Lecture #7 – November 8, 2021

Ultra Wide Band Communications

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Lecture 7

Multi User Interference models for Impulse Radio

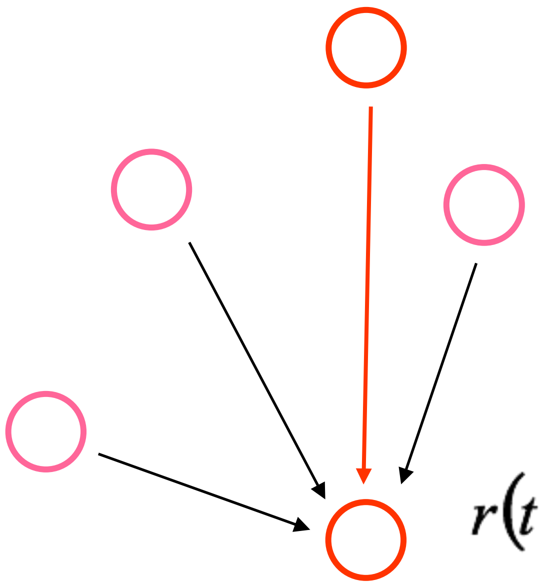
Outline

- Reference multi-user system model
- The effect of MUI on the decision variable
- BER evaluation under the Standard Gaussian Approximation
- BER evaluation under the Pulse Collision model

Reference system model (1/4)

- We consider a network where asynchronous devices transmit IR-UWB signals using binary orthogonal PPM with TH coding
- Propagation is over an AWGN channel

Transmitter



Receiver

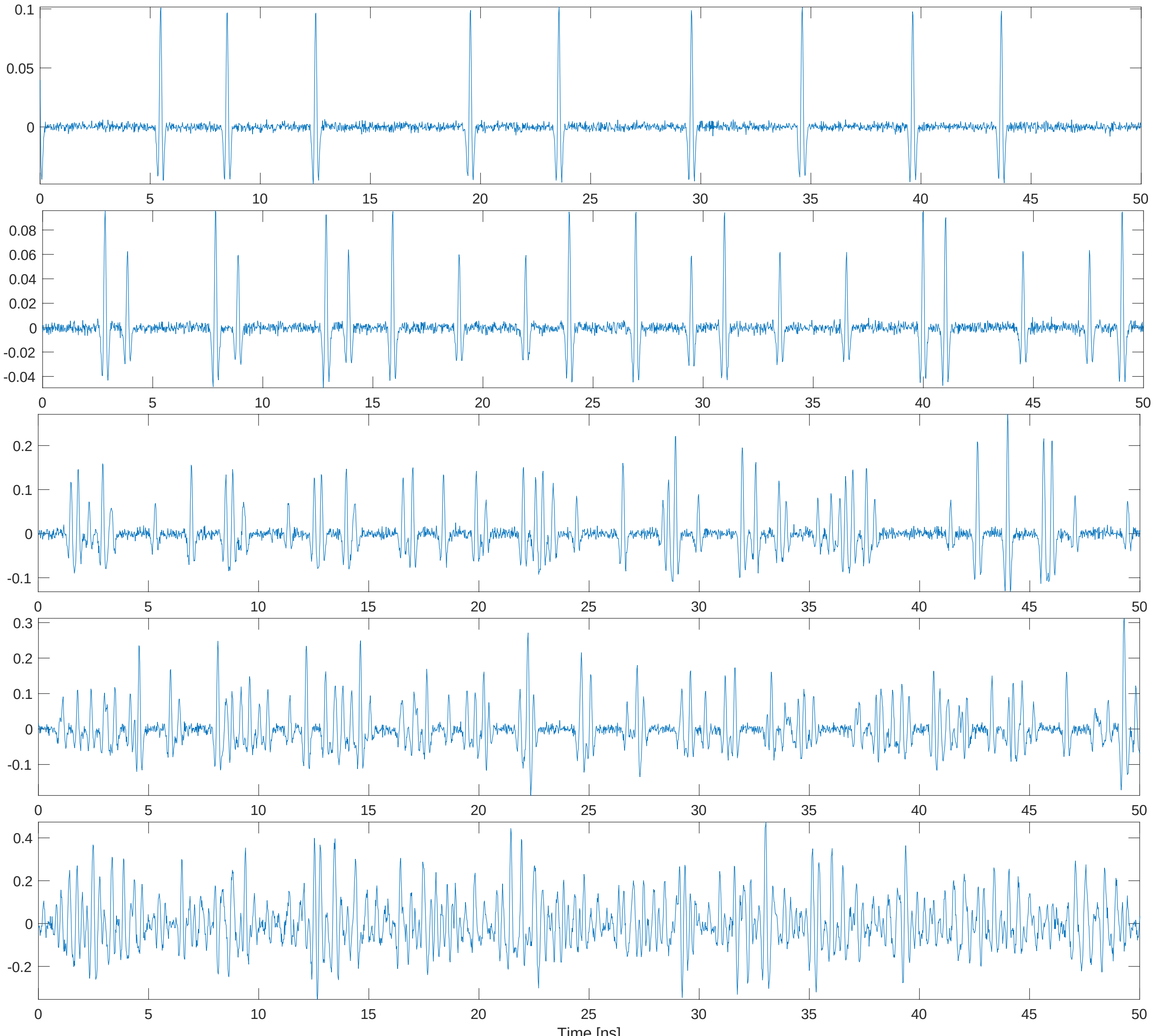
$$s_{TX}(t) = \sqrt{E_{TX}} \sum_j p_0\left(t - jT_S - c_j T_C - \varepsilon b_{\lfloor j/N_S \rfloor}\right)$$

$$r(t) = \alpha s_{TX}(t-\tau) + s_{mui}(t) + n(t) = s_{RX}(t) + s_{mui}(t) + n(t)$$

α : amplitude gain

τ : delay

Reference system model (2/4)



Received "useful" signal plus thermal noise

Received "useful" signal plus thermal noise and **1** interfering signal

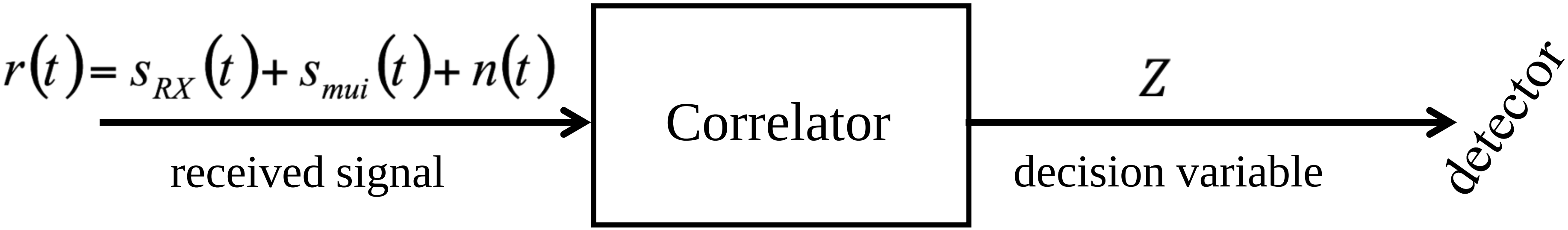
Received "useful" signal plus thermal noise and **5** interfering signal

Received "useful" signal plus thermal noise and **10** interfering signal

Received "useful" signal plus thermal noise and **50** interfering signal

Reference system model (3/4)

Optimum receiver for the AWGN channel



$$s_{RX}(t) = \sqrt{E_{RX}} \sum_j p_0\left(t - jT_S - c_j T_C - \varepsilon b_{\lfloor j/N_S \rfloor} - \tau\right)$$

$$s_{mui}(t) = \sum_{n=1}^{N_i} \sqrt{E^{(n)}} \sum_j p_0\left(t - jT_S - c_j^{(n)} T_C - \varepsilon b^{(n)}_{\lfloor j/N_S^{(n)} \rfloor} - \tau^{(n)}\right)$$

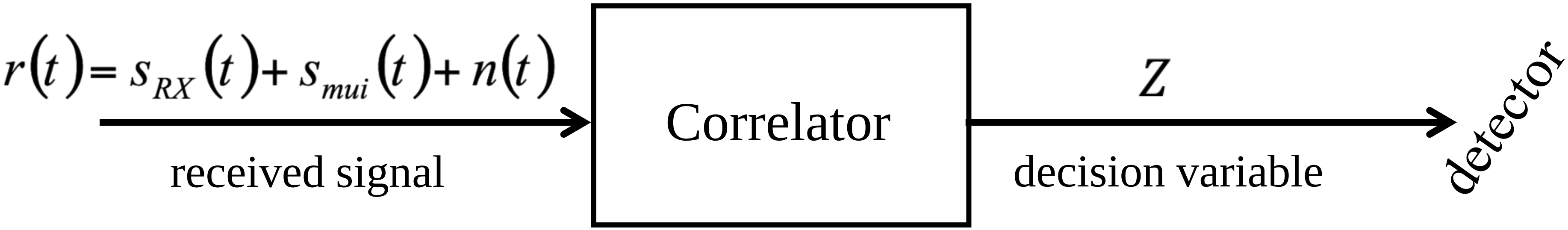
$n(t)$ Additive white Gaussian noise signal with double-sided spectral density $\mathcal{N}_0/2$

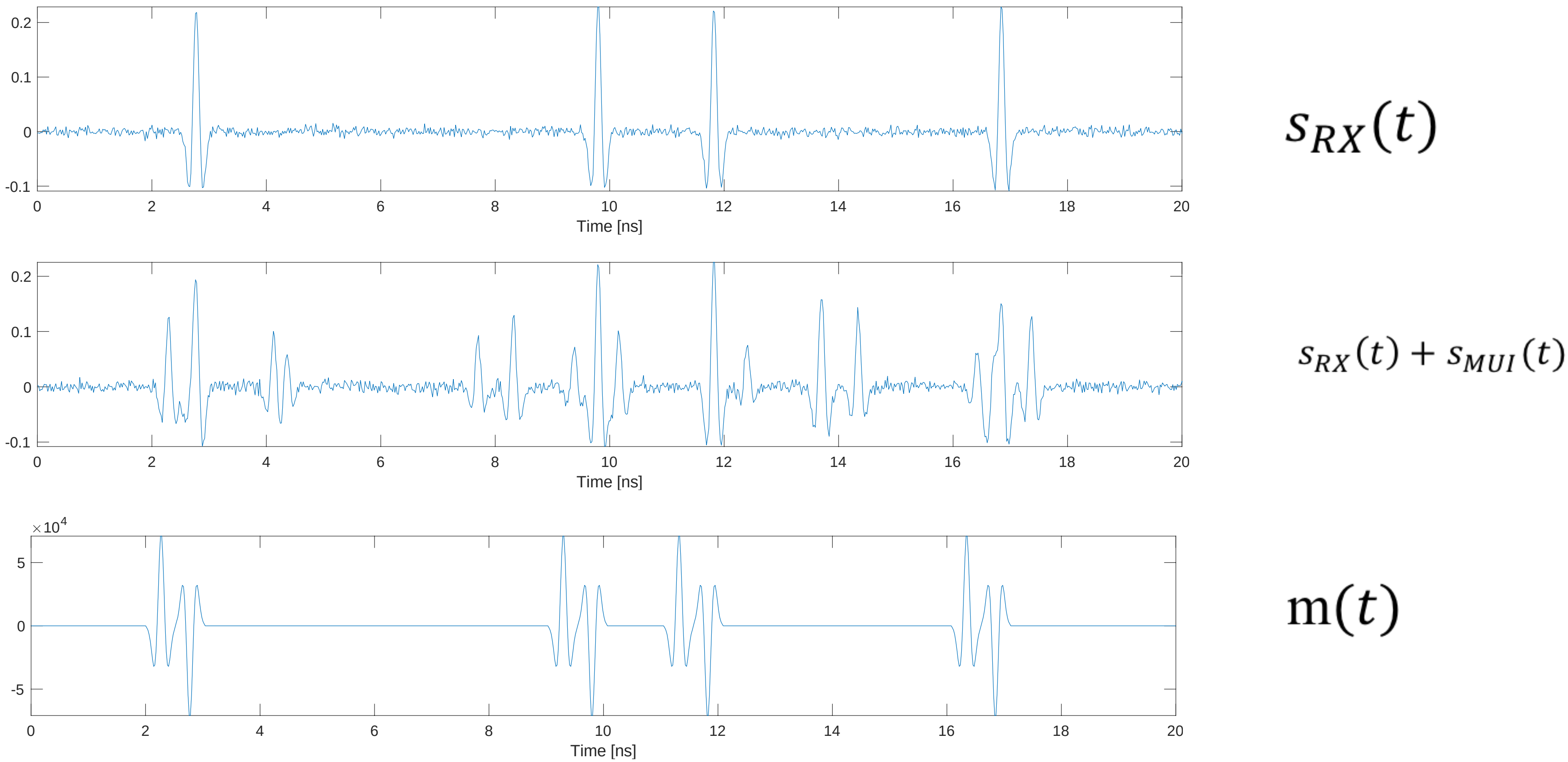
decision variable
$$Z = \int_{\tau}^{N_S T_S + \tau} r(t) m(t - \tau) dt$$

correlator mask
$$m(t) = \sum_{j=1}^{N_S} \left(p_0\left(t - jT_S - c_j T_C\right) - p_0\left(t - jT_S - c_j T_C - \varepsilon\right)\right)$$

Reference system model (4/4)

Optimum receiver for the AWGN channel





The decision variable (1/5)

decision variable

$$Z = \int_{\tau}^{N_S T_S + \tau} r(t) m(t-\tau) dt$$

$$Z = Z_u + Z_{MUI} + Z_n$$

the signal term

received signal

$$r(t) = s_{RX}(t) + s_{mui}(t) + n(t)$$

$$Z_u = \sqrt{E_{RX}} \int_{\tau}^{N_S T_S + \tau} \sum_{j=1}^{N_S} p_0\left(t - jT_S - c_j T_C - \varepsilon b_{\lfloor j/N_S \rfloor} - \tau\right) \cdot \sum_{j=1}^{N_S} \left(p_0\left(t - jT_S - c_j T_C - \tau\right) - p_0\left(t - jT_S - c_j T_C - \varepsilon - \tau\right)\right) dt$$

$$Z_u = N_S \sqrt{E_{RX}} \int_0^{T_S} p_0(t - \varepsilon b) \cdot \left(p_0(t) - p_0(t - \varepsilon)\right) dt$$

$$Z_u = \pm N_S \sqrt{E_{RX}} \left(1 - R_0(\varepsilon)\right) = \begin{cases} + N_S \sqrt{E_{RX}} \left(1 - R_0(\varepsilon)\right) & for\ b = 0 \\ - N_S \sqrt{E_{RX}} \left(1 - R_0(\varepsilon)\right) & for\ b = 1 \end{cases}$$

The decision variable (2/5)

decision variable

$$Z = \int_{\tau}^{N_S T_S + \tau} r(t) m(t-\tau) dt$$

$$Z = Z_u + Z_{MUI} + Z_n$$

the MUI term

received signal

$$r(t) = s_{RX}(t) + s_{mui}(t) + n(t)$$

$$Z_{mui} = \sum_{i=1}^{N_i} \sqrt{E^{(i)}} \sum_{j=0}^{N_S} \int_{-\infty}^{\infty} p_0(t - \theta_j)(p_0(t) - p_0(t - \varepsilon)) dt$$

$\theta_j = \tau_j$ is a random variable uniformly distributed over $[0, T_S)$

The decision variable (3/5)

decision variable

$$Z = \int_{\tau}^{N_S T_S + \tau} r(t) m(t-\tau) dt$$

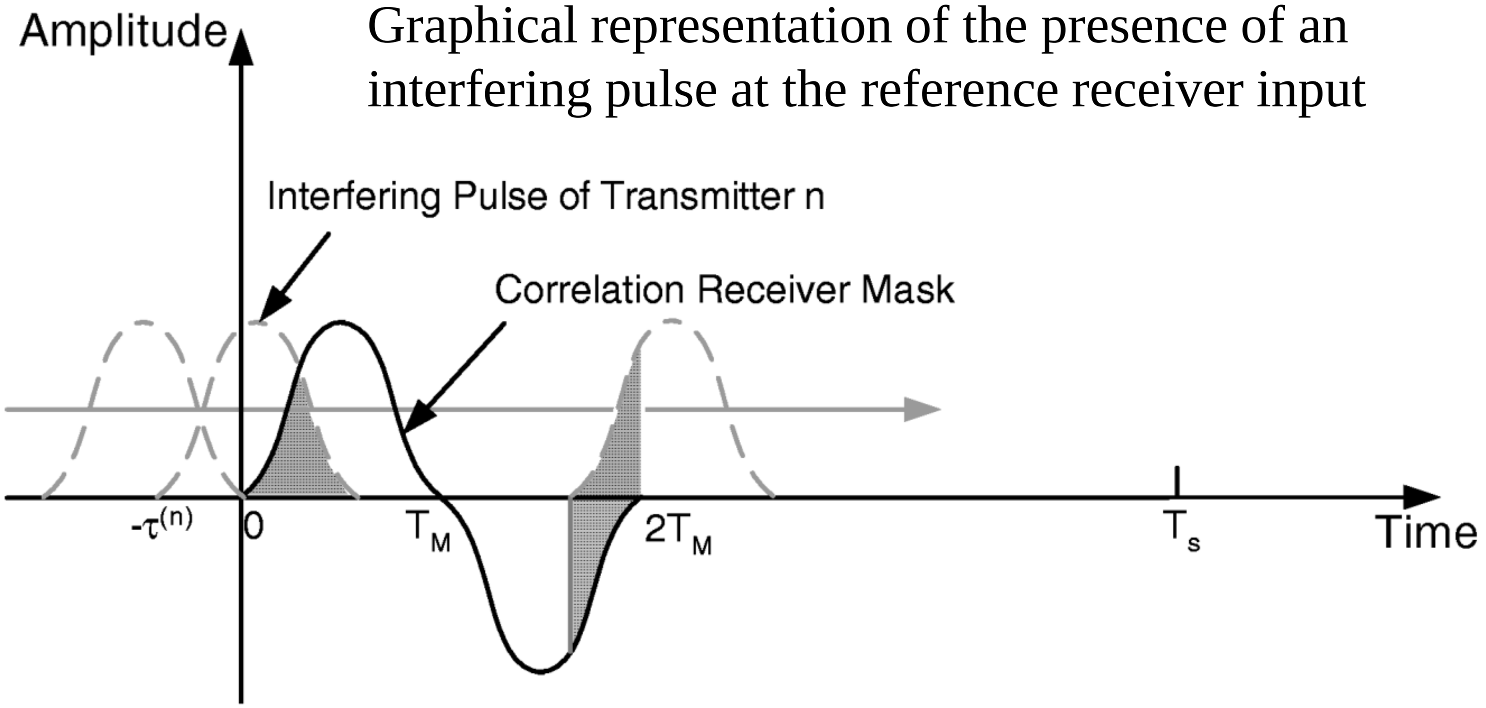
$$Z = Z_u + Z_{MUI} + Z_n$$

the MUI term

received signal

$$r(t) = s_{RX}(t) + s_{mui}(t) + n(t)$$

Graphical representation of the presence of an interfering pulse at the reference receiver input



The decision variable (4/5)

decision variable

$$Z = \int_{\tau}^{N_S T_S + \tau} r(t) m(t - \tau) dt$$

$$Z = Z_u + Z_{MUI} + Z_n$$

the MUI term

received signal

$$r(t) = s_{RX}(t) + s_{mui}(t) + n(t)$$

$$Z_{mui} = \sum_{i=1}^{N_i} \sqrt{E^{(i)}} \sum_{j=0}^{N_S} \int_{-\infty}^{\infty} p_0(t - \theta_j)(p_0(t) - p_0(t - \varepsilon))dt$$

Z_{mui} is a random variable with mean zero and variance:

$$\sigma_{mui}^2 = \sum_{i=1}^{N_i} E^{(i)} N_S \frac{1}{T_S} \underbrace{\int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} p_0(t - \tau)(p_0(t) - p_0(t - \varepsilon))dt \right)^2 d\tau}_{\sigma_M^2}$$

σ_M^2 constant term which depends on both pulse shape and PPM shift

$$\sigma_{mui}^2 = \sum_{i=1}^{N_i} E^{(i)} N_S \frac{\sigma_M^2}{T_S} = \frac{N_S}{T_S} \sigma_M^2 \sum_{i=1}^{N_i} E^{(i)}$$

The decision variable (5/5)

decision variable

$$Z = \int_{\tau}^{N_S T_S+\tau} r(t)m(t-\tau)dt$$

$$Z = Z_u + Z_{MUI} + Z_n$$

the noise term

received signal

$$r(t) = s_{RX}(t) + s_{mui}(t) + n(t)$$

$$Z_n = \int_{\tau}^{N_S T_S+\tau} n(t) \cdot \sum_{j=1}^{N_S} \left(p_0\left(t - jT_S - c_j T_C\right) - p_0\left(t - jT_S - c_j T_C - \varepsilon\right)\right)dt$$

$n(t)$ is a white Gaussian random process with uniform **double-sided** Power Spectral Desnity $\mathcal{N}_0/2$

Z_n is a Gaussian random variable with mean zero and variance:

$$\sigma_n^2 = N_S \mathcal{N}_0 \left(1 - R_0(\varepsilon)\right)$$

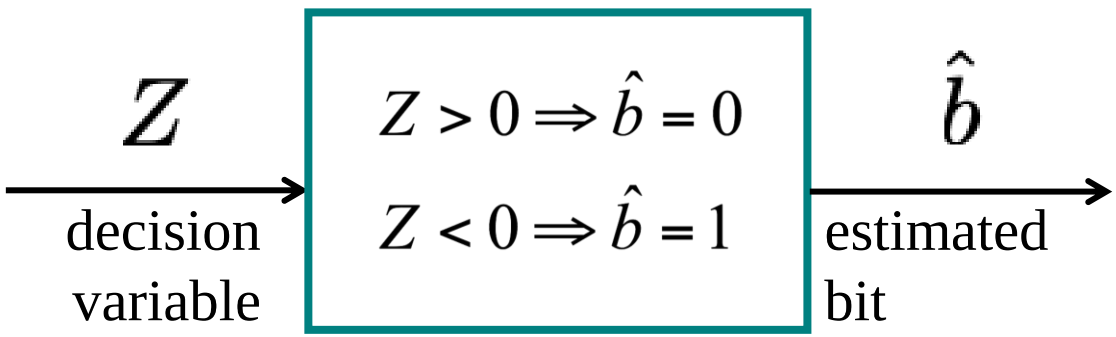
The decision rule

decision variable

$$Z = Z_u + Z_{MUI} + Z_n$$

$$Z = \begin{cases} +N_S\sqrt{E_{RX}}\left(1-R_0(\varepsilon)\right)+Z_{mui}+Z_n & for\ b=0 \\ -N_S\sqrt{E_{RX}}\left(1-R_0(\varepsilon)\right)+Z_{mui}+Z_n & for\ b=1 \end{cases}$$

decision rule



$$\Pr_b = Prob\left(Z < 0 \mid b = 0\right) = Prob\left(N_S\sqrt{E_{RX}}\left(1-R_0(\varepsilon)\right)+Z_{mui}+Z_n < 0\right)$$

average probability of error at the receiver output

The Standard Gaussian Approximation (1/6)

- The **Standard Gaussian Approximation** (**SGA**) hypothesis assumes that Z_{mui}, as well as Z_n, is a Gaussian random process.

average probability of error at the receiver output

$$\Pr_b = Prob\left(N_S\sqrt{E_{RX}}\left(1 - R_0(\varepsilon)\right) + Z_{mui} + Z_n < 0\right)$$

Gaussian random variable with mean zero and variance σ_{MUI}^2

Gaussian random variable with mean zero and variance σ_n^2

$$\Pr_b = Prob\left(N_S\sqrt{E_{RX}}\left(1 - R_0(\varepsilon)\right) + Z_{mui} + Z_n < 0\right)$$

Gaussian random variable with mean zero and variance $\sigma_{MUI}^2 + \sigma_n^2$

The Standard Gaussian Approximation (2/6)

$$\Pr_b = Prob\left(N_S\sqrt{E_{RX}}\left(1-R_0(\varepsilon)\right)+Z_{mui}+Z_n<0\right)$$

Gaussian random variable with mean zero and variance $\sigma_{MUI}^2+\sigma_n^2$

$$\Pr_b = \frac{1}{2}erfc\left(\sqrt{\frac{1}{2}SNR_{spec}}\right)$$

$$SNR_{spec} = \frac{|Z_u|^2}{\sigma_{mui}^2+\sigma_n^2} = \left(\left(\frac{|Z_u|^2}{\sigma_n^2}\right)^{-1}+\left(\frac{|Z_u|^2}{\sigma_{mui}^2}\right)^{-1}\right)^{-1} = \left((SNR_n)^{-1}+(SNR_{mui})^{-1}\right)^{-1}$$

$$SNR_n = \frac{N_S^2E_{RX}\left(1-R_0(\varepsilon)\right)^2}{N_S\mathcal{N}_0\left(1-R_0(\varepsilon)\right)} = N_S\frac{E_{RX}}{\mathcal{N}_0}\left(1-R_0(\varepsilon)\right) = \frac{E_b}{\mathcal{N}_0}\left(1-R_0(\varepsilon)\right)$$

$$SNR_{mui} = \frac{N_S^2E_{RX}\left(1-R_0(\varepsilon)\right)^2}{\frac{N_S}{T_S}\sigma_M^2\sum_{i=1}^{N_i}E^{(i)}} = \frac{T_SN_S\left(1-R_0(\varepsilon)\right)^2}{\sigma_M^2\sum_{i=1}^{N_i}\frac{E^{(i)}}{E_{RX}}} = \frac{\left(1-R_0(\varepsilon)\right)^2}{R_b\sigma_M^2\sum_{i=1}^{N_i}\frac{E^{(i)}}{E_{RX}}}$$

The Standard Gaussian Approximation (3/6)

$$\Pr_b = \frac{1}{2} erfc\left(\sqrt{\frac{1}{2}\left(\left(N_S \frac{E_{RX}}{\mathcal{N}_0}\left(1 - R_0(\varepsilon)\right)\right)^{-1} + \left(\frac{\left(1 - R_0(\varepsilon)\right)^2}{R_b \sigma_M^2 \sum_{i=1}^{N_i} \frac{E^{(i)}}{E_{RX}}} \right)^{-1} \right)^{-1}} \right)$$

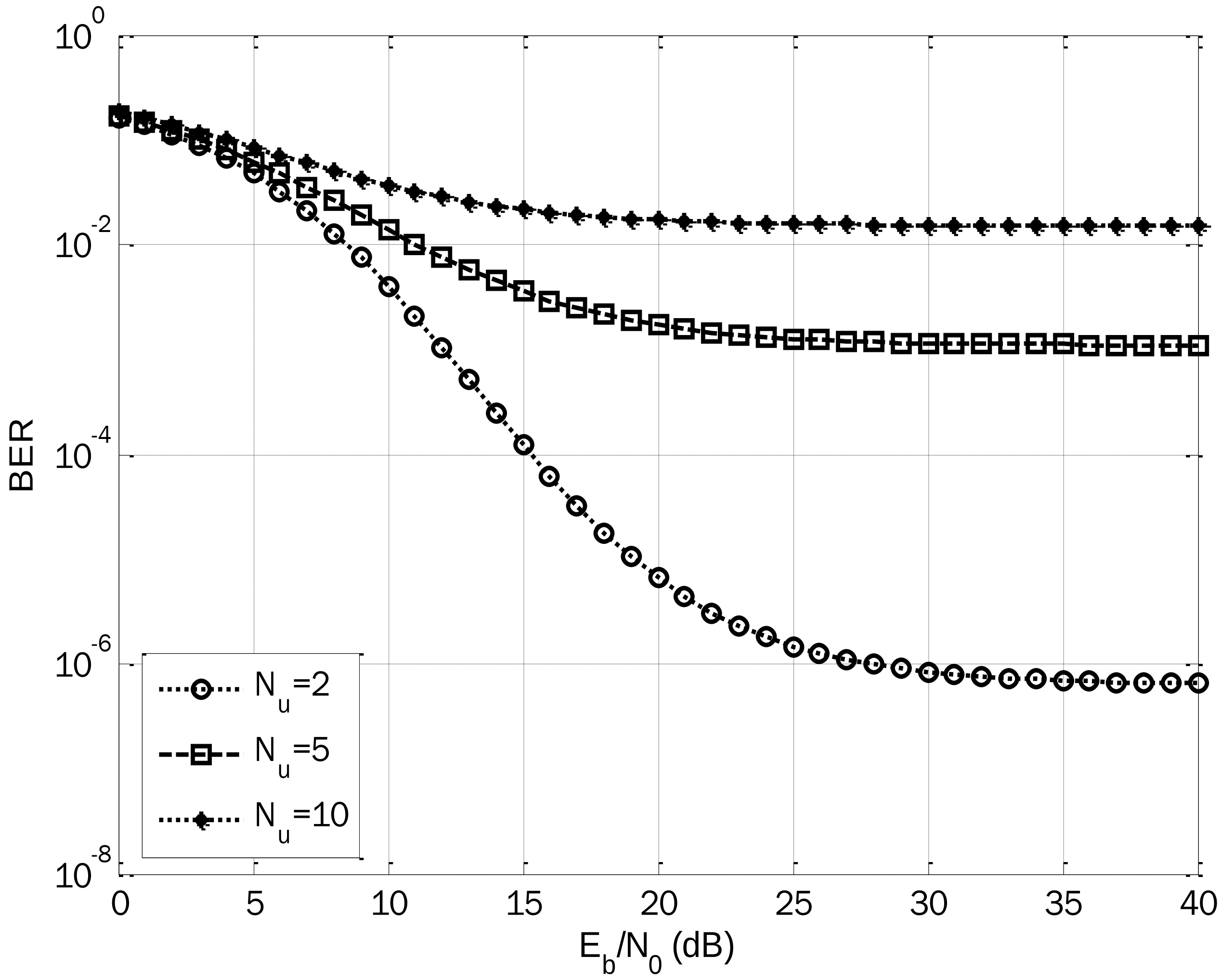
average probability of error for a multi-user 2PPM-TH-IR-UWB system

$$\Pr_b = \frac{1}{2} erfc\left(\sqrt{\frac{1}{2}\left(\left(N_S \frac{E_{RX}}{\mathcal{N}_0}\left(1 - R_0(\varepsilon)\right)\right)^{-1} + \left(\frac{\left(1 - R_0(\varepsilon)\right)^2}{R_b \sigma_M^2 N_i} \right)^{-1} \right)^{-1}} \right)$$

average probability of error for a multi-user 2PPM-TH-IR-UWB system with power control at the reference receiver

- By applying the same analytical passages, the above results can be easily extended to the PAM case.
- In both cases, the MUI term can be controlled by fixing the maximum value of the user bit rate R_b and/or the maximum number of asynchronous transmission in the network.

The Standard Gaussian Approximation (4/6)



10 interferers

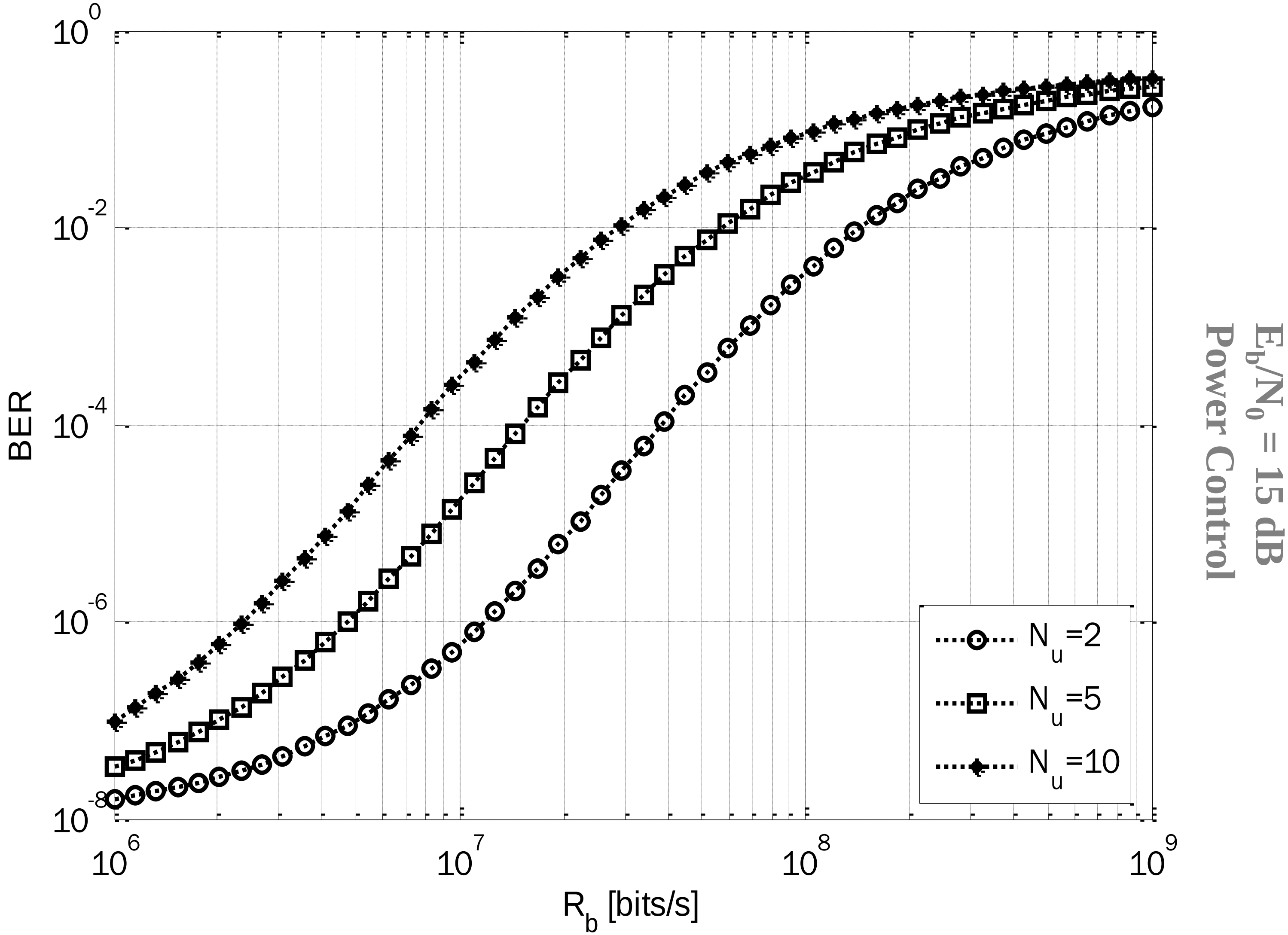
5 interferers

2 interferers

R_b = 40 Mb/s
Power Control

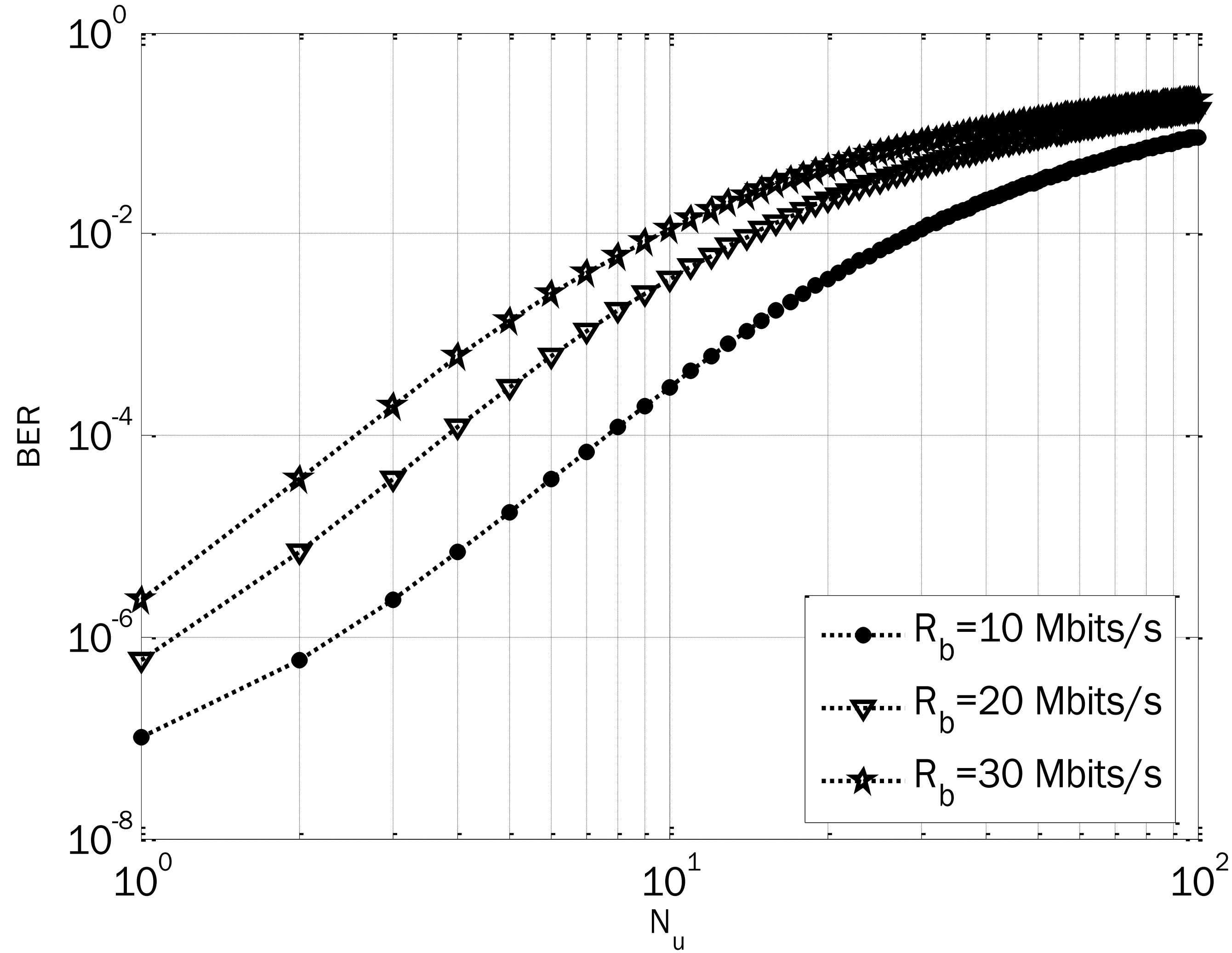
BER as a function of the Signal to Thermal Noise Ratio

The Standard Gaussian Approximation (5/6)



BER as a function of the user bit rate

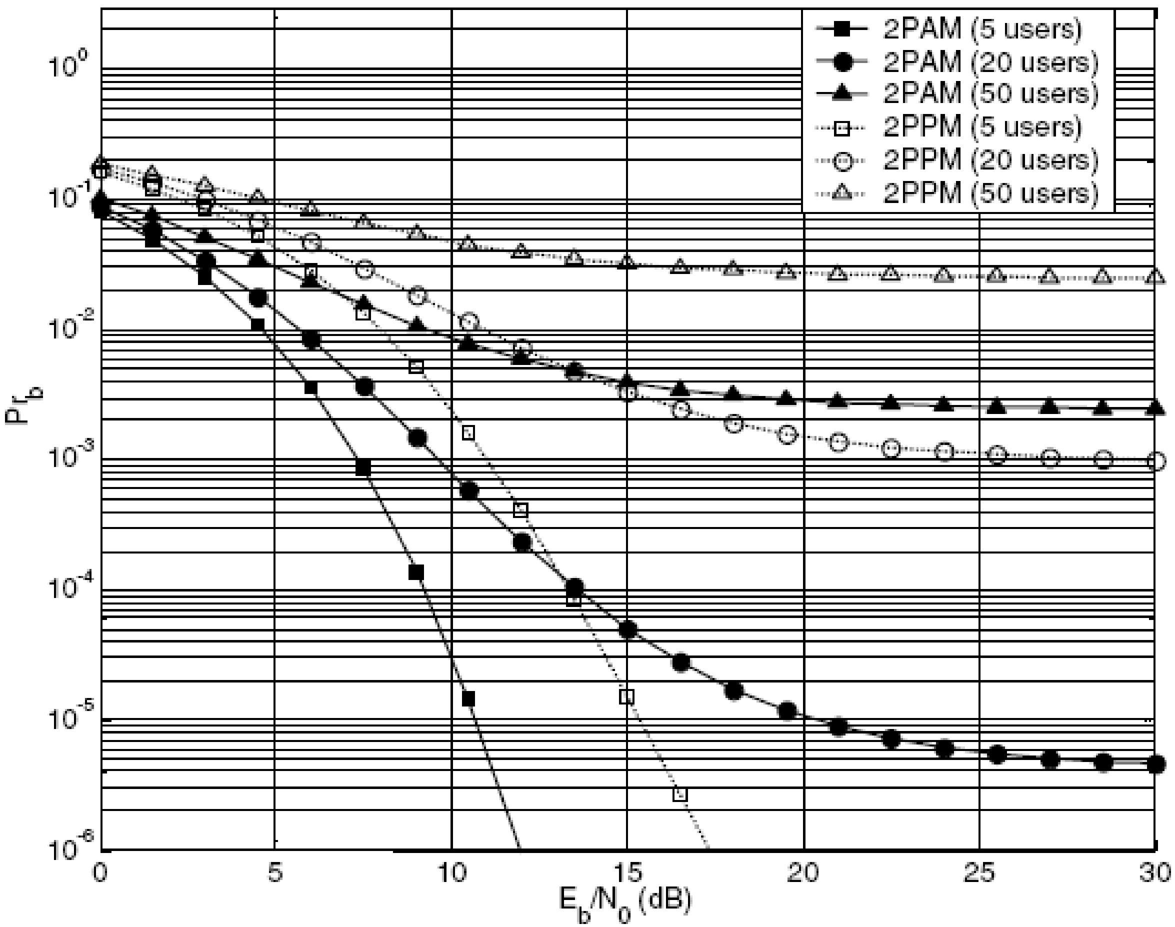
The Standard Gaussian Approximation (6/6)



E_b/N_0 = 15 dB
Power Control

BER as a function of the number of interferers

PAM vs. PPM



Theoretical probability of error on the bit $\mathbf{Pr_b}$ versus the signal to noise ratio at the receiver $\mathbf{E_b/\mathcal{N}_0}$ in presence of multiple uncoordinated users.

White markers are for 2PPM-TH-UWB

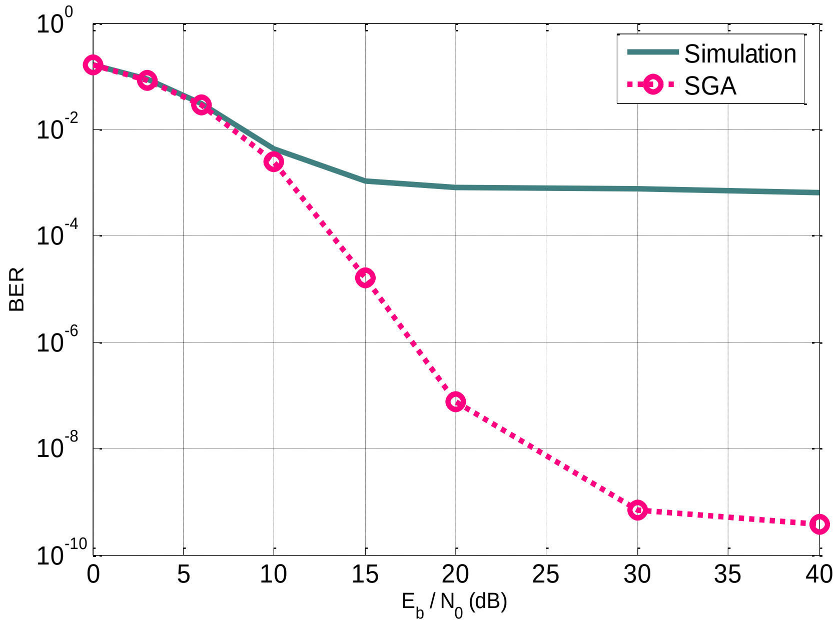
Black markers are for 2PAM-TH-UWB

Squares: 5 users
Circles: 20 users
Triangles: 50 users

On the validity of the SGA for IR systems

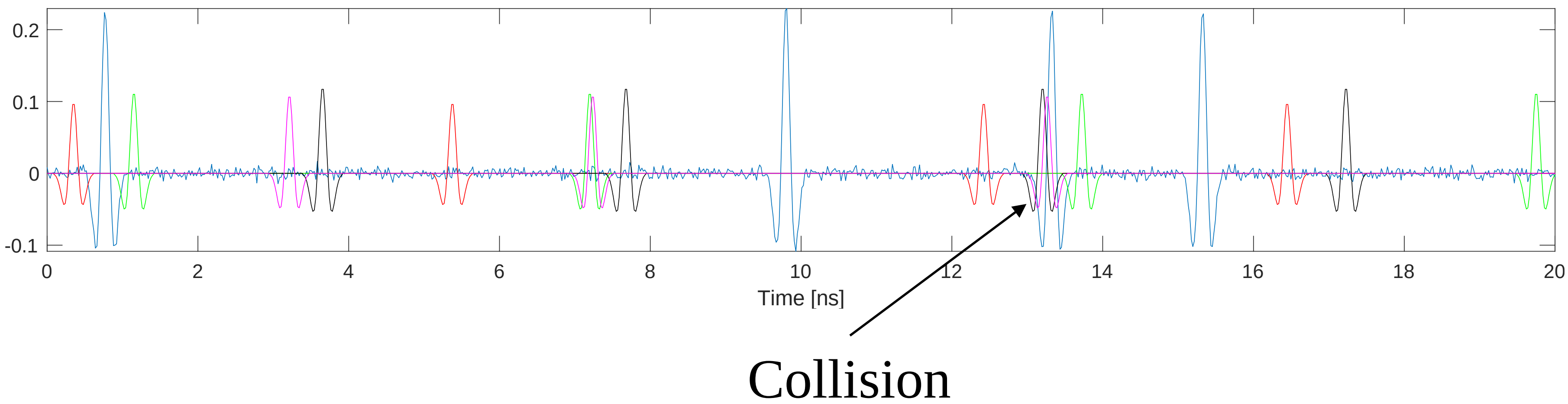
- The SGA derives from the central limit theorem, and thus it is valid only **asymptotically**
- The SGA was demonstrated to provide optimistic BER predictions, in particular when low data rates and sparse topologies are taken into account.

Simulation vs. theoretical prediction for a network scenario with 4 total users transmitting at 22.22 Mb/s.



The Pulse Collision Approach (1/4)

- When IR systems are taken into account, interference at the reference receiver is provoked by **collisions** occurring between pulses belonging to different transmissions.



- Based on the above observation, we proposed a novel approach for estimating BER in multi user UWB networks (the **Pulse Collision** approach).

The Pulse Collision Approach (2/4)

- System performance can be evaluated by first computing the probability of pulse collisions at the receiver, and by then estimating the probability of error in presence of collisions.

$$Pr_{PC} = 1 - e^{\left(-2(N_U - 1)\frac{T_M}{T_s}\right)}$$

Collision probability among N_U asynchronous users, when modeling the pulse interarrival process as a Poisson Process. T_M is the pulse duration, T_S is the average pulse repetition period.

$$Pr_{PE} = 0.5 Pr_{PE}$$

Approximated Pulse Error probability

$$Pr_b = \sum_{i=\lceil \frac{N_s}{2} \rceil}^{N_s} \binom{N_s}{i} (Pr_{PE})^i (1 - Pr_{PE})^{N_s - i}$$

Average Probability of Error on the bit, when assuming N_S pulses transmitted per bit and hard decision detection at the receiver

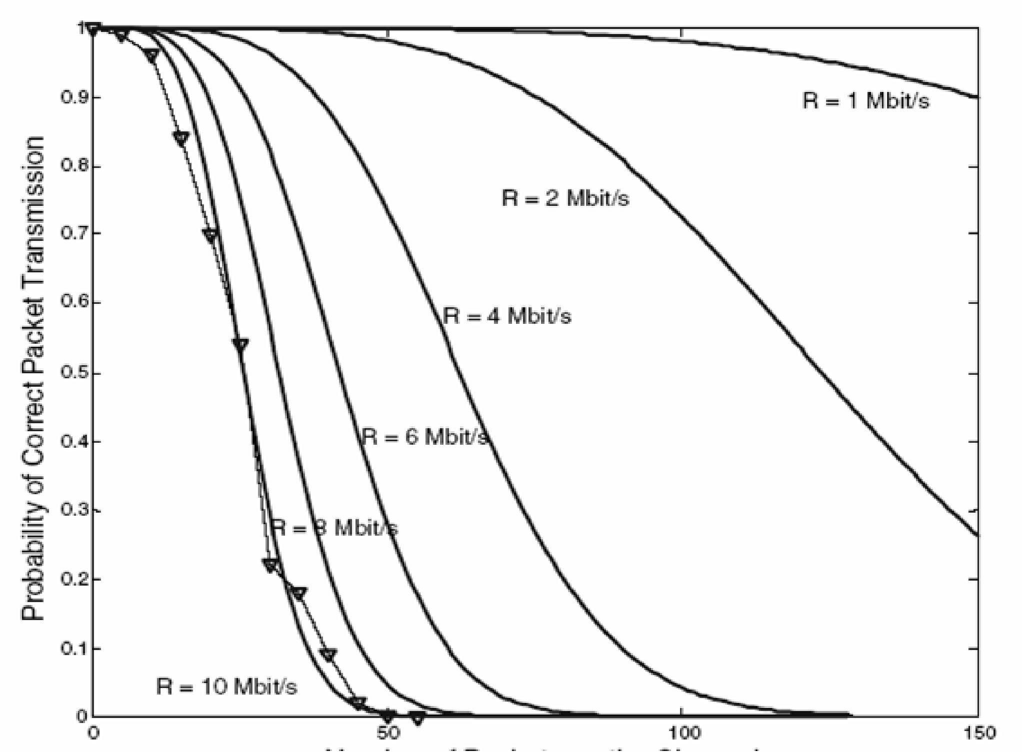
The Pulse Collision Approach (3/4)

$$Pr_b = \sum_{i=\lceil \frac{N_s}{2} \rceil}^{N_s} \binom{N_s}{i} (Pr_{PE})^i (1 - Pr_{PE})^{N_s - i}$$

Average Probability of Error on the bit

$$Pr_{Succ} = (1 - Pr_b)^L$$

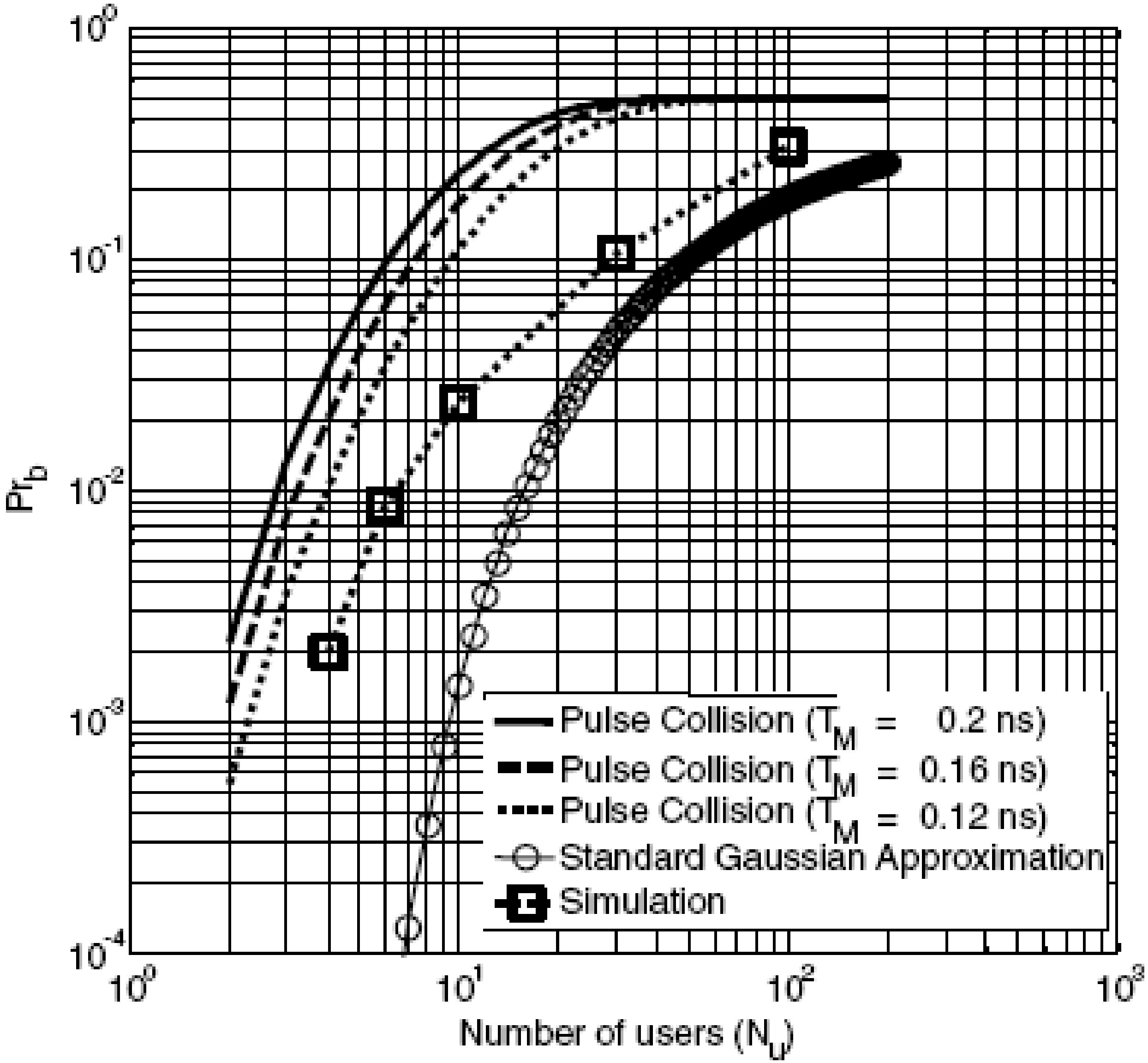
Average Probability of Success for the transmission of a packet having length of *L* bits



Probability of correct packet transmission vs. number of packets on the channel for different values of bit rate R.

Solid lines represent theoretical performance; dashed line shows the simulation performance in the case R_b = 10 Mbits/s

The Pulse Collision Approach (4/4)



Comparison of the probability of error Pr_b vs. number of users N_u derived from theoretical models (PC and SGA) and from simulation. Different time durations of the pulse are considered for the PC model