Lecture #6 –November 05, 2021

Ultra Wide Band Radio Fundamentals

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Lecture 8

Pulse Shaping in UWB

Outline

- The Pulse
- Spectrum shaping by Pulse Shaping
  - Pulse width variation
  - Pulse differentiation
  - Linear combination
    - by random selection
    - by LSE minimization

The Pulse (1/4)

- The choice of the impulse response of the pulse shaper filter is crucial since it affects the PSD of the transmitted signal.
- The pulse shape that can be generated in the easiest way by a pulse generator has a bell shape such as a Gaussian.

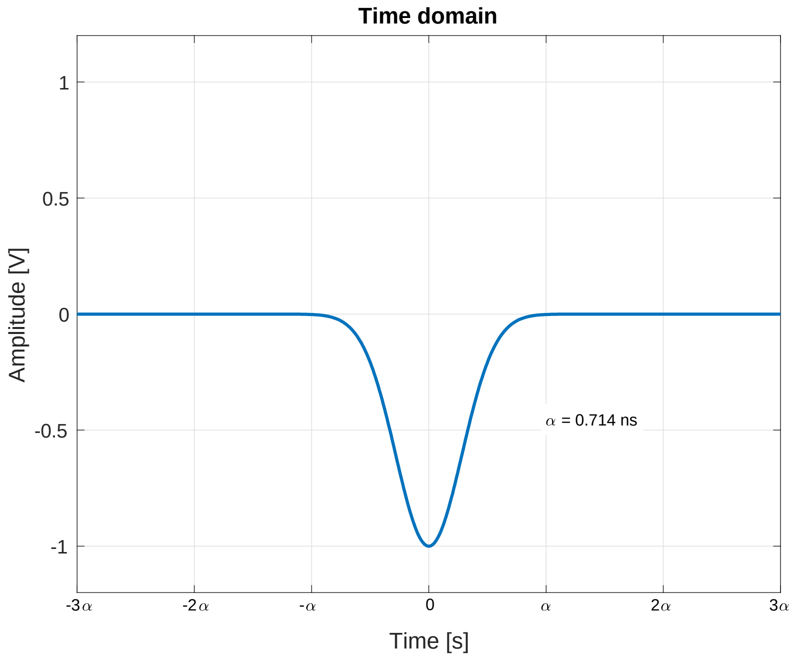
Analytical expression of a Gaussian pulse

$$p(t) = \pm \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{t^2}{2\sigma^2}} = \pm \frac{\sqrt{2}}{\alpha} e^{-\frac{2\pi t^2}{\alpha^2}}$$

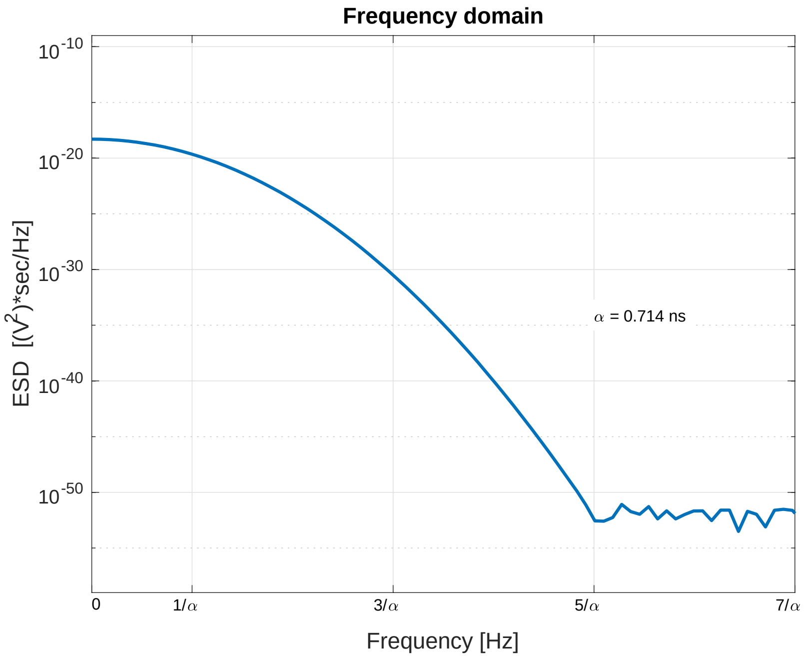
$\alpha^2 = 4\pi\sigma^2$ is the shape factor

The Pulse (2/4)

Gaussian pulse waveform



Energy Spectral Density



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$\alpha^2 = 4\pi\sigma^2$ is the shape factor

The Pulse (3/4)

- To be radiated in an efficient way, a basic feature of the pulse is to have a zero DC (direct current) offset.
- Several pulse waveforms might be considered, provided that this condition is verified.
- Gaussian derivatives are suitable.
- Actually, the most currently adopted pulse shape is modeled as the second derivative of a Gaussian function

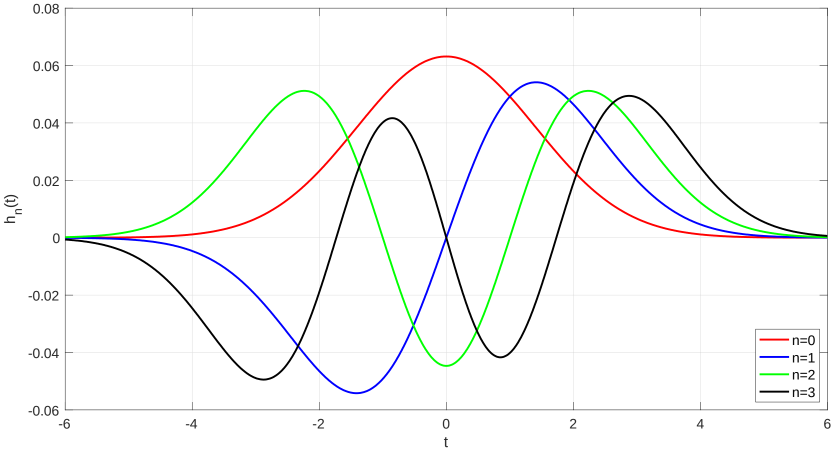
Second derivative of a Gaussian pulse

$$\frac{d^2 p(t)}{dt^2} = \left(1 - 4\pi \frac{t^2}{\alpha^2}\right) e^{-\frac{2\pi t^2}{\alpha^2}}$$

- Other pulse shapes have also been proposed such as:

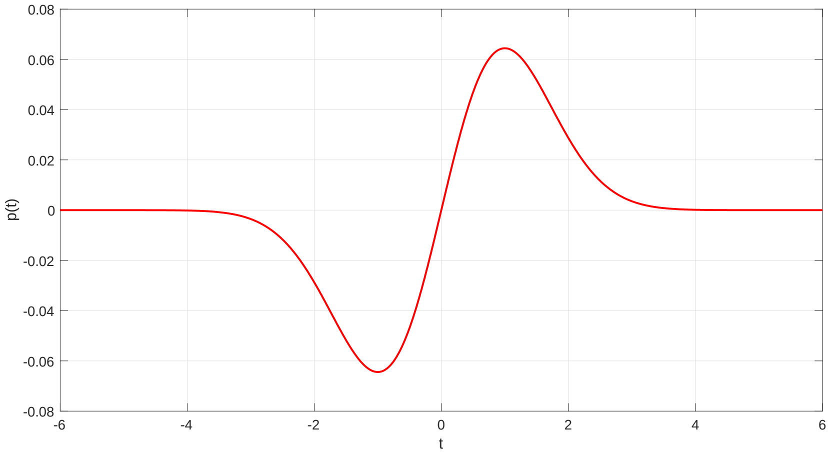
Hermite pulses

$$h_n(t) = (-1)^n \, e^{\frac{t^2}{4}} \frac{d^n}{(dt)^n} \left(e^{\frac{-t^2}{2}} \right)$$



Rayleigh pulses

$$p(t;\sigma) = \frac{t}{\sigma^2} e^{-\frac{t^2}{2\sigma^2}}$$

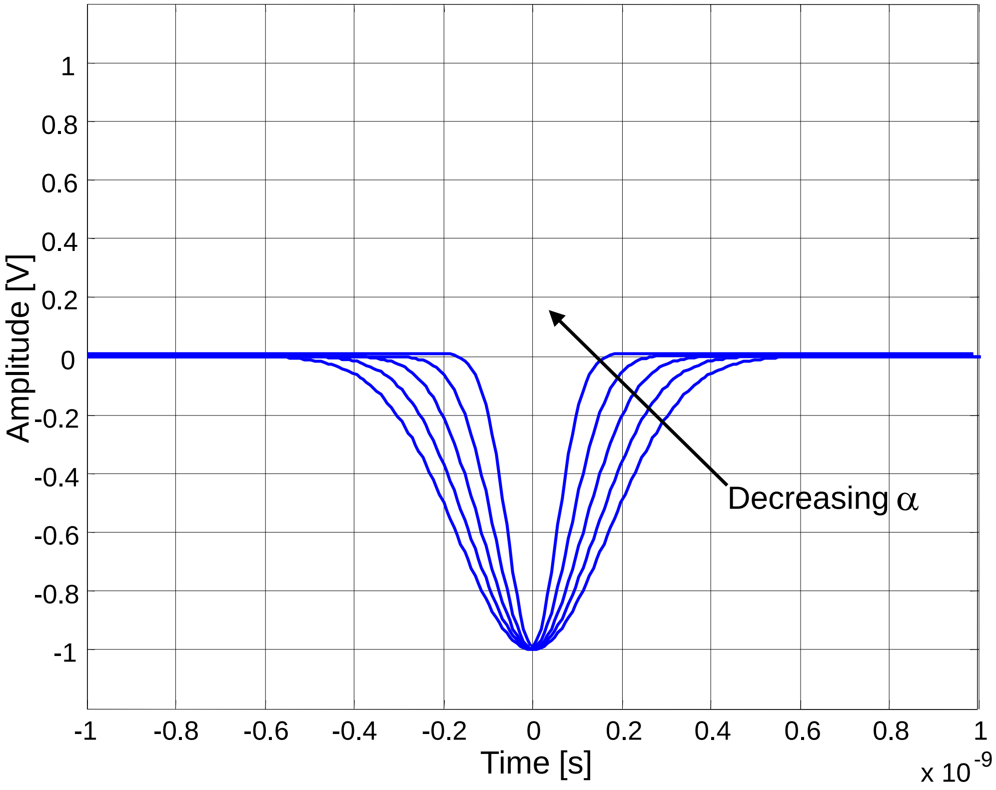


Spectrum Shaping by Pulse Shaping

- Shaping the spectrum by changing the pulse waveform is an interesting feature of Impulse Radio.
- Pulse shaping can be used to meet the FCC emission masks
- Basically, the spectrum may be shaped in three different ways:
  - Pulse width variation
  - Pulse differentiation
  - Combination of base functions

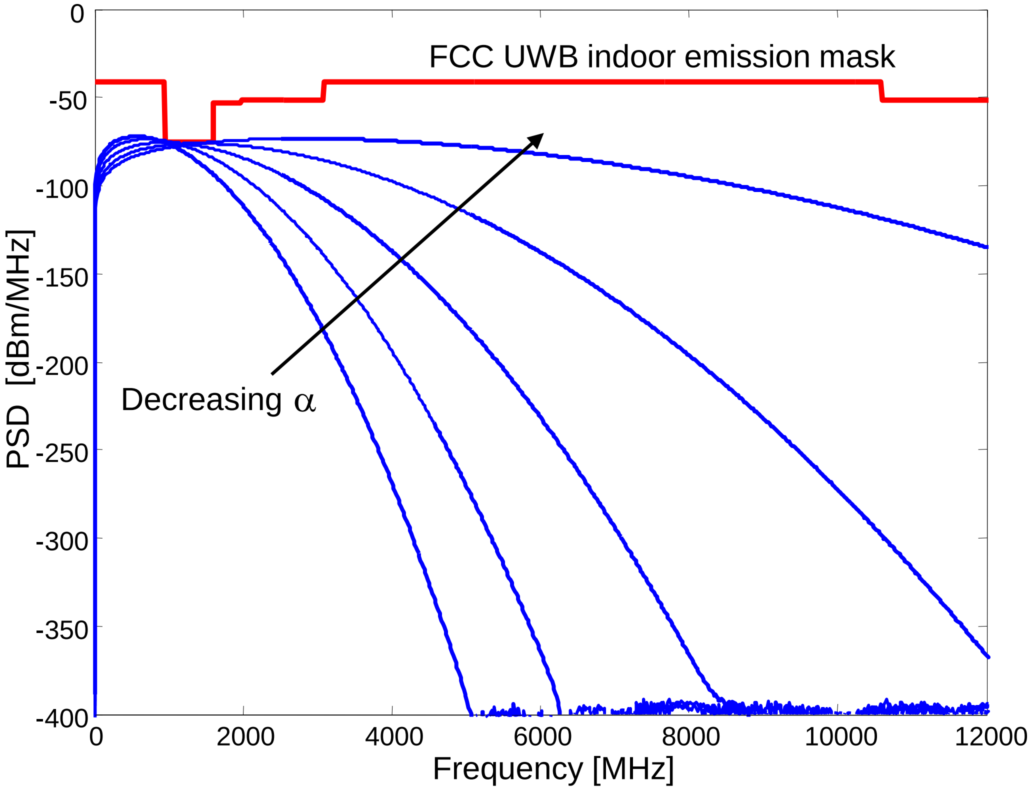
Pulse width variation (1/2)

- Pulse width variation is obtained by varying the shape factor α
- The smaller α, the shorter the pulse



Pulse width variation (2/2)

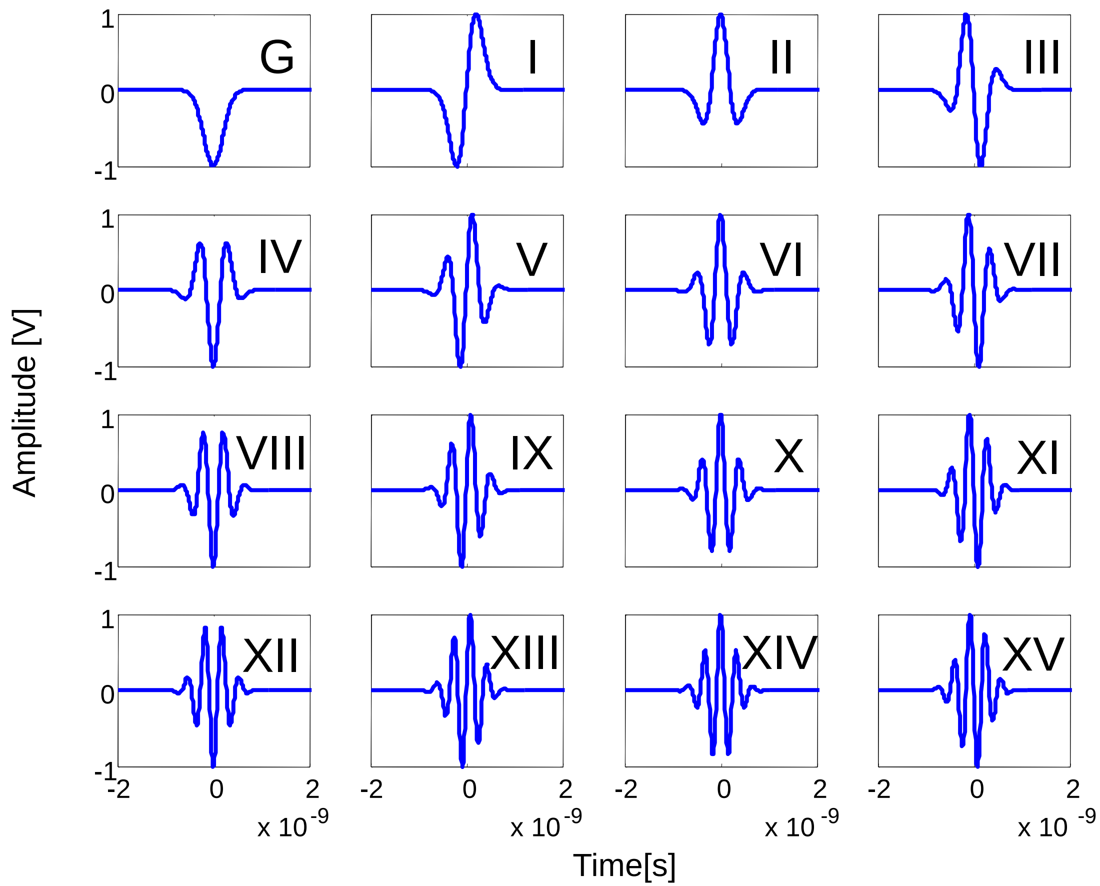
- A *shorter* pulse corresponds to a *larger* bandwidth occupation:



- Pulse width variation increases efficiency, but we can do better

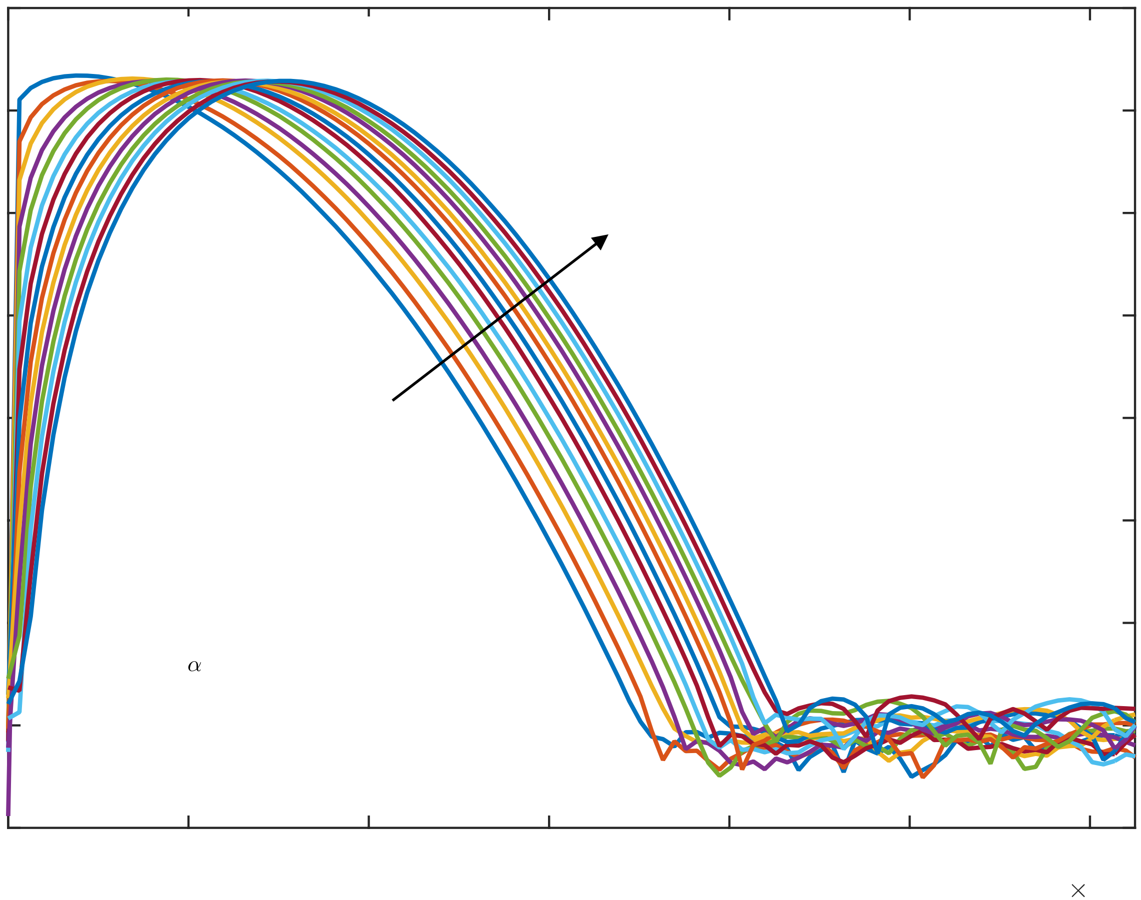
Pulse differentiation (in time) (1/3)

- Starting from the Gaussian pulse, we can generate new waveforms by pulse differentiation



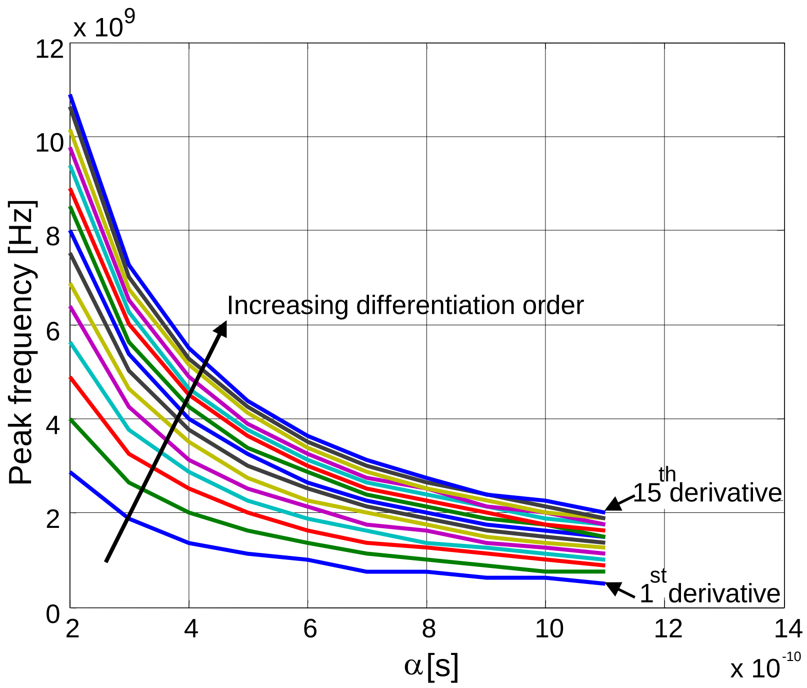
Pulse differentiation (in frequency) (1/3)

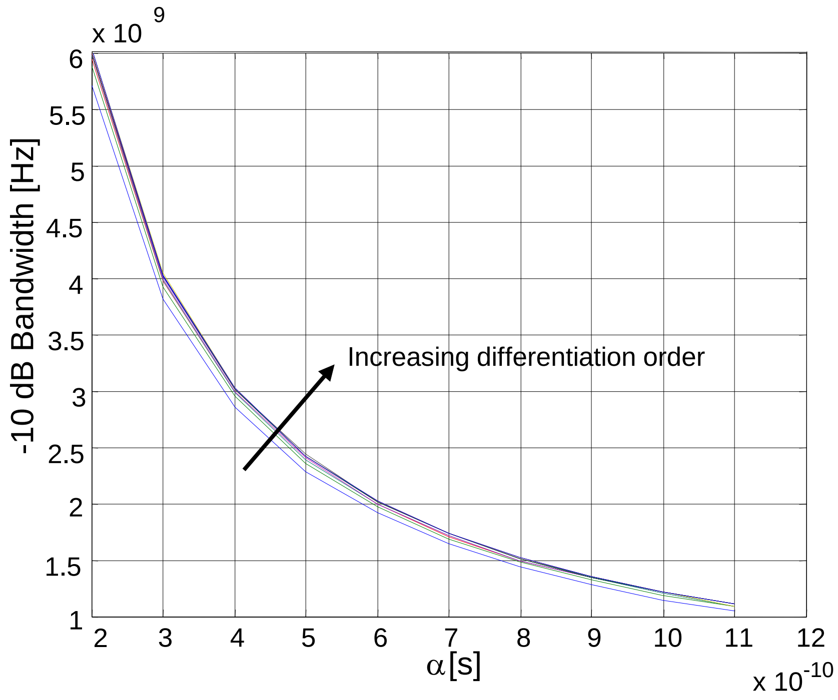
- Starting from the Gaussian pulse, we can generate new waveforms by pulse differentiation



Pulse differentiation (2/3)

- Differentiation leads to higher peak frequency in the PSD of the signal and to a larger -10 dB bandwidth:



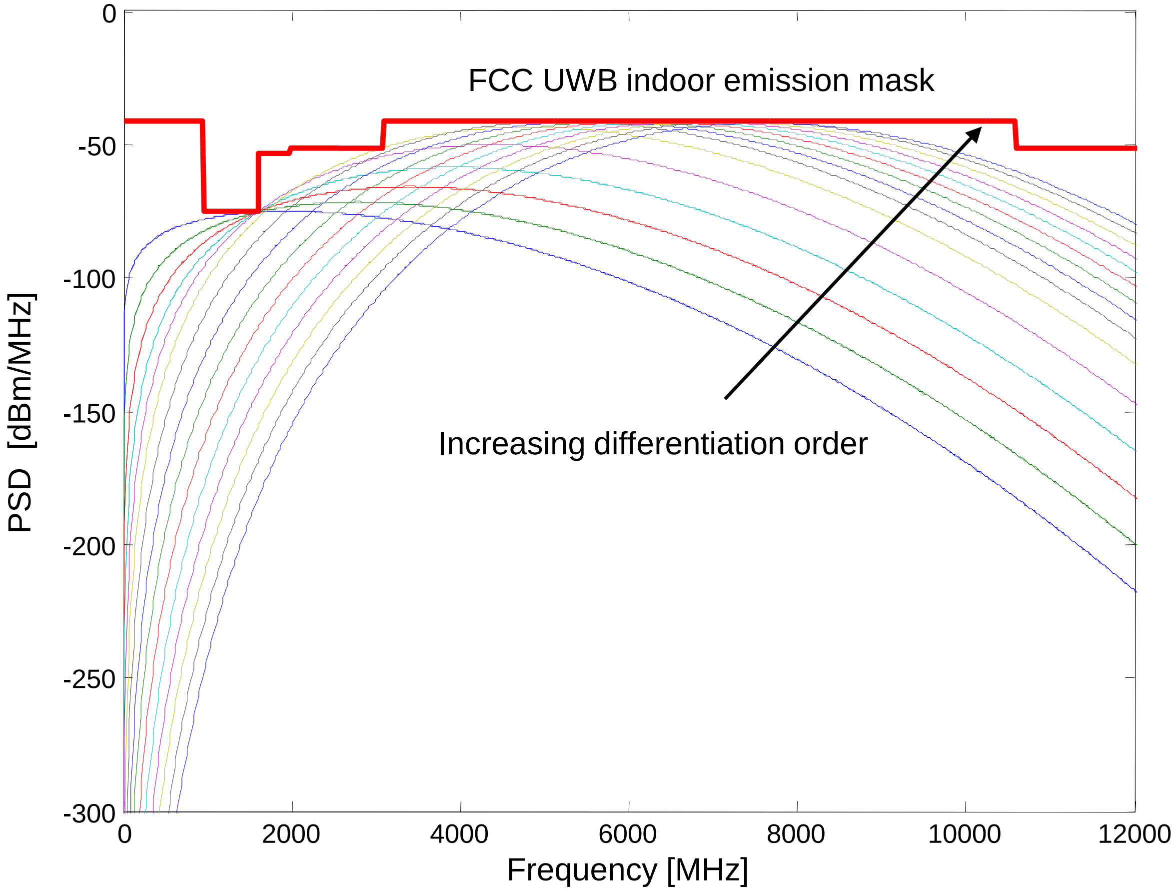


- The peak frequency follows the law: where *k* is the differentiation order

$$f_{peak,k} = \sqrt{k}\frac{1}{\alpha\sqrt{\pi}}$$

Pulse differentiation (3/3)

- The overall effect is to move power at higher frequencies:



Combining pulse width variation and differentiation (1/2)

- Pulse width variation and differentiation allow to modify the PSD of the emitted signal
- A single waveform *p(t)* does not allow to achieve efficient power use at all frequencies
- A set of different waveforms $p_i(t)$ (each corresponding to a different derivative with a different shape factor α_i) can be used to increase efficiency

Combining pulse width variation and differentiation (2/2)

- The transmitted waveform can be thus obtained as a linear combination of pulses having different pulse shapes or corresponding to different derivatives

$$p_{transmitted}(t) = \sum_{i=1}^{N} c_i p_i(t)$$

- Two typical approaches for determining the coefficients c_i are:
  - Random selection
  - Least Mean Square minimization

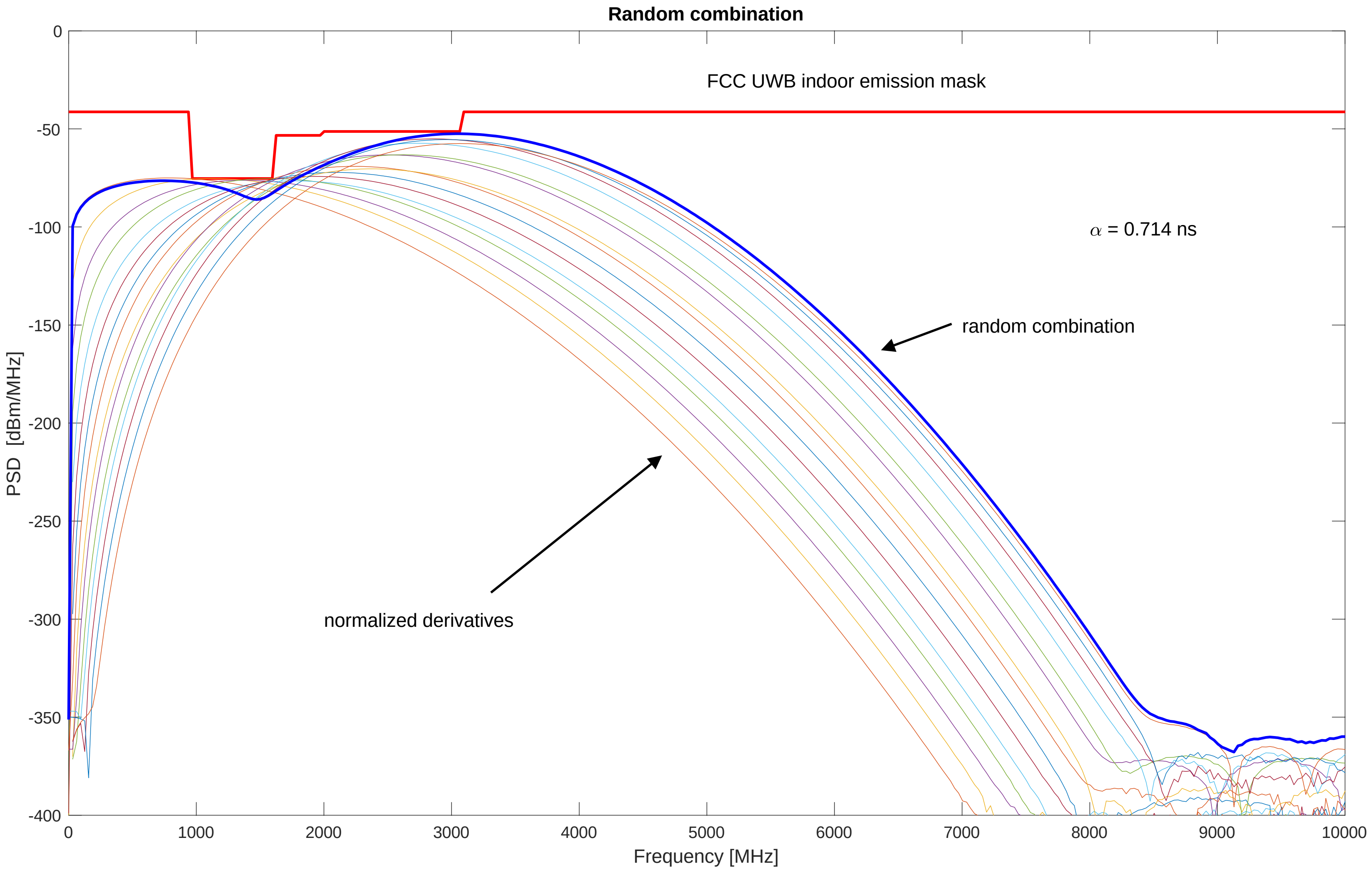
Random selection of coefficients (1/3)

Procedure for the random selection of the coefficients

1. Choose a set of Base Functions (BF).
2. Generate in a random way a set of coefficients, named S.
3. Check if the PSD of the linear combination of the functions obtained with coefficients S satisfies the emission limits.
4. If the emission limits in Step 3 are met and this is the first set S verifying the limits, then initialize the procedure by setting SB = S. If the emission limits in Step 3 are met and the procedure was already initialized, then compare S with SB; if S leads to a better waveform than SB according to well-defined distance metrics, set SB = S.
5. Repeat Steps 1–4 until the distance between the mask and PSD of the generated waveform falls below a fixed threshold.

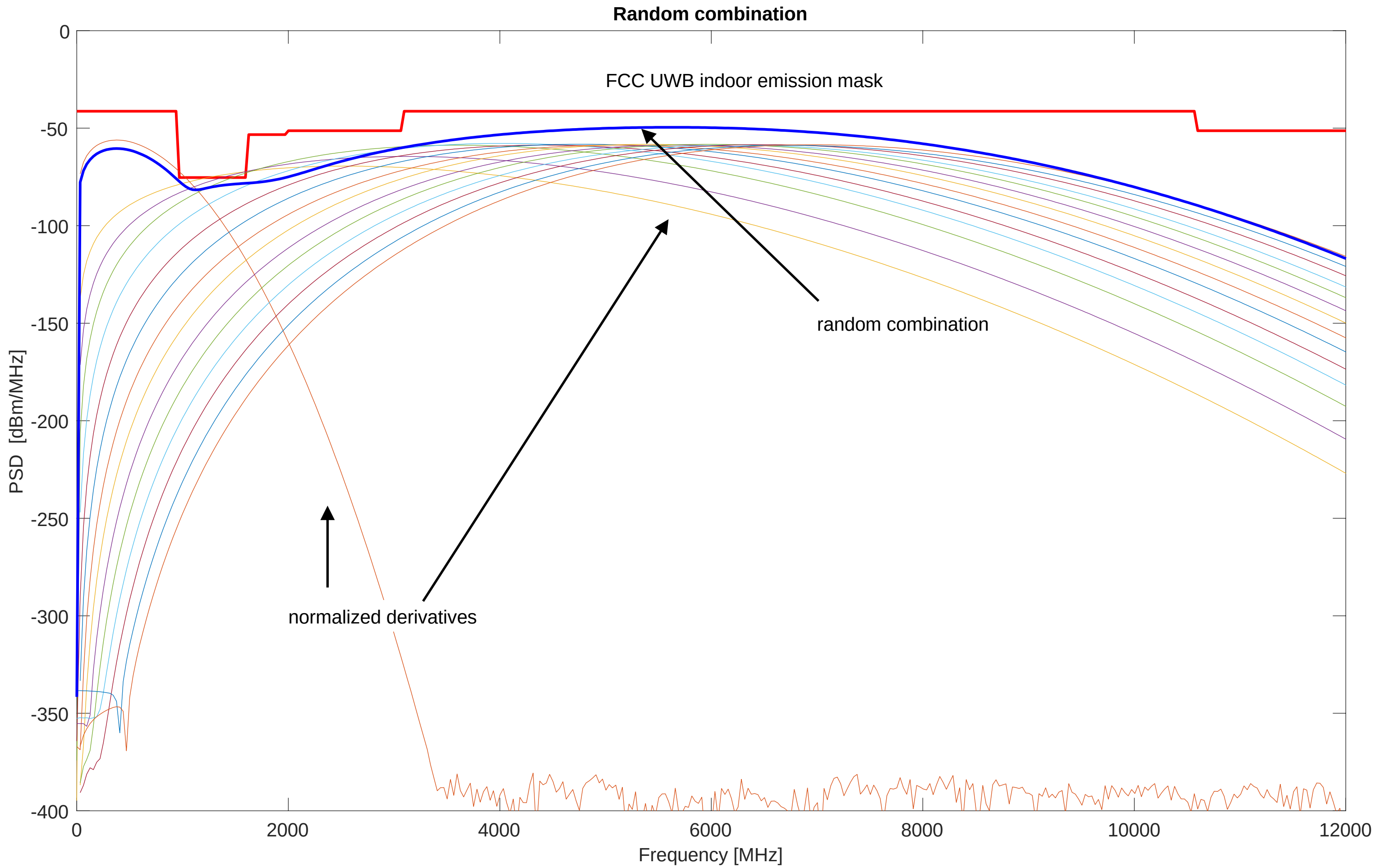
Random selection of coefficients (2/3)

- Example 1: all functions have same α



Random selection of coefficients (3/3)

- Example 2: first derivative has larger α:



Least Square Error minimization (1/4)

- Random selection is only one of the possible strategies for the set of coefficients in the linear combination.
- A more systematic way of selecting such coefficients is to apply standard procedures for error minimization such as the **Least Square Error** (**LSE**).
- With LSE, the following *error function* must be minimized:

$$e_s(t)=\int_{-\infty}^{+\infty}|e(t)|^2\,dt=\int_{-\infty}^{+\infty}\left|f(t)-\sum_{k=1}^{N}a_k f_k(t)\right|^2 dt$$

where *f(t)* is the target function

Least Square Error minimization (2/4)

- Since requirements are specified in terms of meeting a PSD, the error function writes as follows

$$e = \int_{-\infty}^{+\infty} \left| P_M(f) - F(f) \right|^2 df$$

$P_M(f)$ represents the emission mask
$F(f)$ is the PSD of the linear combination

- Since the envelope of the PSD envelope is mainly determined by the Fourier transform of the pulse shaper $p(t)$, one can express the optimization problem in the time domain

$$e = \int_{-\infty}^{+\infty} \left| m(t) - \sum_{k=1}^{N} a_k f_k(t) \right|^2 dt$$

$m(t)$ represents the mask bound in the time domain

Least Square Error minimization (4/4)

- As a general remark, note that the LSE method cannot guarantee by itself compliance with the emission mask.
- The optimization procedure is based on an average quadratic distance and does not impose bounds on a frequency-by-frequency basis.
- To guarantee compliance with the mask for each frequency, error minimization must be performed by imposing a bound on the PSD of the linear combination

Least Square Error minimization (3/4)

Envelope of the PSD of the linear combination of Gaussian waveforms vs. FCC indoor emission mask