Lecture #8 November 15, 2021

Ultra Wide Band Radio Fundamentals

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Lecture 8

Ranging and Positioning
with UWB

Outline

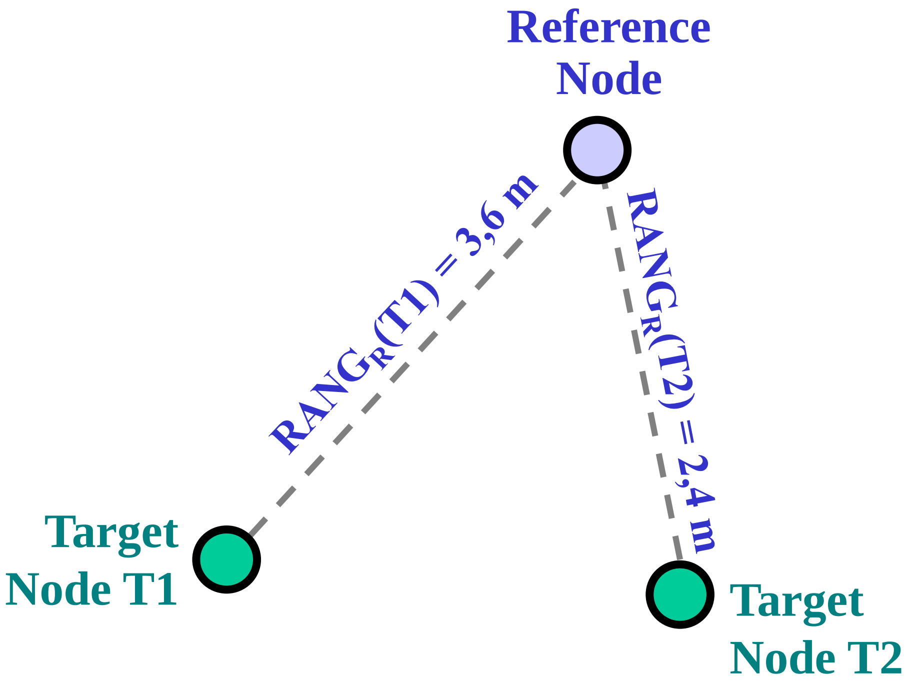
- Ranging
  - RSSI vs. TOA
  - The early-late gate synchronizer
  - Ranging in UWB
- Positioning
  - Node-centered and relative positioning
  - Spherical positioning
  - Positioning with LSE minimization

Position-aware distributed wireless networks

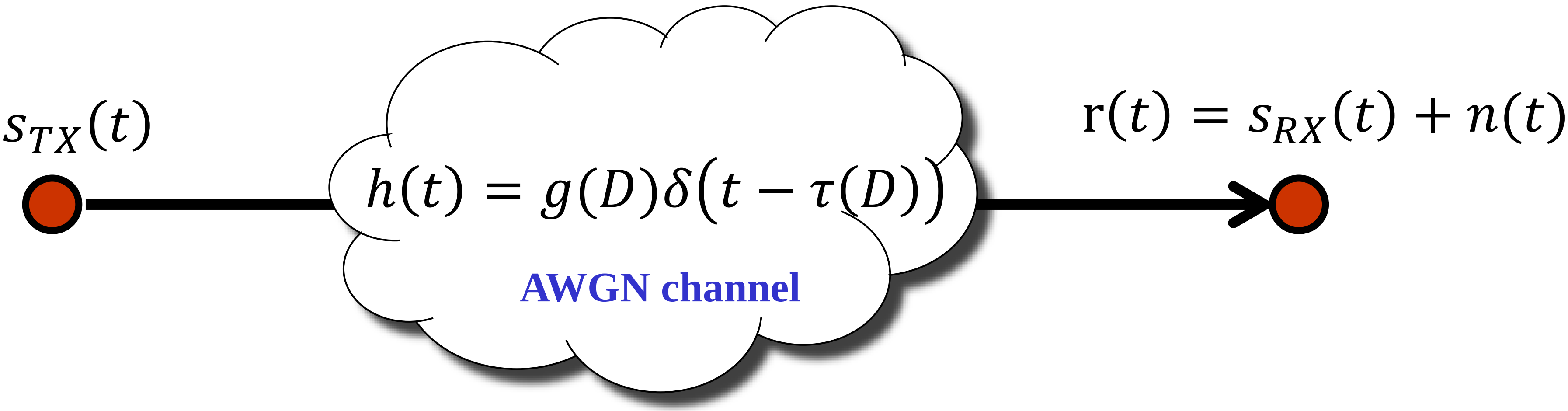
- Positioning can play a major role in the advanced design of wireless communication networks
- **Resource sharing** and **routing**, as an example, can be optimized when positioning information is available throughout the network
- Knowing how to locate and track both static and moving objects with high precision is thus an extremely appealing feature in the design of next generation wireless networks

Ranging (1/4)

- **Ranging** is defined as the action of computing the distance of a target node from a reference node



Ranging (2/4)



Received Signal $r(t) = g(D)s_{TX}\big(t - \tau(D)\big) + n(t)$

Distance D can be estimated from channel attenuation $A(D)=[g(D)]^{-2}$

Received Signal Strength Indicator (RSSI)

Distance D can be estimated from channel delay $\tau(D)$

Time of Arrival (TOA)

Ranging (3/4)

- With **RSSI**, the receiver measures the power of the received signal and derives the distance from the measured attenuation
- RSSI requires an accurate propagation model.
- RSSI is not a very accurate method when terminal mobility and unpredictable variations in channel behaviour are taken into account
- Adoption of RSSI is confined to applications requiring <u>coarse ranging</u>

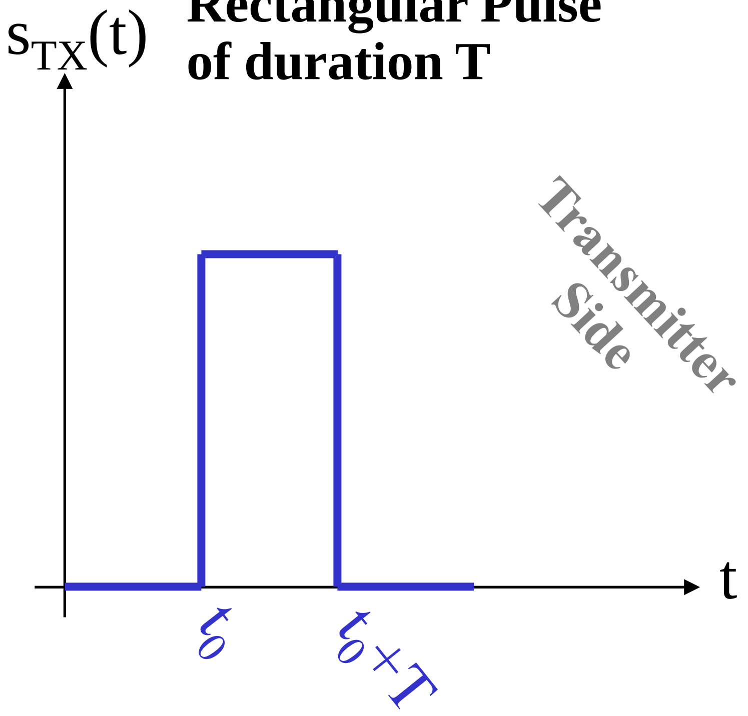
Ranging (4/4)

- The **TOA** technique computes distance based on the estimation of the propagation delay between transmitter and receiver
- TOA is the most commonly used distance estimation method in the radar field
- Delay estimation is a key topic in wireless communications since it is required for achieving symbol synchronization between transmitter and receiver
- Most of the solution for delay estimation are based on the Maximum Likelihood (ML) estimator

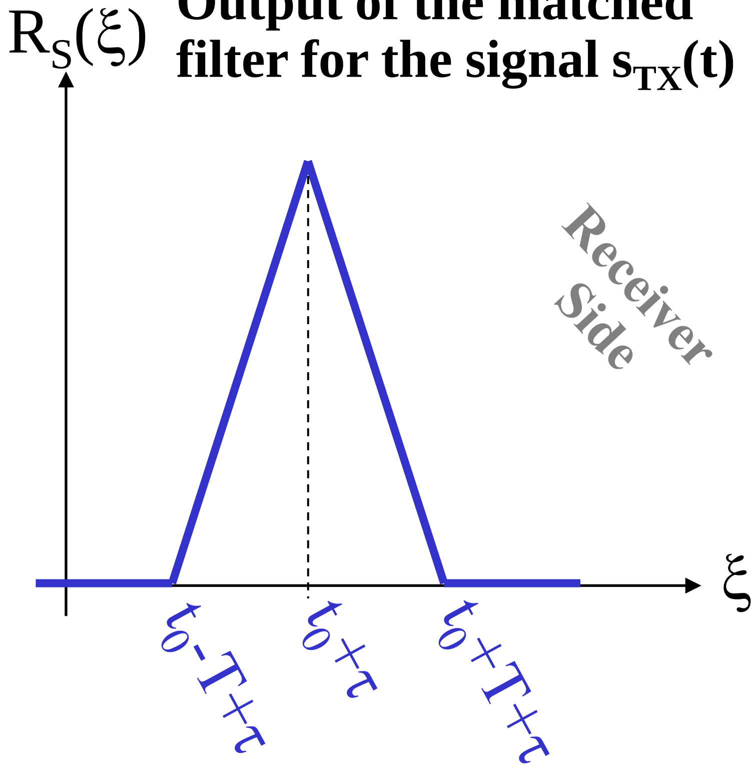
The early-late gate synchronizer (1/4)

- A common synchronization scheme that approximates the ML estimator is the **early-late gate synchronizer**

Rectangular Pulse of duration T

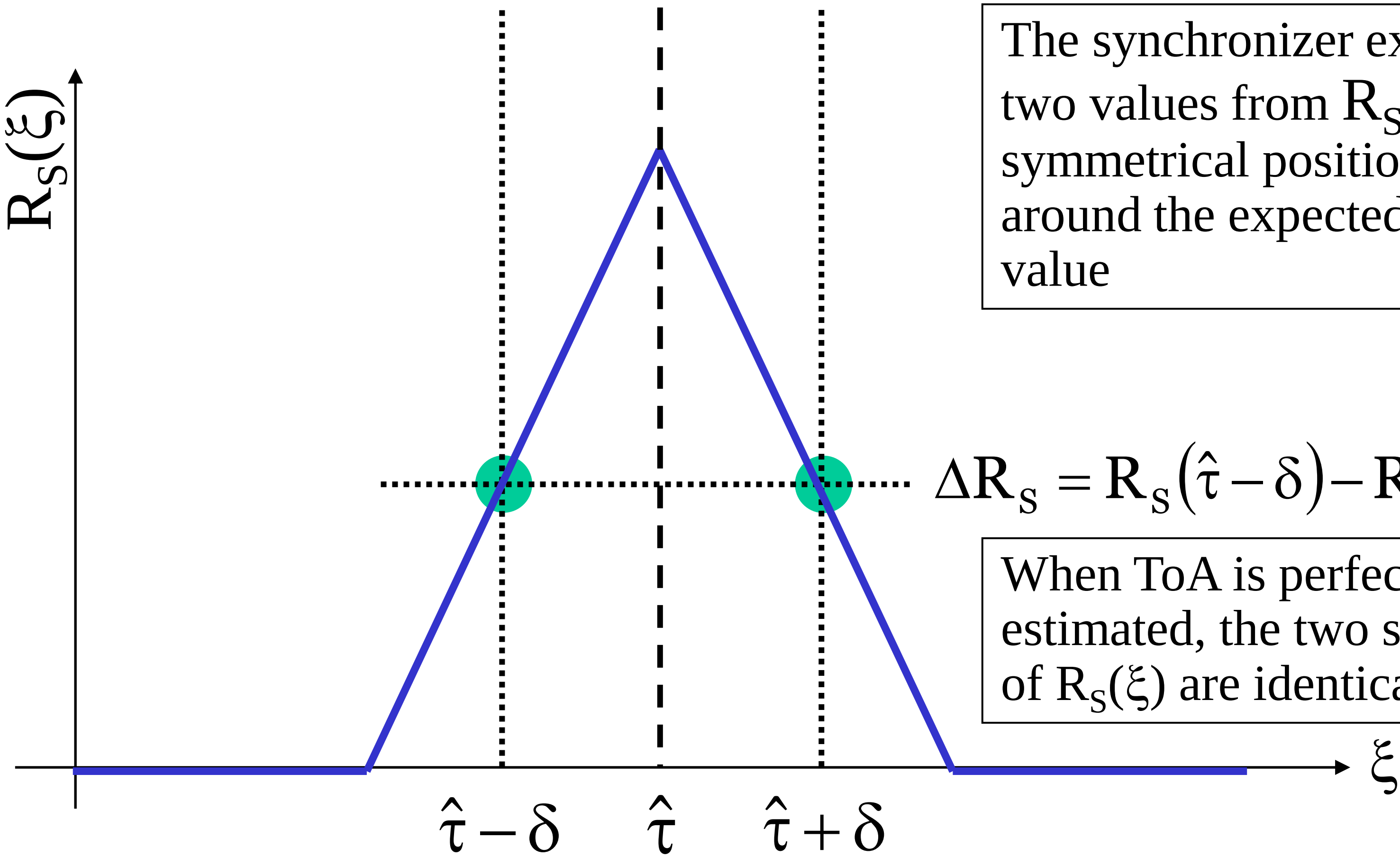


Output of the matched filter for the signal $s_{TX}(t)$



The early-late gate synchronizer (2/4)

- The early-late gate synchronizer exploits the symmetry of $R_S(\xi)$



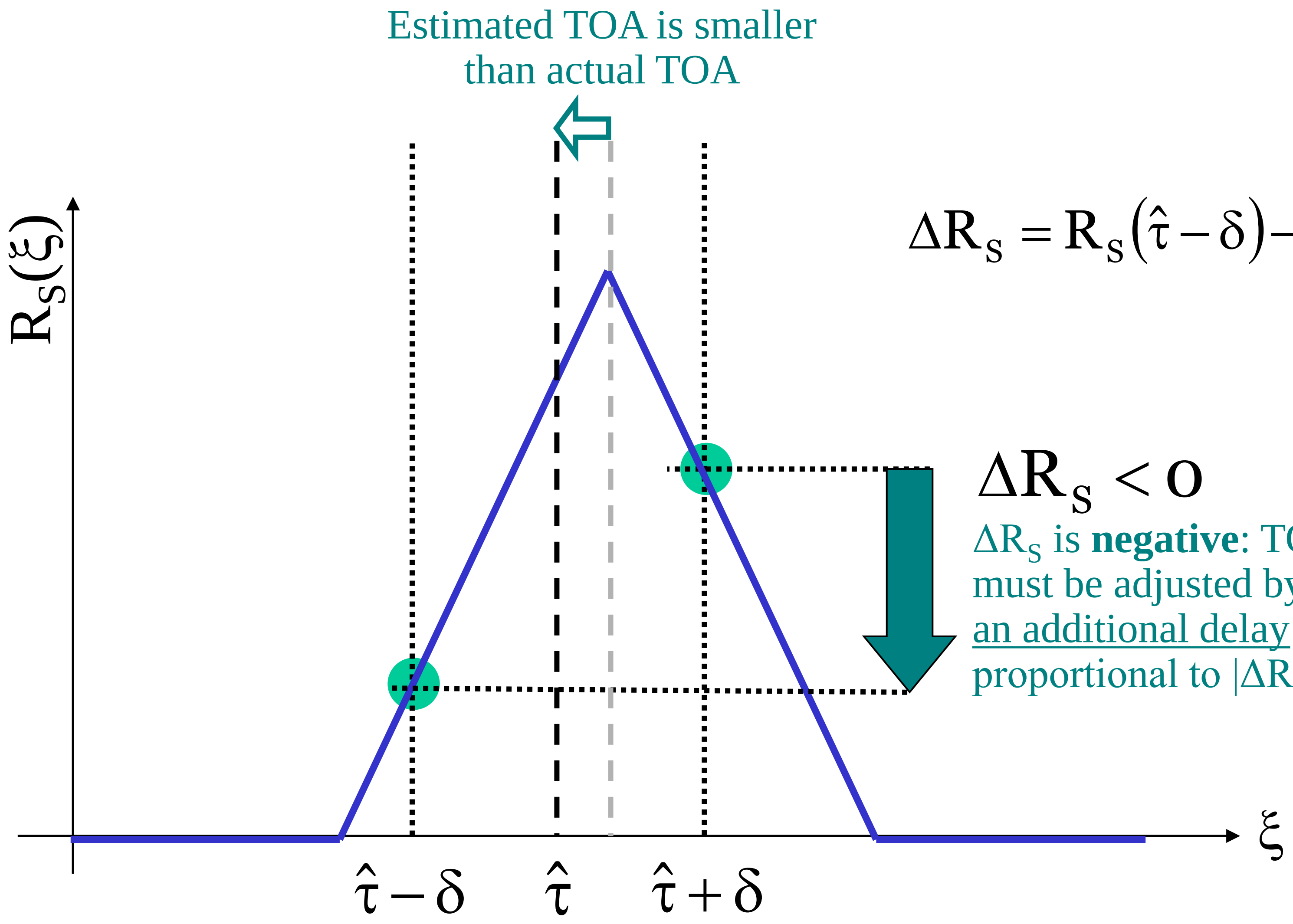
The synchronizer extracts two values from $R_S(\xi)$ at symmetrical positions around the expected peak value

$$\Delta R_S = R_S(\hat{\tau}-\delta) - R_S(\hat{\tau}+\delta) = 0$$

When ToA is perfectly estimated, the two samples of $R_S(\xi)$ are identical

The early-late gate synchronizer (3/4)

In the case of imperfect TOA estimation, the two samples of $R_S(\xi)$ are not identical



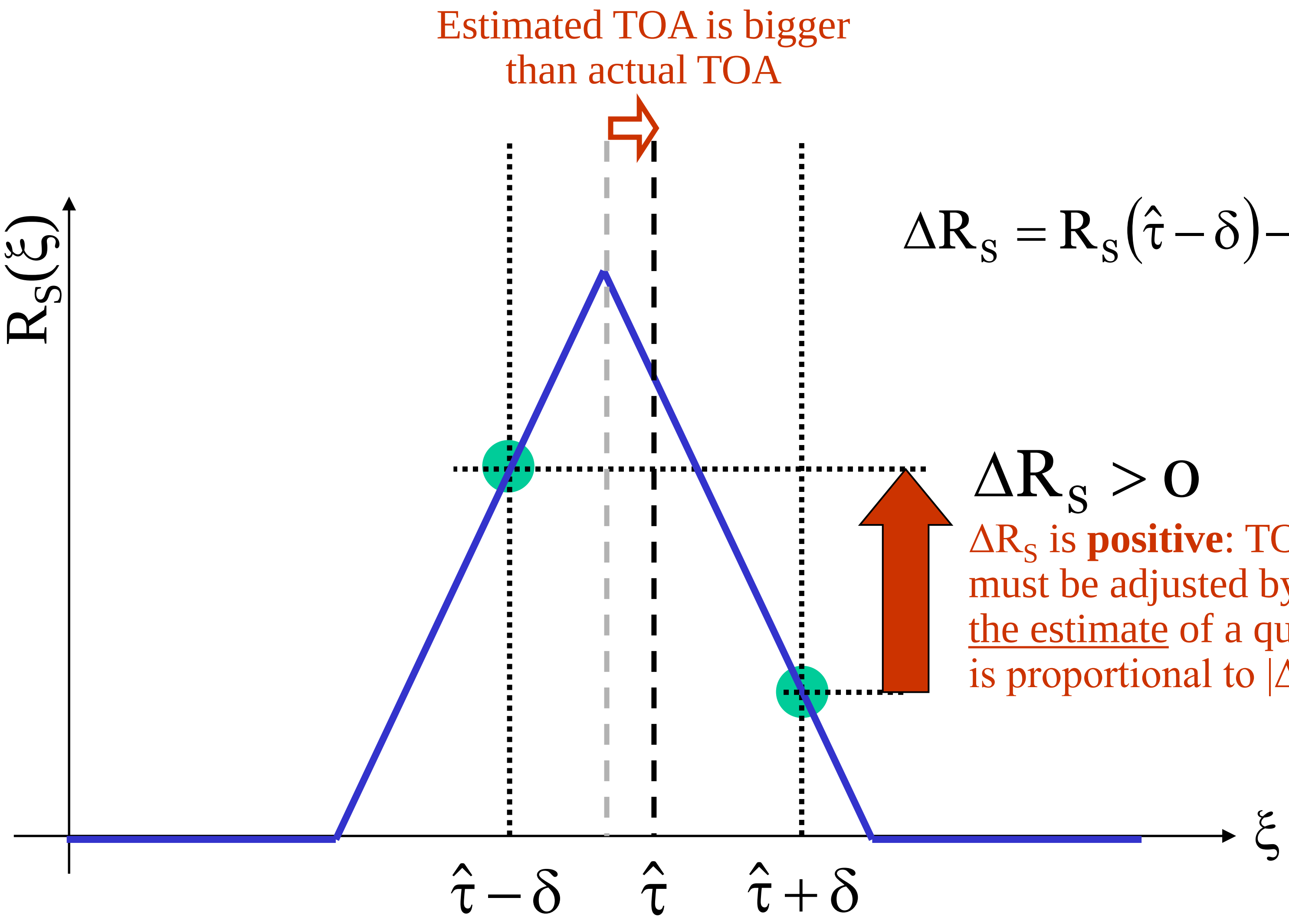
$$\Delta R_S = R_S(\hat{\tau}-\delta) - R_S(\hat{\tau}+\delta)$$

$$\Delta R_S < 0$$

ΔR_S is **negative**: TOA estimate must be adjusted by introducing an additional delay which is proportional to $|\Delta R_S|$

The early-late gate synchronizer (4/4)

In the case of imperfect TOA estimation, the two samples of $R_S(\xi)$ are not identical



$$\Delta R_S = R_S(\hat{\tau}-\delta) - R_S(\hat{\tau}+\delta)$$

$$\Delta R_S > 0$$

ΔR_S is **positive**: TOA estimate must be adjusted by decreasing the estimate of a quantity which is proportional to $|\Delta R_S|$

Ranging in UWB (1/3)

- The accuracy of the TOA estimation is related to the bandwidth of the signal and SNR at the receiver
- The lower limit for the variance of the TOA estimation error is in fact given by the Cramer-Rao lower bound:

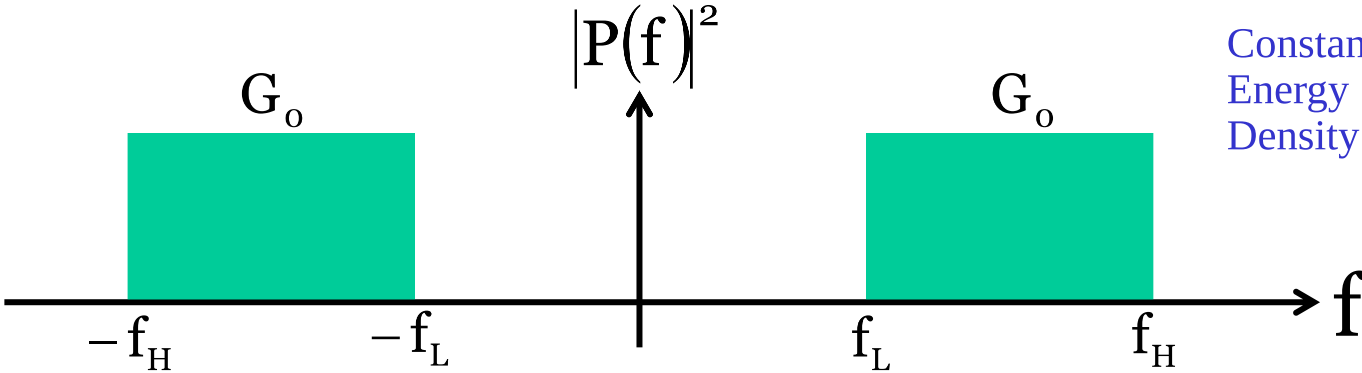
Variance of TOA estimation error

$$\sigma_{\hat{\tau}}^2 = \frac{\mathcal{N}_0}{2\int_{-\infty}^{+\infty} (2\pi f)\left|P(f)\right|^2 df}$$

- TOA technique is thus particularly suited for UWB radio, thanks to the ultra-wide bandwidth

Ranging in UWB (2/3)

Example of application of the Cramer-Rao lower bound



Constant bilateral Energy Spectral Density

$$\sigma_{\hat{\tau}}^2 = \frac{\mathcal{N}_o}{2\int_{-\infty}^{+\infty} (2\pi f)|P(f)|^2\, df} \longrightarrow \sigma_{\hat{\tau}}^2 = \frac{\mathcal{N}_o}{\frac{8}{3}\pi^2 2G_o \underbrace{(f_H - f_L)}_{\text{BANDWITH } \mathbf{B}} (f_H{}^2 + f_H f_L + f_L{}^2)}$$

Special case of an UWB pulse FCC compliant

$$f_L = 3.1\,\text{GHz}$$
$$f_H = 10.6\,\text{GHz}$$
$$B = 7.5\,\text{GHz}$$
$$2G_o = 9.86 \cdot 10^{-24}\ \text{J/Hz}$$
$$\mathcal{N}_o = 2 \cdot 10^{-20}\ \text{W/Hz}$$

$$\longrightarrow \sigma_{\hat{\tau}} = 2.44\,\mu\text{m}$$

Mean Standard Deviation of TOA estimation error

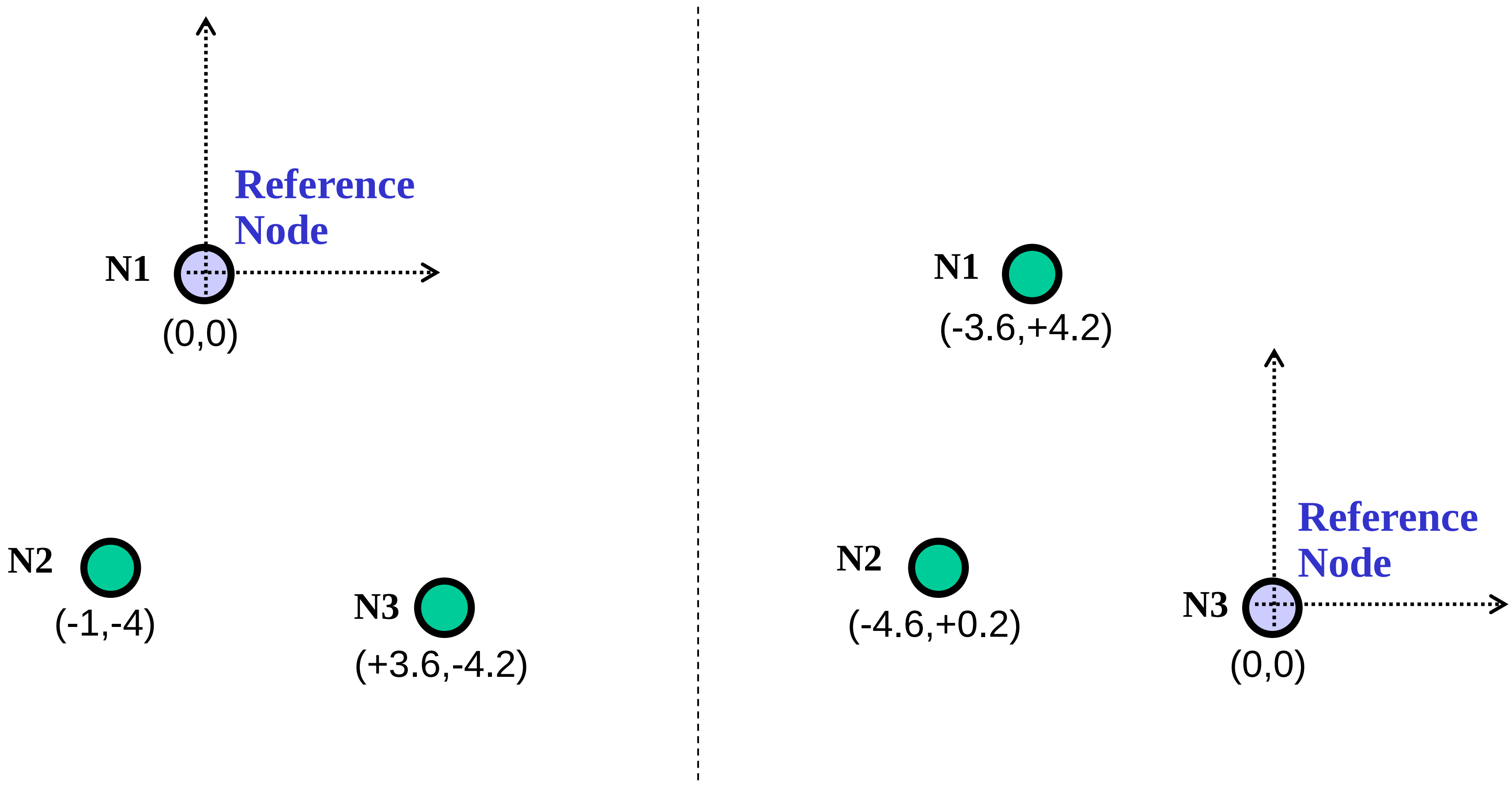
Ranging in UWB (3/3)

- The Cramer-Rao lower bound provides only a theoretical bound for ranging estimation error

- Ranging accuracy is typically limited by:
  - Receiver hardware limitations
  - Reduced efficiency in the generation of the transmitted signal
  - Presence of multipath propagation
  - Presence of MUI

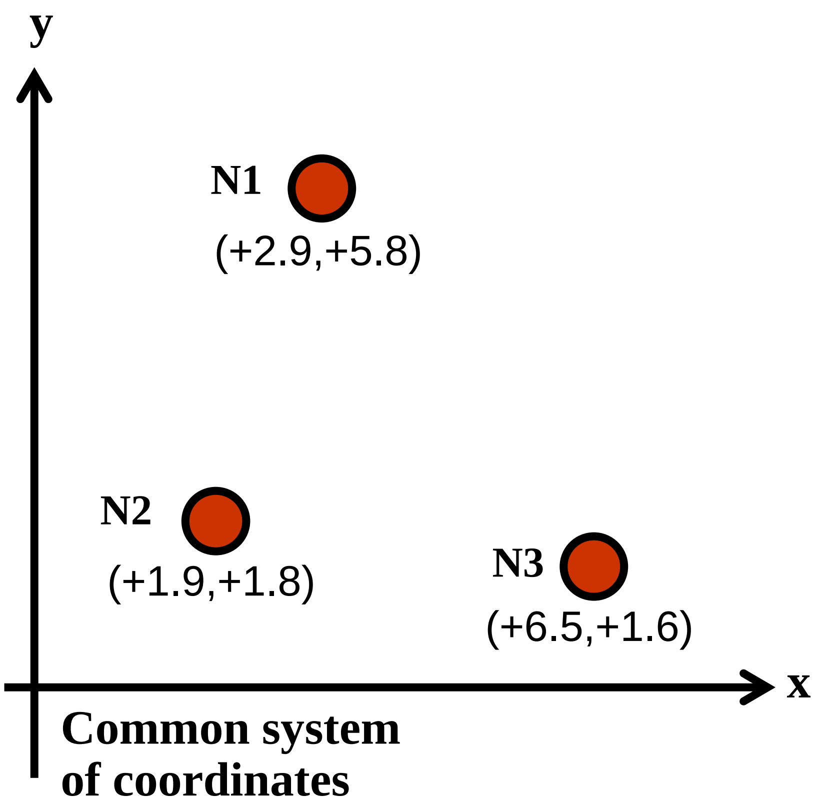
Positioning (1/4)

- **Node-centered positioning** is defined as the action of computing the positions of a set of target nodes with respect to a reference node



Positioning (2/4)

- **Relative positioning** indicates the action of computing the position of a set of nodes with respect to a common system of coordinates

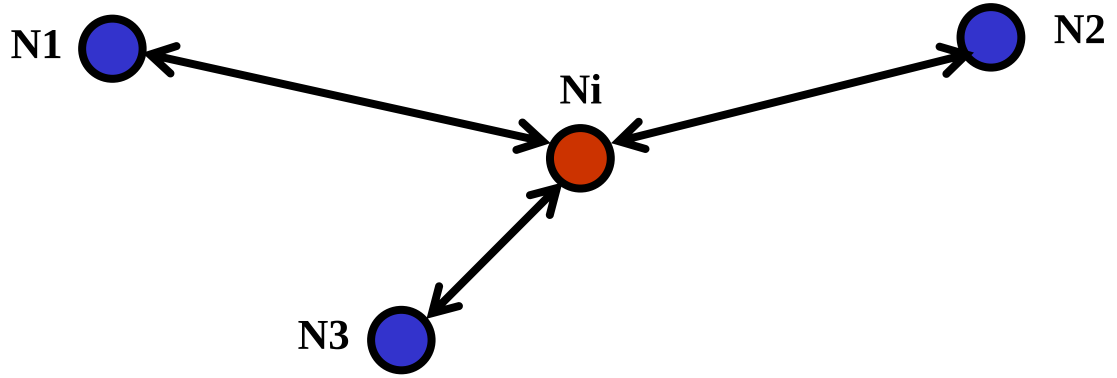


Positioning (3/4)

- Both node-centered and relative positioning require ranging for retrieving distances
- The degree of accuracy in distance estimation has an impact on positioning accuracy
- The distance estimation technique must be selected according to <u>requirements imposed by the application</u>

Positioning (4/4)

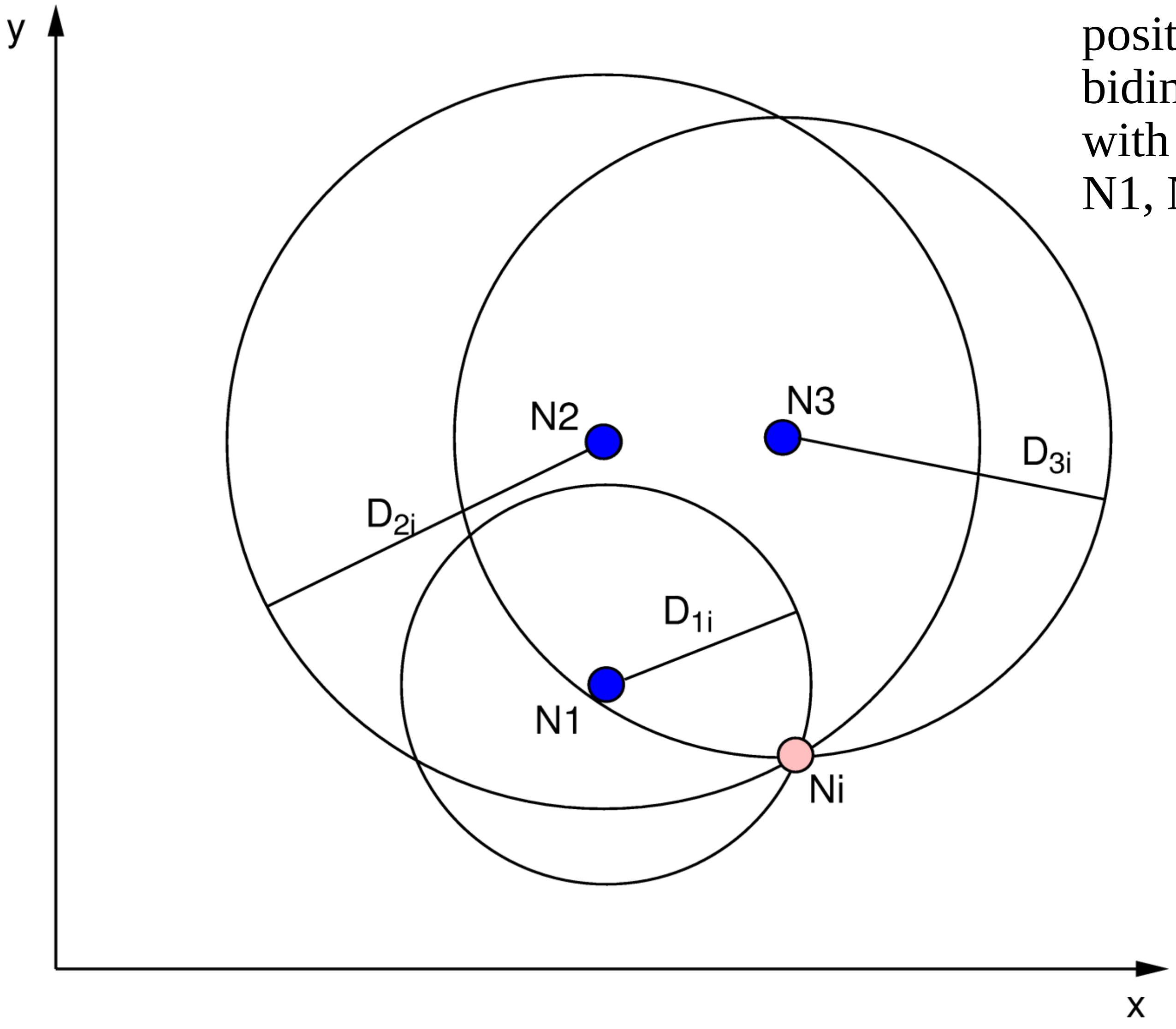
- The ranging procedure provides an estimation of distances between pairs of nodes of a given network
- Assume that a node Ni knows its distance from all the other nodes
- Among these nodes, Ni can choose k reference nodes (N1, …, Nk) to form a reference system, in which it estimates its position



Spherical Positioning (1/5)

- **Spherical Positioning** technique is based on the observation that in a tridimensional space (x,y,z), each distance between Ni and the reference Nj determines a sphere of radius *Dji* centered in Nj
- Position of Ni is thus determined by the intersection of the k spheres of radii (*D1i*,…,*Dki*) centered in the reference nodes (N1, …, Nk)
- Since the intersection of 4 spheres is required for determining a single point in the tridimensional space, at least 4 reference nodes are required in 3D positioning, (or 3 nodes in 2D positioning)

Spherical Positioning (2/5)



Example of spherical positioning of Ni in a bidimensional space with reference nodes N1, N2, and N3

Spherical Positioning (3/5)

$$\begin{Bmatrix} \sqrt{(X_1 - X_i)^2 + (Y_1 - Y_i)^2 + (Z_1 - Z_i)^2} \\ \sqrt{(X_2 - X_i)^2 + (Y_2 - Y_i)^2 + (Z_2 - Z_i)^2} \\ \cdots \\ \sqrt{(X_k - X_i)^2 + (Y_k - Y_i)^2 + (Z_k - Z_i)^2} \end{Bmatrix} = \begin{Bmatrix} D_{1i} \\ D_{2i} \\ \cdots \\ D_{ki} \end{Bmatrix}$$

System of equations which determines the position (X_i,Y_i,Z_i) of the target node Ni in 3D-positioning (k≥4)

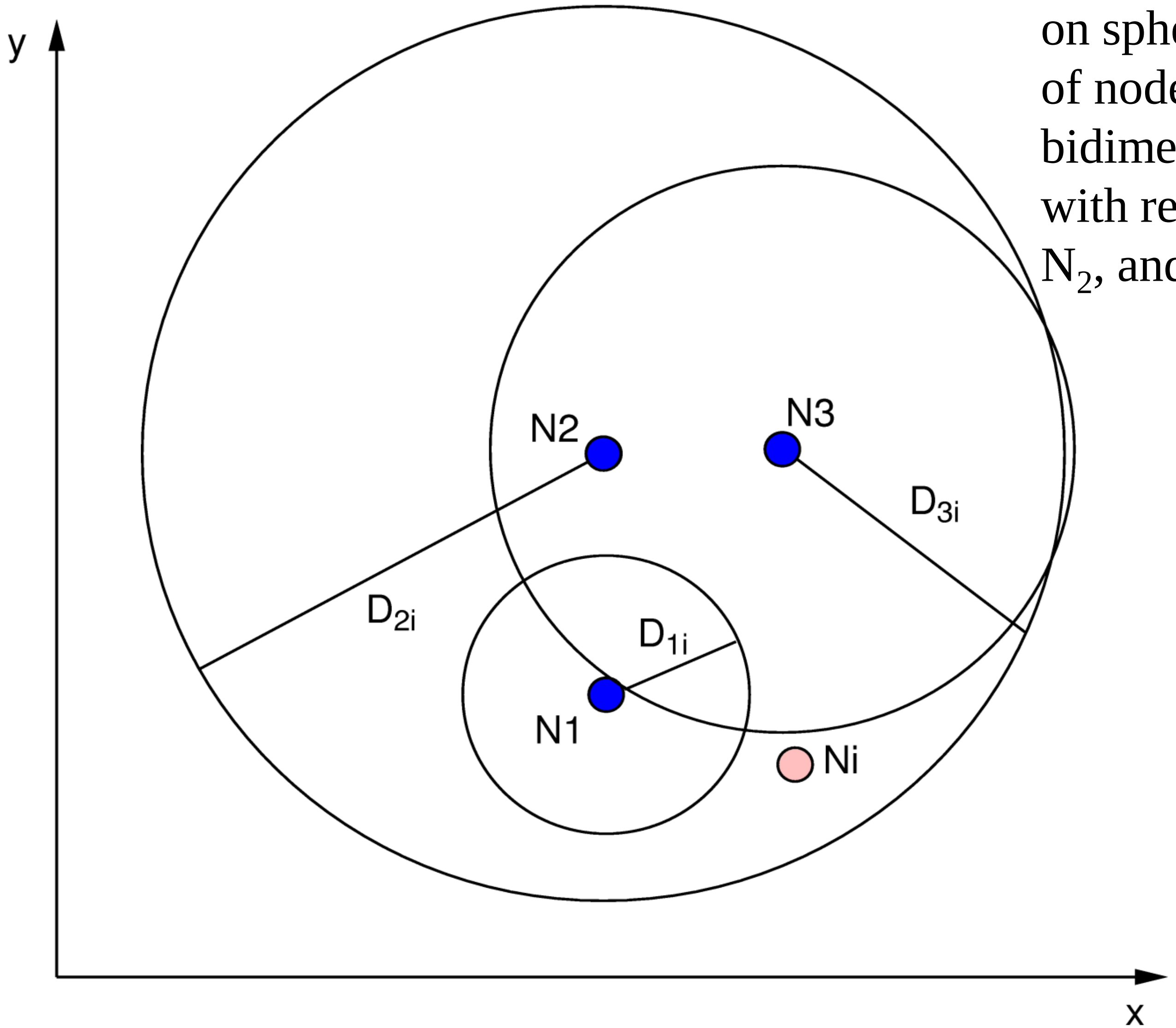
$$\begin{Bmatrix} \sqrt{(X_1 - X_i)^2 + (Y_1 - Y_i)^2} \\ \sqrt{(X_2 - X_i)^2 + (Y_2 - Y_i)^2} \\ \cdots \\ \sqrt{(X_k - X_i)^2 + (Y_k - Y_i)^2} \end{Bmatrix} = \begin{Bmatrix} D_{1i} \\ D_{2i} \\ \cdots \\ D_{ki} \end{Bmatrix}$$

System of equations which determines the position (X_i,Y_i) of the target node Ni in 2D-positioning (k≥3)

Spherical Positioning (4/5)

- The spherical positioning technique requires error-free ranging information to provide a position of the target node
- Ranging estimates, however, are affected by errors due to thermal noise, multipath,…
- In the presence of ranging errors, the analytical solution of the system of equations which determines the position of node Ni may not exist

Spherical Positioning (5/5)



Effect of ranging errors on spherical positioning of node N_i in a bidimensional space with reference nodes N_1, N_2, and N_3

Hyperbolic positioning (1/3)

- Spherical positioning can be used only when a common time reference is available to N_i and all reference nodes $\{N_1,\ldots,N_k\}$.
- Hyperbolic positioning only requires a common time reference to be available between the reference nodes, and compensates for an unknown delay δ between the common time reference and the time reference of target node N_i by working on time differences:

Unknown delay

$$D_{ni} - D_{(n-1)i} = c\left(\tau_{ni} + \delta\right) - c\left(\tau_{(n-1)i} + \delta\right) = c\left(\tau_{ni} - \tau_{(n-1)i}\right)$$

- In conditions of perfect distance measurements, hyperbolic positioning leads to the same result of spherical positioning
- It can be shown however that ranging errors have a stronger effect on hyperbolic positioning

Hyperbolic positioning (2/3)

- Given a target node N_i its position in a tridimensional space is determined as the intersection of hyperboloids in space, as described by the following equations:

$$\left\{\begin{array}{c} \sqrt{(X_2-X_i)^2+(Y_2-Y_i)^2+(Z_2-Z_i)^2}-\sqrt{(X_1-X_i)^2+(Y_1-Y_i)^2+(Z_1-Z_i)^2} \\ \sqrt{(X_3-X_i)^2+(Y_3-Y_i)^2+(Z_3-Z_i)^2}-\sqrt{(X_2-X_i)^2+(Y_2-Y_i)^2+(Z_2-Z_i)^2} \\ \cdots \\ \sqrt{(X_k-X_i)^2+(Y_k-Y_i)^2+(Z_k-Z_i)^2}-\sqrt{(X_{k-1}-X_i)^2+(Y_{k-1}-Y_i)^2+(Z_{k-1}-Z_i)^2} \end{array}\right\} =$$

$$= \left\{\begin{array}{c} D_{2i}-D_{1i} \\ D_{3i}-D_{2i} \\ \cdots \\ D_{ki}-D_{(k-1)i} \end{array}\right\}$$

with $k \geq 4$

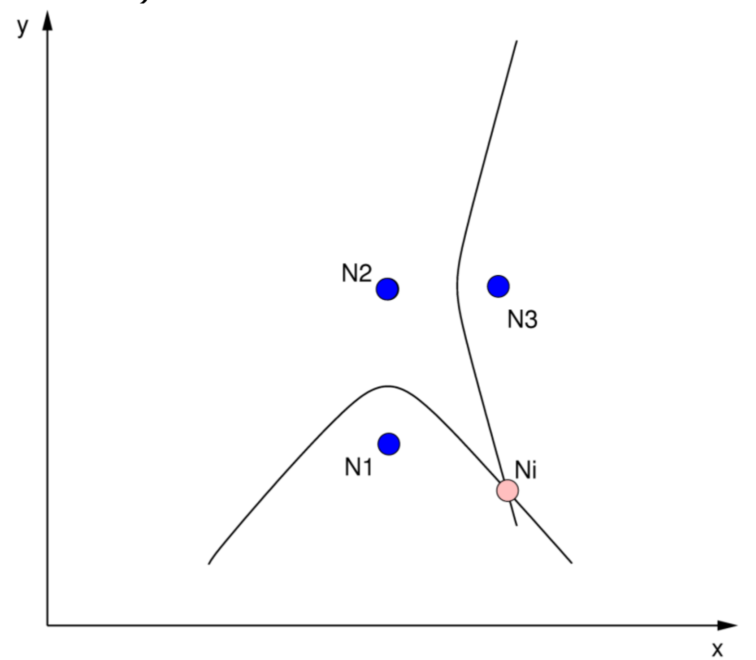
Hyperbolic positioning (3/3)

- In a bidimensional space, one has:

$$\left\{ \begin{array}{c} \sqrt{(X_2 - X_i)^2 + (Y_2 - Y_i)^2} - \sqrt{(X_1 - X_i)^2 + (Y_1 - Y_i)^2} \\ \sqrt{(X_3 - X_i)^2 + (Y_3 - Y_i)^2} - \sqrt{(X_2 - X_i)^2 + (Y_2 - Y_i)^2} \\ \cdots \\ \sqrt{(X_k - X_i)^2 + (Y_k - Y_i)^2} - \sqrt{(X_{k-1} - X_i)^2 + (Y_{k-1} - Y_i)^2} \end{array} \right\} = \left\{ \begin{array}{c} D_{2i} - D_{1i} \\ D_{3i} - D_{2i} \\ \cdots \\ D_{ki} - D_{(k-1)i} \end{array} \right\}$$

with $k \geq 3$

- The solution is thus given by the intersection of two hyperboles in the plane:



Positioning with LSE minimization (1/7)

- The effect of ranging errors on positioning can be reduced by adopting minimization procedures such as the Least Square Error (LSE)

$$-2\begin{bmatrix} (X_1 - X_k) & (Y_1 - Y_k) & (Z_1 - Z_k) \\ (X_2 - X_k) & (Y_2 - Y_k) & (Z_2 - Z_k) \\ \cdots & \cdots & \cdots \\ (X_{k-1} - X_k) & (Y_{k-1} - Y_k) & (Z_{k-1} - Z_k) \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \begin{bmatrix} D_{1i}^2 - D_{ki}^2 - X_1^2 + X_k^2 - Y_1^2 + Y_k^2 - Z_1^2 + Z_k^2 \\ D_{2i}^2 - D_{ki}^2 - X_2^2 + X_k^2 - Y_2^2 + Y_k^2 - Z_2^2 + Z_k^2 \\ \cdots \\ D_{(k-1)i}^2 - D_{ki}^2 - X_{(k-1)}^2 + X_k^2 - Y_{(k-1)}^2 + Y_k^2 - Z_{(k-1)}^2 + Z_k^2 \end{bmatrix}$$

Set of linear equations in the example case of 3D-positioning

$$\mathrm{P} = \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix}$$

Vector P *contains the coordinates of node i*

Positioning with LSE minimization (2/7)

$$AP = b$$

Positioning Problem in Linear Form

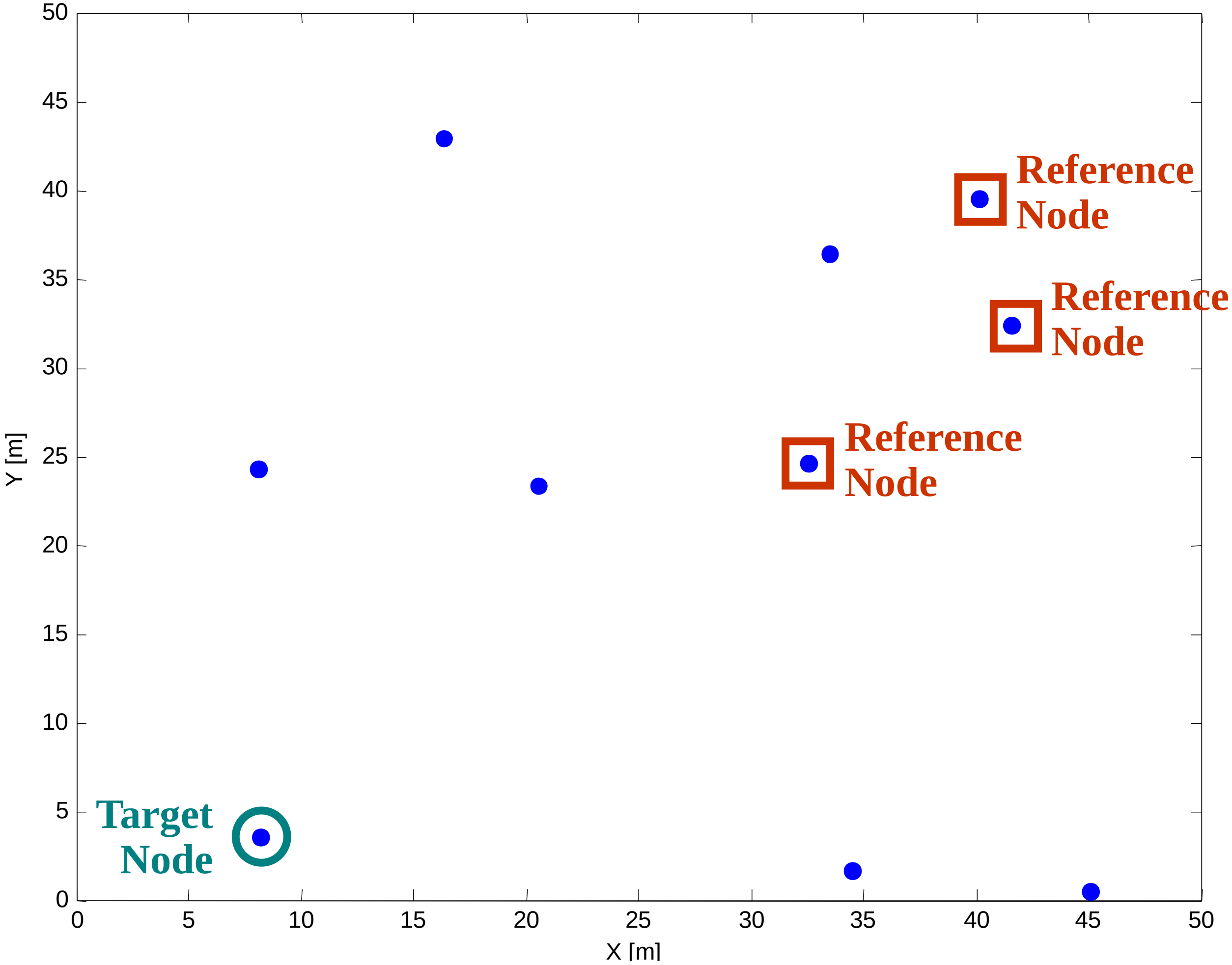
- The above system of equations can be solved in the sense of LSE minimization

$$P = A^{-1}b$$

- In this case, the adoption of a redundant set of ranging measurements reduces the variance of the positioning error

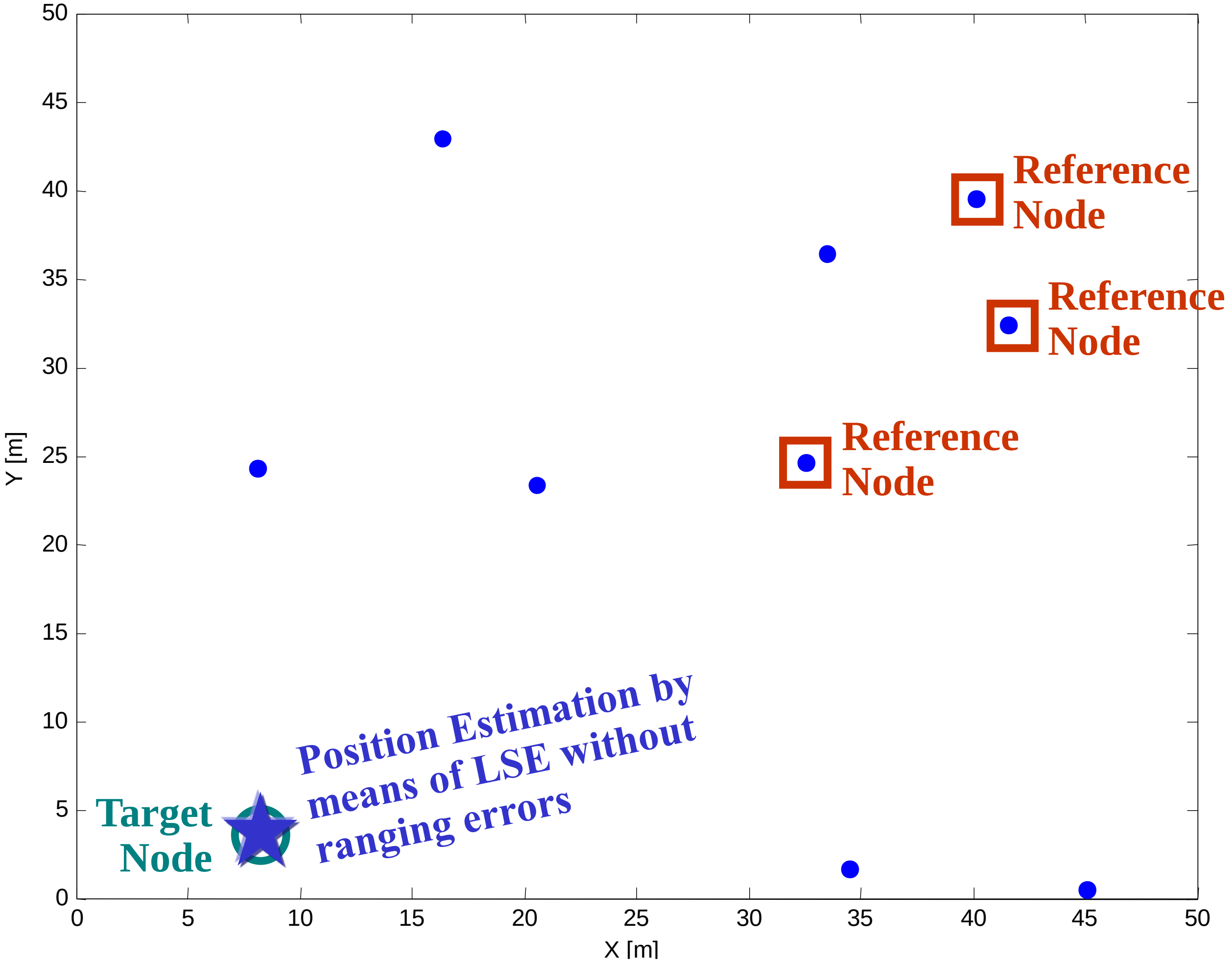
Positioning with LSE minimization (3/7)

An example: set of 10 nodes in an area 50x50 m^2



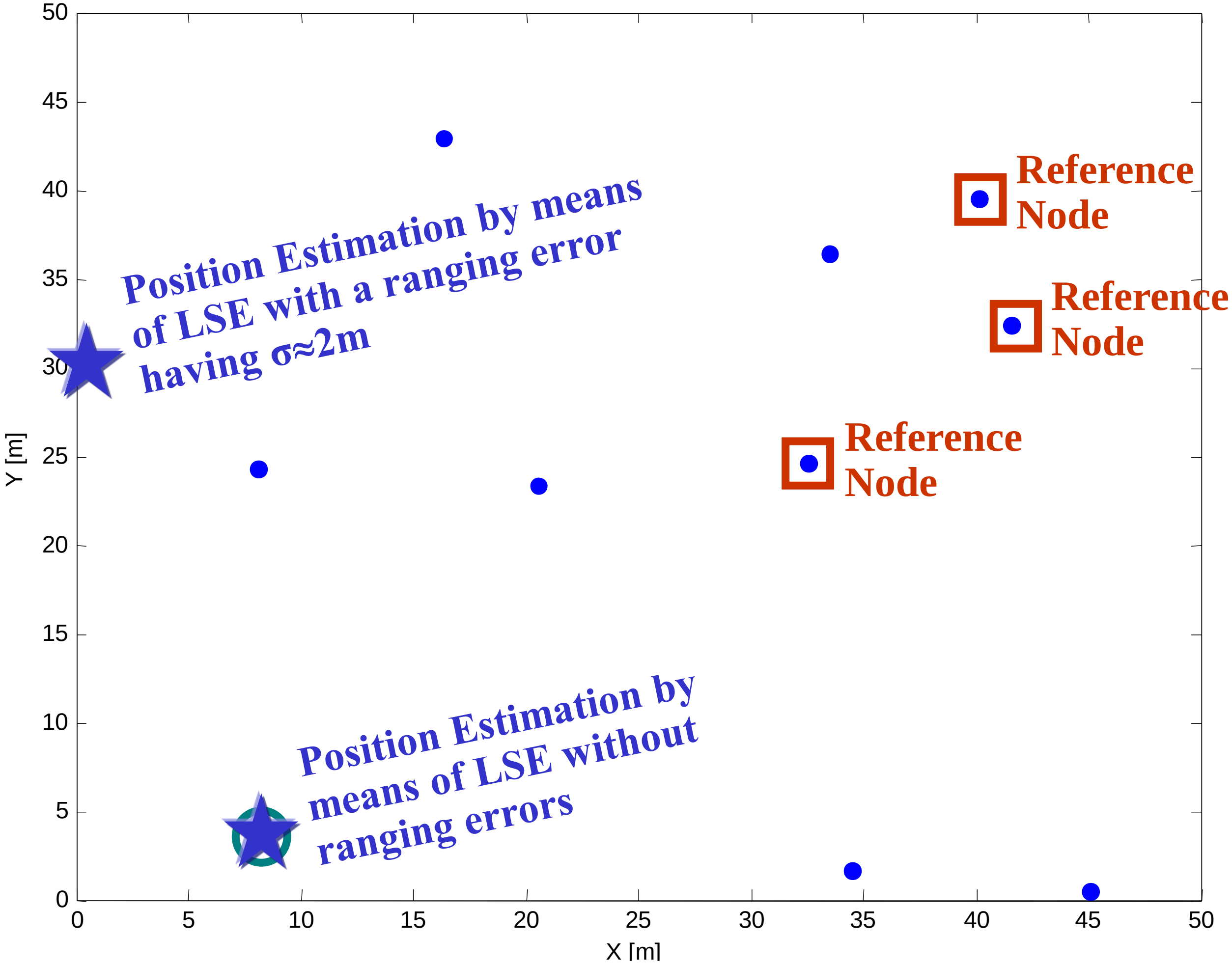
Positioning with LSE minimization (4/7)

An example: set of 10 nodes in an area 50x50 m^2



Positioning with LSE minimization (5/7)

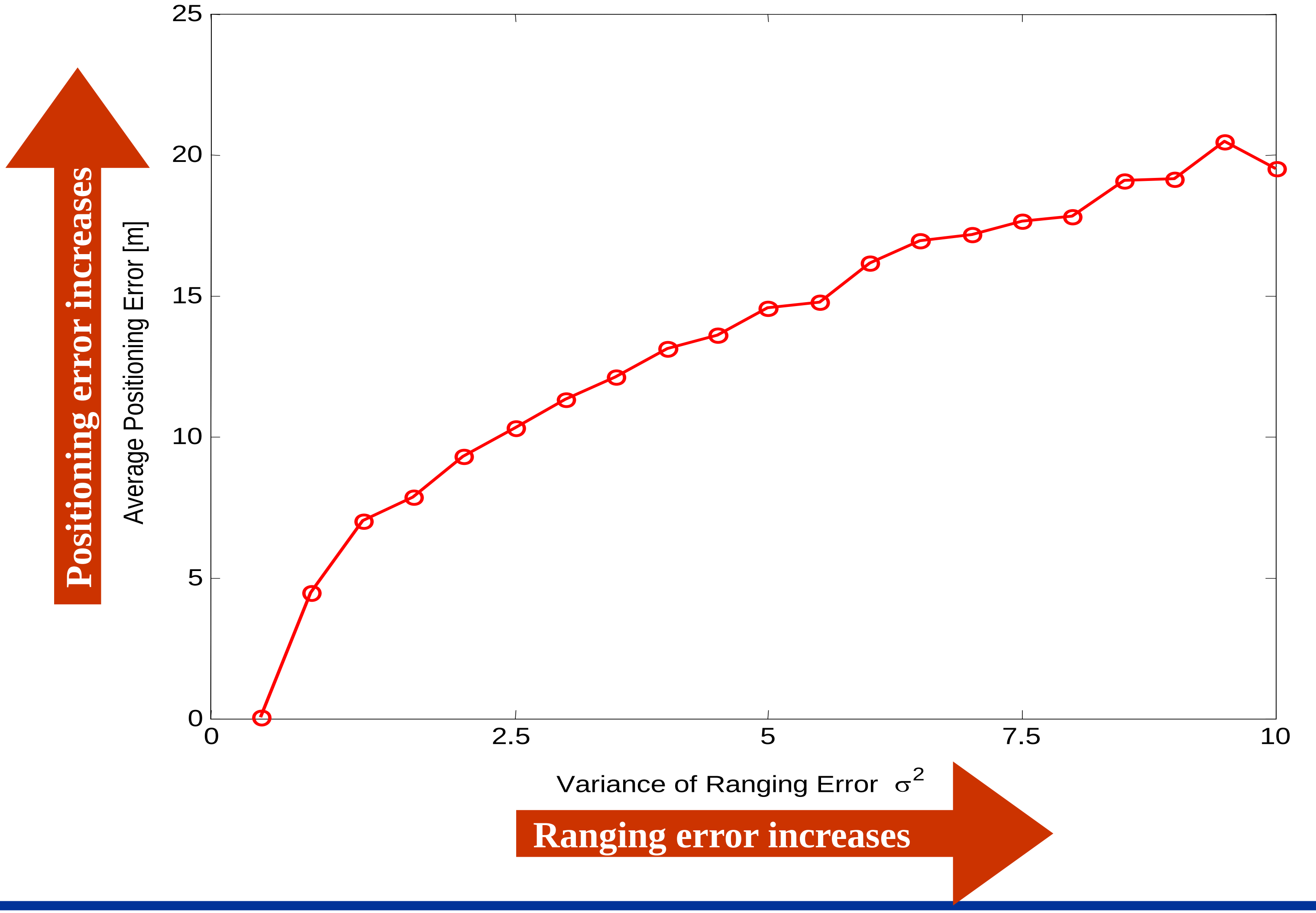
An example: set of 10 nodes in an area 50x50 m^2



Positioning with LSE minimization (6/7)

An example: set of 10 nodes in an area 50x50 m^2

Average Positioning Error vs. variance of ranging errors



Positioning with LSE minimization (7/7)

An example: set of 10 nodes in an area 50x50 m^2

Average Positioning Error vs. Number of reference nodes