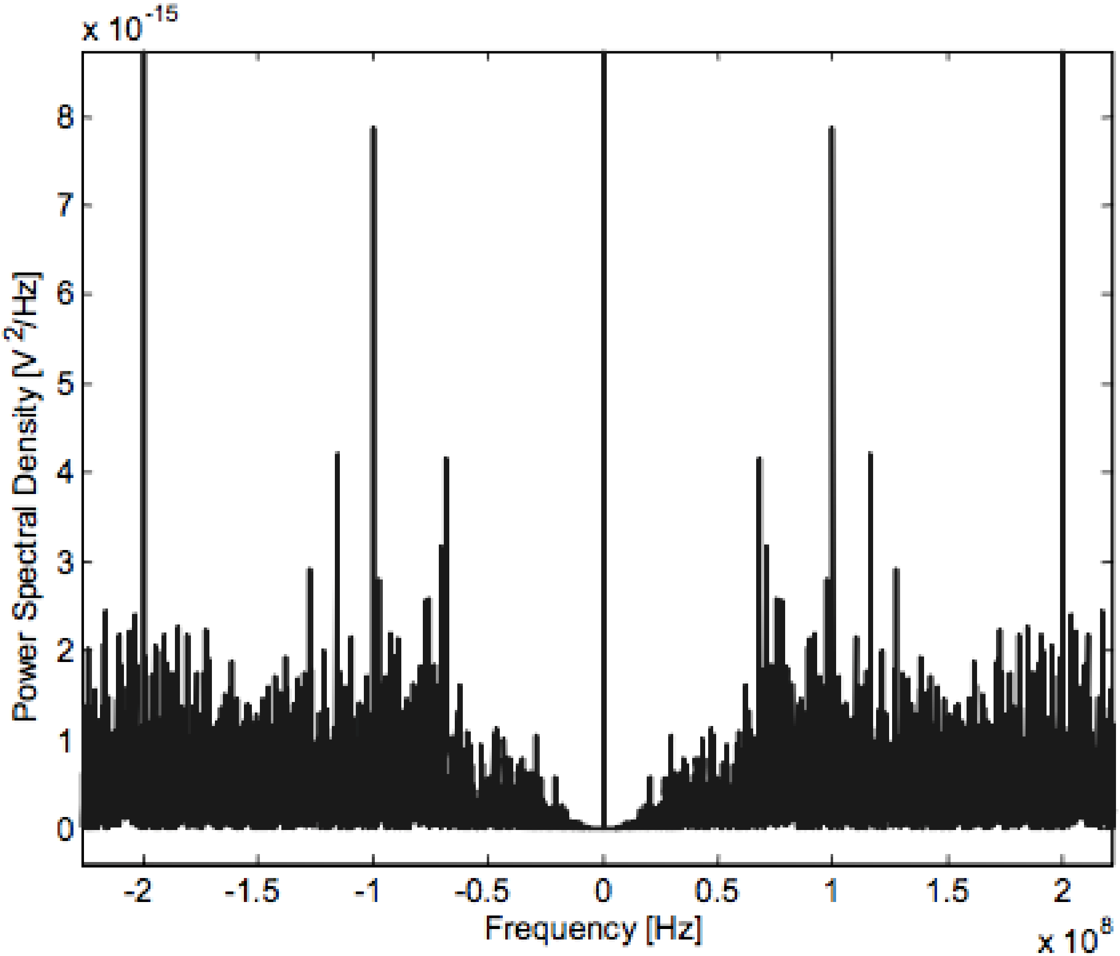
The PSD of PPM analog waveform (13/13)

Case 3

PSD of PPM signals with a random modulating signal

$$P_{x_{PPM}}(f)=\frac{|P(f)|^2}{T_S}\left[1-|W(f)|^2+\frac{|W(f)|^2}{T_S}\sum_{n=-\infty}^{+\infty}\delta\left(f-\frac{n}{T_S}\right)\right]$$

- The PSD is dominated by a strong peak at zero, and composed by spectral lines and spurious contributions between lines, for e.g. in the region of ±200MHz



The PSD of TH-UWB signals (1/5)

2PPM-TH-UWB signal

$$s_{PPM}(t) = \sum_{j=-\infty}^{+\infty} p\left(t - jT_S \underbrace{- c_j T_C - a_j \varepsilon}_{\theta_j}\right)$$

$\boldsymbol{\theta}_j$ time dither term

Analog PPM wave

$$x_{PPM}(t) = \sum_{j=-\infty}^{+\infty} p\left(t - jT_S - m(jT_S)\right)$$

- Since the shift due to PPM is much smaller than the shift introduced by the code, the time dither process θ is considered quasi-periodic and closely follows the periodicity of the TH code.
- We can make <u>a first reasonable hypothesis</u> that $s_{PPM}(t)$ is modulated by a periodic signal with period $N_P T_S = T_p = N_S T_S = T_b$ (if $N_P = N_S$), $\mathrm{T_b}$ is the bit interval.
- Under such assumption, the PSD is **discrete** with lines at multiples of $1/N_P T_S = 1/T_b$

The PSD of TH-UWB signals (2/5)

2PPM-TH-UWB signal

$$s_{PPM}(t) = \sum_{j=-\infty}^{+\infty} p\left(t - jT_S - \underbrace{c_j T_C - a_j \varepsilon}_{\theta_j}\right)$$

θ_j time dither term

Analog PPM wave

$$x_{PPM}(t) = \sum_{j=-\infty}^{+\infty} p\left(t - jT_S - m(jT_S)\right)$$

- When considering the presence of ε, signal $s_{PPM}(t)$ is no longer periodic.
- An analytical expression for the PSD can be still provided, however, when considering the special case $N_P = N_S$

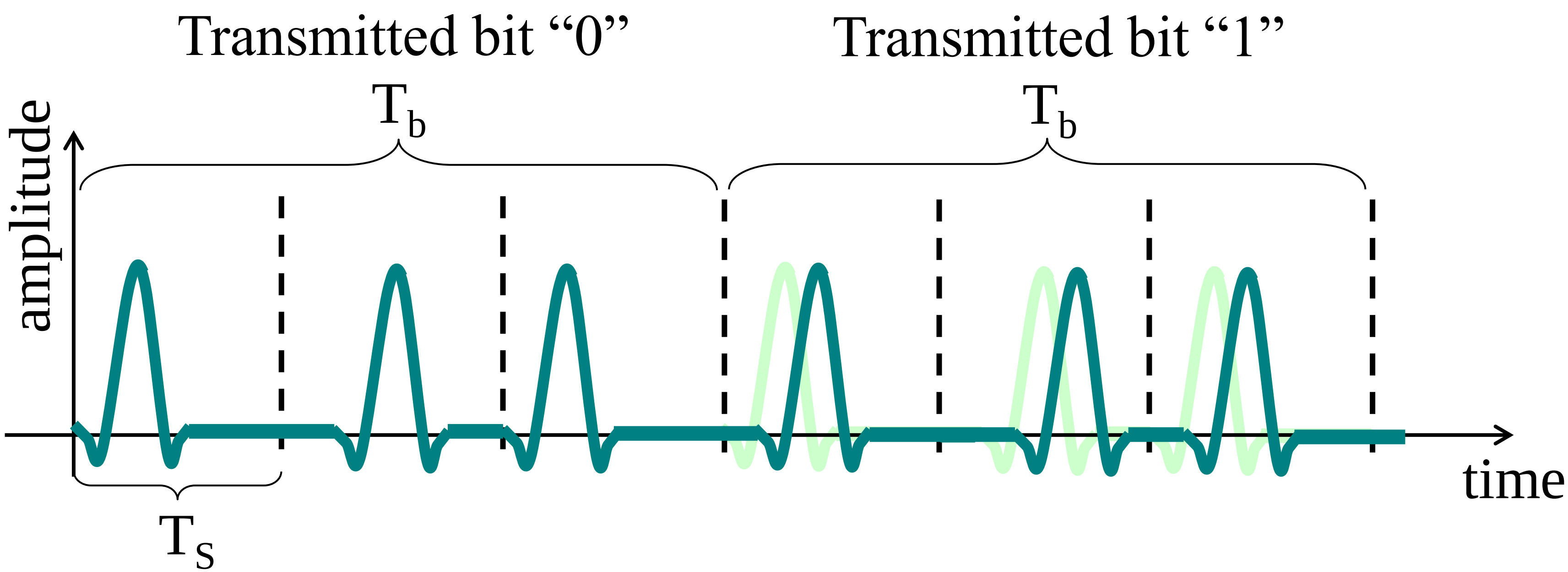
$$s_{PPM}(t) = \sum_{j=-\infty}^{+\infty} p\left(t - jT_S - c_j T_C - a_j \varepsilon\right) = \sum_{j=-\infty}^{+\infty} v\left(t - jT_b - b_j \varepsilon\right)$$

$$v(t) = \sum_{j=0}^{N_S-1} p\left(t - jT_S - c_j T_C\right)$$

v(t) is the basic Multi-pulse including the repetition code

The PSD of TH-UWB signals (3/5)

Example of 2PPM-TH-UWB signal with $N_S = N_P = 3$



$$s_{PPM}(t) = \sum_{j=-\infty}^{+\infty} p\left(t - jT_S - c_j T_C - a_j \varepsilon\right) = \sum_{j=-\infty}^{+\infty} v\left(t - jT_b - b_j \varepsilon\right)$$

$s_{PPM}(t)$ is a PPM modulated waveform in which the shift is ruled by the sequence of data symbols **b,** that is, the **b** process emitted by the source.

Assumptions: **b** is SSS, b_j are statistically independent and have a common pdf w

The PSD of TH-UWB signals (4/5)

From Slide 15:

$$P_{x_{PPM}}(f) = \frac{|P(f)|^2}{T_S}\left[1 - |W(f)|^2 + \frac{|W(f)|^2}{T_S}\sum_{n=-\infty}^{+\infty}\delta\left(f - \frac{n}{T_S}\right)\right]$$

PSD of a PPM wave having a random modulating signal $m(t)$.

$W(f)$ is the Fourier transform of the probability density function of the samples of $m(t)$

$P(f)$ is the Fourier transform of the pulse waveform $p(t)$

Now:

PSD of a 2PPM-TH-UWB signal with $N_S = N_P$

$$P_{x_{PPM}}(f) = \frac{|P_v(f)|^2}{T_b}\left[1 - |W(f)|^2 + \frac{|W(f)|^2}{T_b}\sum_{n=-\infty}^{+\infty}\delta\left(f - \frac{n}{T_b}\right)\right]$$

$W(f)$ is the Fourier transform of the probability density function of the random bits b_j

$P_v(f)$ is the Fourier transform of the multi-pulse waveform $v(t)$

$P_v(f)$ is dependent on $P(f)$

The PSD of TH-UWB signals (5/5)

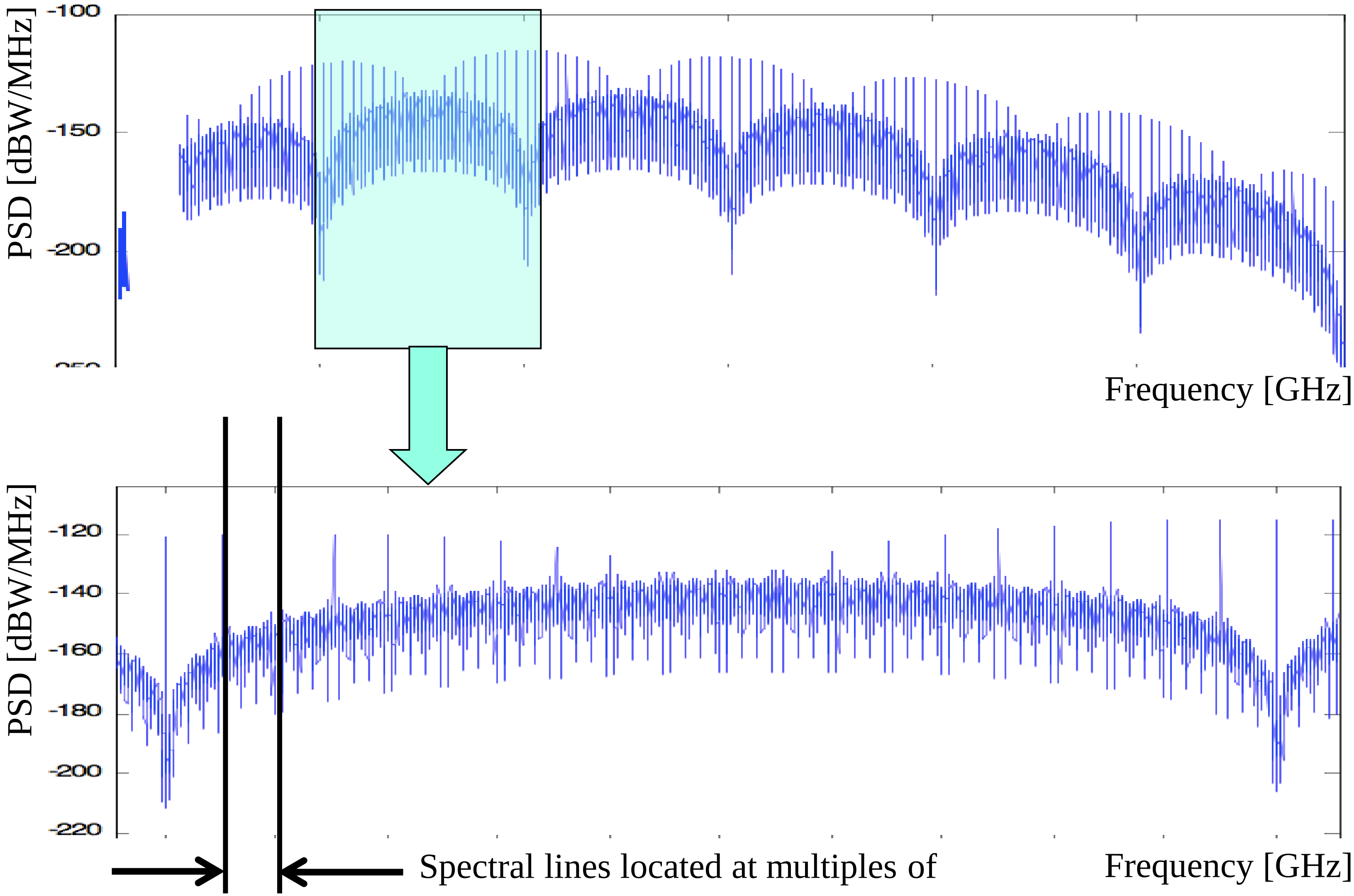
$$P_{x_{PPM}}(f) = \frac{|P_v(f)|^2}{T_b}\left[1 - |W(f)|^2 + \frac{|W(f)|^2}{T_b}\sum_{n=-\infty}^{+\infty}\delta\left(f - \frac{n}{T_b}\right)\right]$$

PSD of a 2PPM-TH-UWB signal with $N_S = N_P$

- According to the above equation, the TH code affects the PSD through the Fourier transform of the multi-pulse $P_v(f)$
- The PSD is composed of:
  - a continuous part, which is shaped by $P_v(f)$ and $W(f)$.
  - a discrete part, consisting of spectral lines located at multiples of the bit rate $1/T_b$, and weighted by the statistical properties of the source represented by $|W(f)|^2$.

Example (1/4)

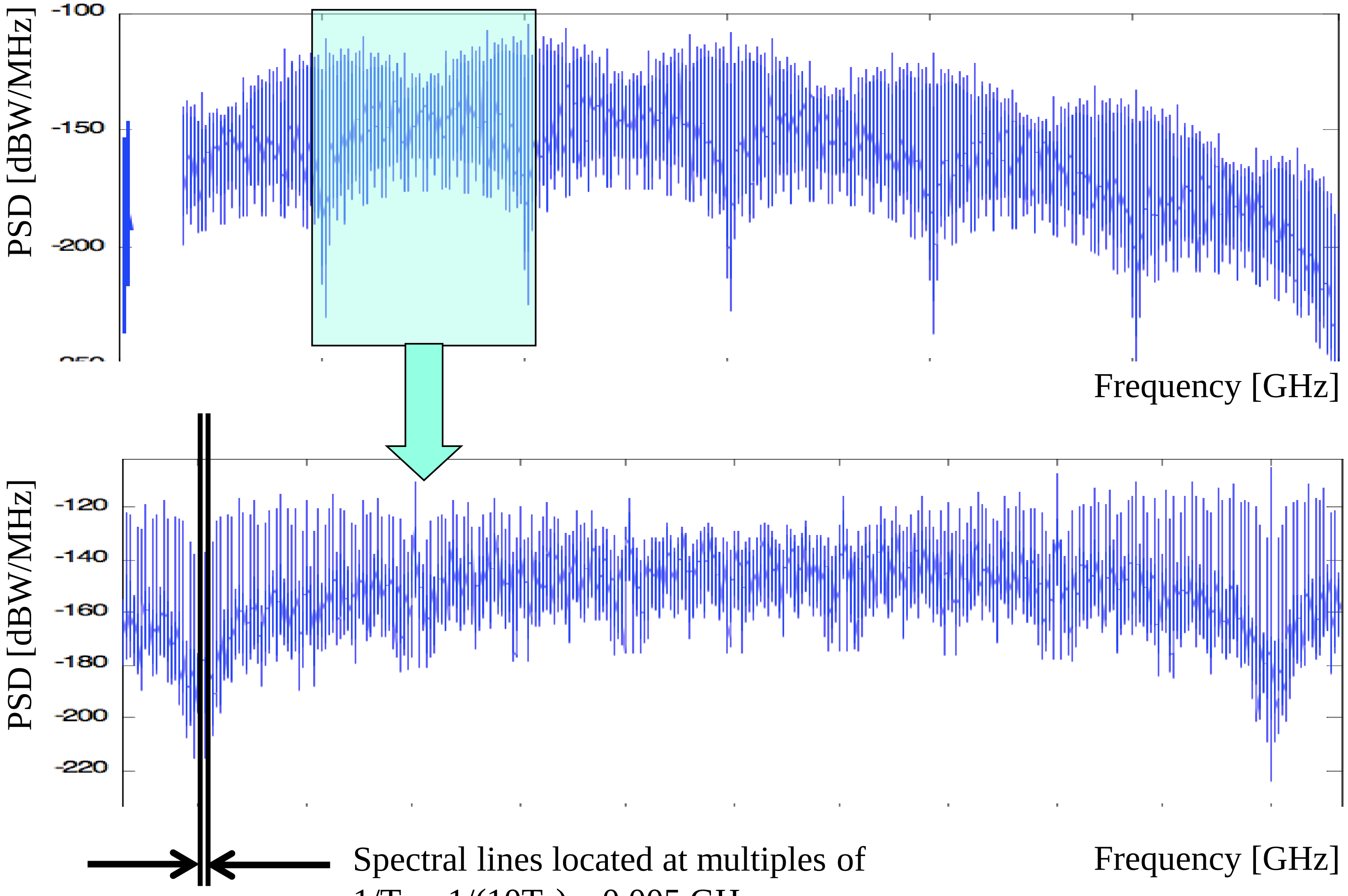
Power Spectral Density of a 2PPM-TH-UWB signal with T_S = 20 ns, and N_S=N_P=**1**



Spectral lines located at multiples of
$1/T_b = 1/T_S = 0.05$ GHz

Examples (2/4)

Power Spectral Density of a 2PPM-TH-UWB signal with T_S = 20 ns, and N_S=N_P=**10**



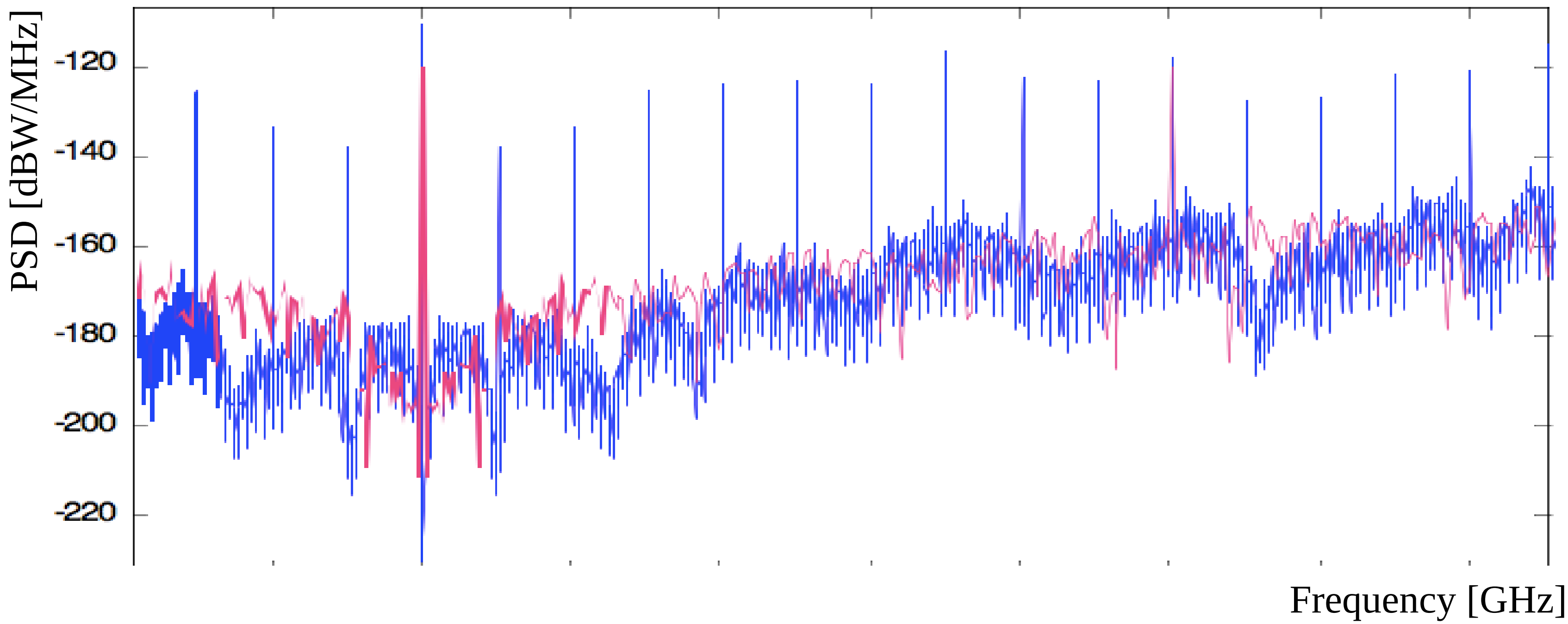
Spectral lines located at multiples of
$1/T_b = 1/(10T_S) = 0.005$ GHz

Examples (3/4)

Comparison between the PSD of two 2PPM-TH-UWB signals with same T_S = 20 ns

Pink Line: $N_S = N_P = \mathbf{1}$

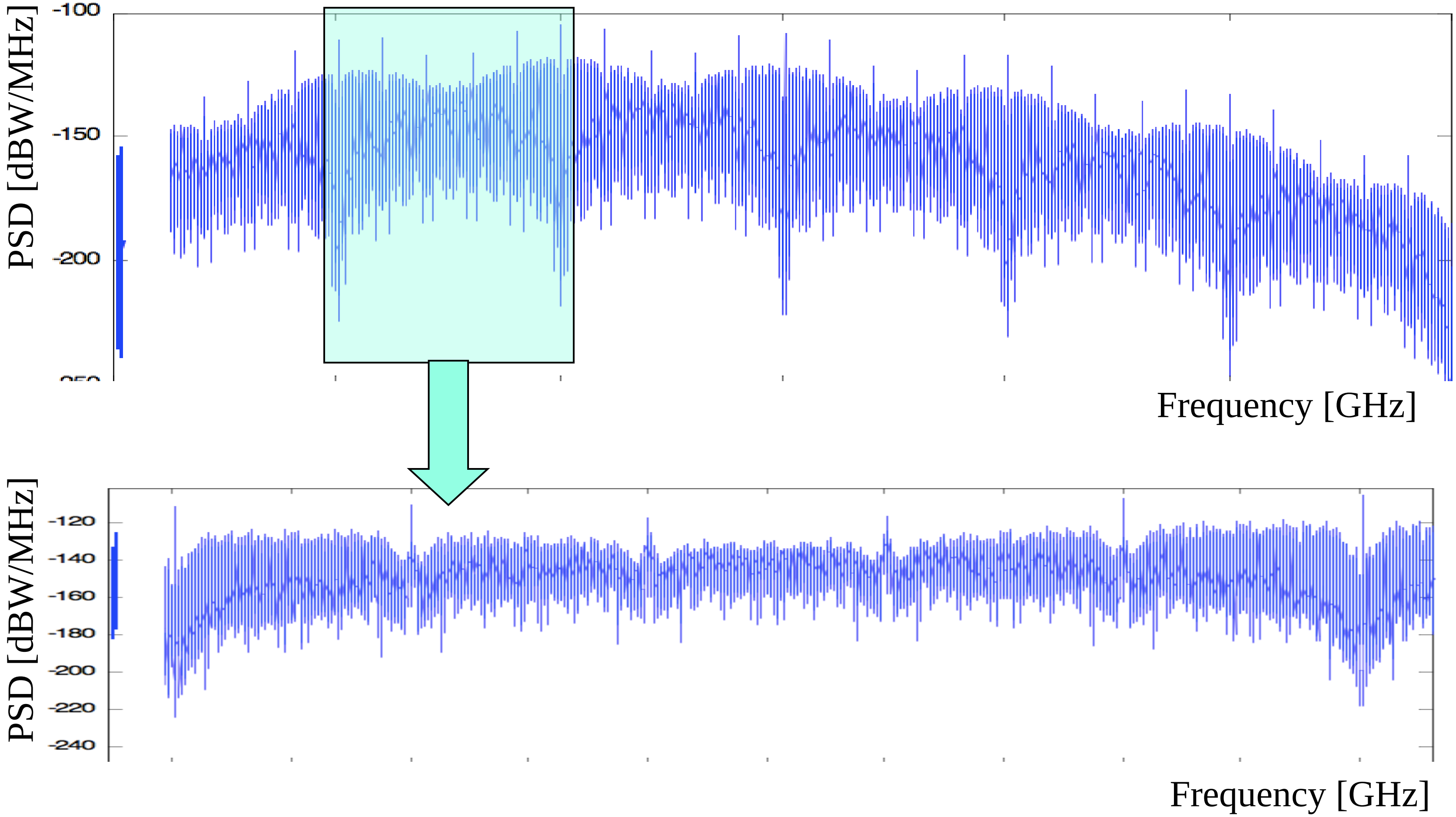
Blue Line: $N_S = N_P = \mathbf{10}$



- If N_P is constrained to be equal to N_S, the effect of increasing N_P is to reduce the distance between adjacent spectral lines

Examples (4/4)

Power Spectral Density of a 2PPM-TH-UWB signal with T_S = 20 ns, N_S= **10** and N_P=**100**



- The discrete part of the PSD can be mitigated by increasing N_P with a fixed N_S (*PSD whitening*).

The PSD of DS-UWB signals (1/2)

- The PSD of a DS-UWB signal is more easily derived with respect to the TH-UWB case since pulses occur at multiples of T_s.

$$x_{PAM}(t) = \sum_{j=-\infty}^{+\infty} d_j p(t - jT_S)$$

Analytical expression of a 2PAM-DS-UWB signal

$$d_j = a_j c_j$$

$$P_{x_{PAM}}(f) = \frac{1}{T_S} |P(f)|^2 P_C(f)$$

PSD of a 2PAM-DS-UWB signal

$P(f)$ is the Fourier transform of $p(t)$

$P_C(f)$ is the **code spectrum**, that is, the discrete time Fourier transform of the autocorrelation function of the random process $\{d_j\}$

The PSD of DS-UWB signals (2/2)

$$P_{x_{PAM}}(f) = \frac{1}{T_S} |P(f)|^2 P_C(f)$$

PSD of a 2PAM-DS-UWB signal

$$P_C(f) = \sum_{m=-\infty}^{+\infty} R_d(m)\, e^{-j2\pi f m T_S}$$

code spectrum

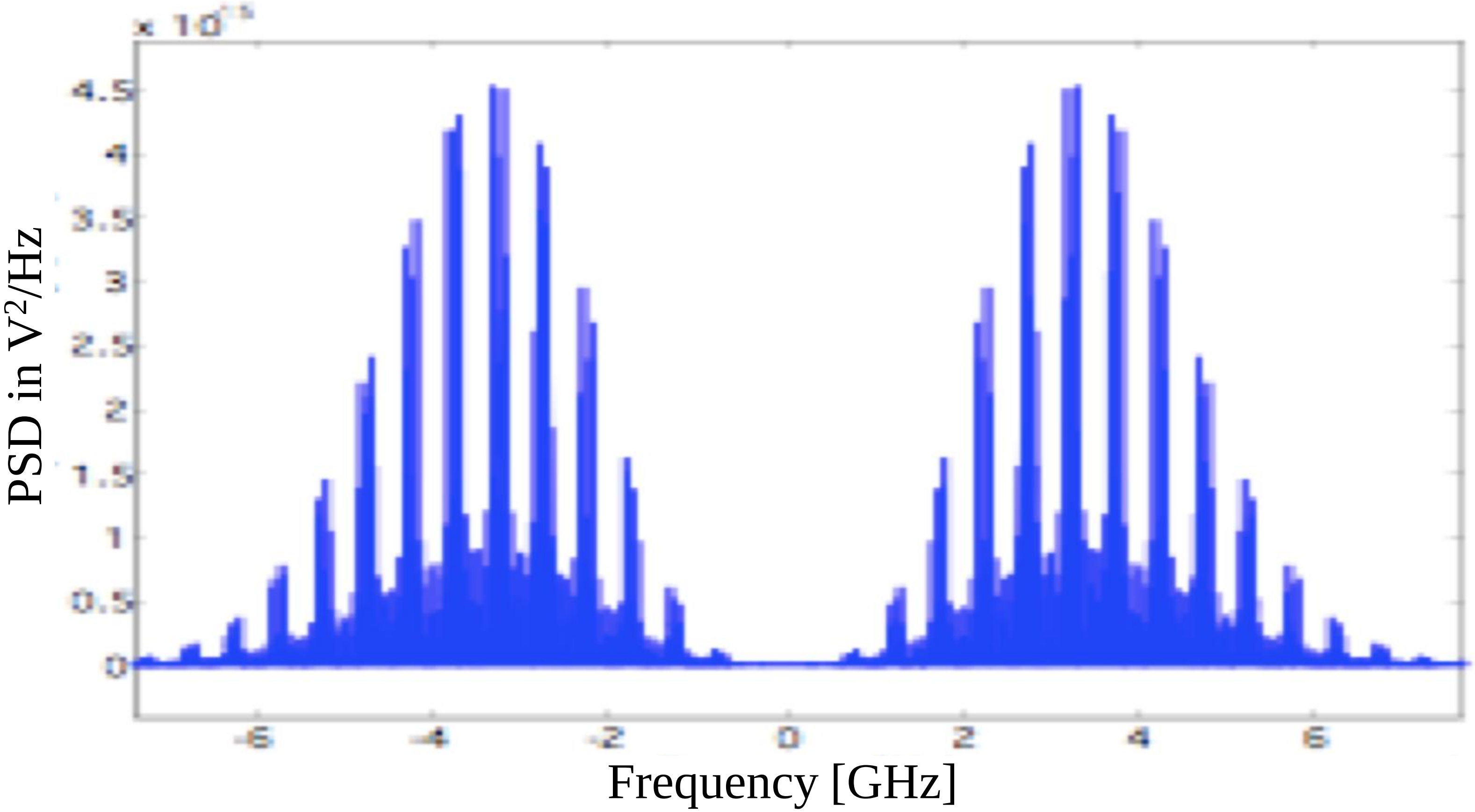
$$R_d(m) = \langle d_{j+m} d_j \rangle$$

Autocorrelation of the sequence $\{d_j\}$

- If sequence $\{d_j\}$ is composed of independent symbols, $R_d(m)$ is different from 0 only for $m = 0$
- In this case, $P_C(f)$ is independent of f, and the spectrum is entirely governed by the properties of the pulse $p(t)$.

Examples (1/3)

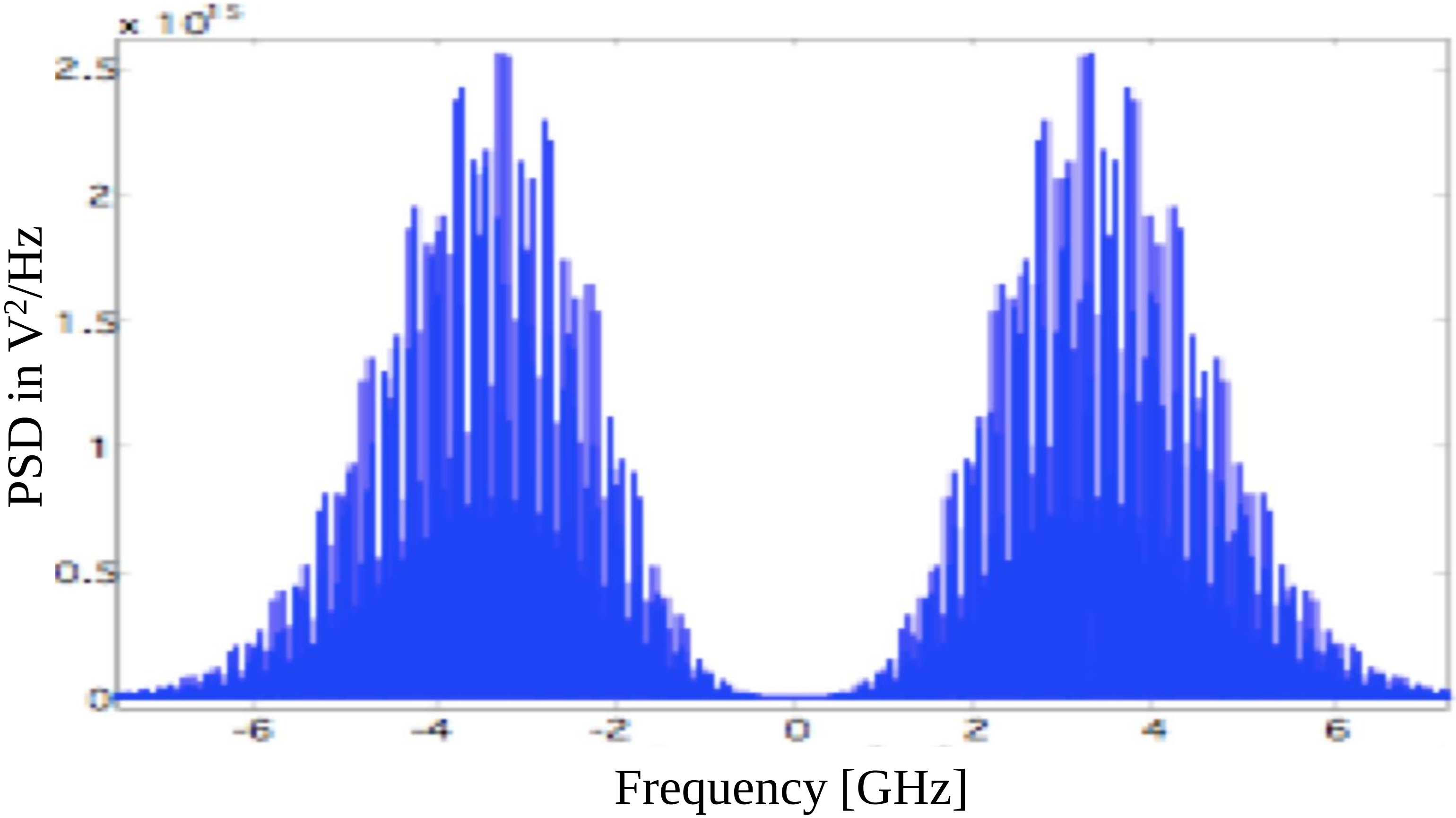
Power Spectral Density of a 2PAM-DS-UWB signal with T_S = 2 ns, N_S= **10** and N_P=**10**



- The envelope of the PSD has the shape of $P(f)$.
- Due to the effect of code spectrum $P_C(f)$, transmitted power concentrates on spectral peaks

Examples (2/3)

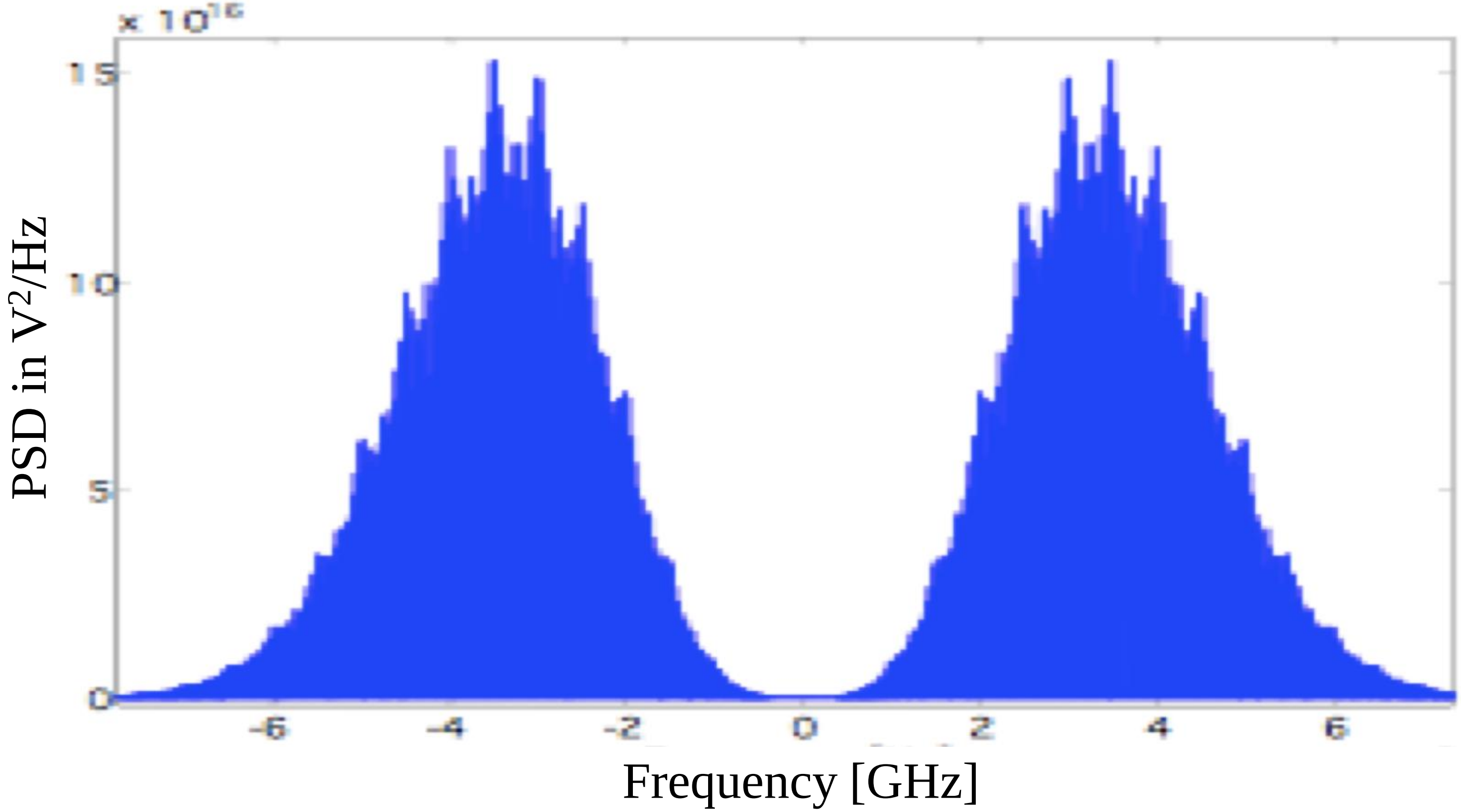
Power Spectral Density of a 2PAM-DS-UWB signal with T_S = 2 ns, N_S= **10** and N_P=**50**



- Signal power is better distributed over the spectrum, that is, the amplitude of the spectral peaks with N_P=50 is reduced with respect to the case of N_P=10

Examples (3/3)

Power Spectral Density of a 2PAM-DS-UWB signal with T_S = 2 ns, N_S= **10** and $N_P \rightarrow \infty$



- In this case, the code sequence is composed of independent symbols, and the PSD approaches the shape of the Fourier transform of the basic pulse.

The PSD of MB-UWB signals (1/2)

- The PSD of a MB-OFDM signal can be found by adding up the PSDs of individual sub-carriers for a generic OFDM symbol .

$$\underline{x}(t) = rect\left(\frac{t}{T}\right)\sum_{m=0}^{N-1} c_m e^{j2\pi f_m t}$$

Complex envelope of an OFDM symbol

$$P_{f_m}(f) = sinc\left(\frac{\pi(f - f_m)}{\Delta f}\right)$$

Spectrum centered on the m-th subcarrier

$$P(f) = \left(\sum_{m=0}^{N-1} \sigma_{c_m}^2\right)\sum_{m=0}^{N-1} P_{f_m}(f)$$

Spectrum of an OFDM symbol

$\sigma_{c_m}^2$ is the variance of the complex term c_m

The PSD of MB-UWB signals (2/2)

Power Spectral Density (in logarithmic units) of a MB-OFDM signal compliant with the UWB signal format proposed to the IEEE 802.15.TG3a by the MB coalition

The OFDM signal is composed of 128 sub-carriers equally spaced by 4.1254 MHz, and located around a central frequency f_c = 3.432 GHz