Ultra Wide Band Communications

Dr. Giuseppe Caso

Prof. Maria-Gabriella Di Benedetto

University of Rome
La Sapienza

School of Engineering
Ranging and Positioning with UWB
Outline

• Ranging
  – RSSI vs. TOA
  – The early-late gate synchronizer
  – Ranging in UWB

• Positioning
  – Node-centered and relative positioning
  – Spherical positioning
  – Positioning with LSE minimization
Position-aware distributed wireless networks

• Positioning can play a major role in the advanced design of wireless communication networks

• **Resource sharing** and **routing**, as an example, can be optimized when positioning information is available throughout the network

• Knowing how to locate and track both static and moving objects with high precision is thus an extremely appealing feature in the design of next generation wireless networks
• **Ranging** is defined as the action of computing the distance of a target node from a reference node.
Received Signal $r(t) = A(D)s_{TX}(t - \tau(D)) + n(t)$

Distance D can be estimated from channel attenuation $A(D)$

Received Signal Strength Indicator (RSSI)

Distance D can be estimated from channel delay $\tau(D)$

Time of Arrival (TOA)
• With **RSSI**, the receiver measures the power of the received signal and derives the distance from the measured attenuation.

• RSSI requires an accurate propagation model.

• RSSI is not a very accurate method when terminal mobility and unpredictable variations in channel behaviour are taken into account.

• Adoption of RSSI is confined to applications requiring coarse ranging.
The **TOA** technique computes distance based on the estimation of the propagation delay between transmitter and receiver.

TOA is the most commonly used distance estimation method in the radar field.

Delay estimation is a key topic in wireless communications since it is required for achieving symbol synchronization between transmitter and receiver.

Most of the solution for delay estimation are based on the **Maximum Likelihood (ML)** estimator.
The early-late gate synchronizer (1/4)

- A common synchronization scheme that approximates the ML estimator is the **early-late gate synchronizer**

\[
\begin{align*}
\text{Rectangular Pulse of duration } T &= s_{TX}(t) \\
\text{Output of the matched filter for the signal } s_{TX}(t) &= R_{S}(\xi)
\end{align*}
\]
The early-late gate synchronizer exploits the symmetry of $R_S(\xi)$

The synchronizer extracts two values from $R_S(\xi)$ at symmetrical positions around the expected peak value.

When ToA is perfectly estimated, the two samples of $R_S(\xi)$ are identical.
In the case of imperfect TOA estimation, the two samples of $R_S(\xi)$ are not identical.

Estimated TOA is smaller than actual TOA.

\[ \Delta R_S = R_S(\hat{\tau} - \delta) - R_S(\hat{\tau} + \delta) \]

$\Delta R_S < 0$ indicates that the TOA estimate must be adjusted by introducing an additional delay which is proportional to $|\Delta R_S|$.
In the case of imperfect TOA estimation, the two samples of $R_S(\xi)$ are not identical.

Estimated TOA is bigger than actual TOA

$$\Delta R_S = R_S(\hat{\tau} - \delta) - R_S(\hat{\tau} + \delta)$$

$\Delta R_S > 0$

$\Delta R_S$ is positive: TOA estimate must be adjusted by decreasing the estimate of a quantity which is proportional to $|\Delta R_S|$.
Ranging in UWB (1/3)

- The accuracy of the TOA estimation is related to the bandwidth of the signal and SNR at the receiver.

- The **lower limit** for the **variance of the TOA estimation error** is in fact given by the Cramer-Rao lower bound:

\[
\sigma_{\hat{t}}^2 = \frac{N_0}{2 \int_{-\infty}^{\infty} (2\pi f)|P(f)|^2 \, df}
\]

- TOA technique is thus particularly suited for UWB radio, thanks to the ultra-wide bandwidth.
Example of application of the Cramer-Rao lower bound

$$\sigma^2_\hat{\tau} = \frac{N_0}{2 \int_{-\infty}^{+\infty} (2\pi f) |P(f)|^2 \, df}$$

$$\sigma^2_\hat{\tau} = \frac{8 \pi^2 2G_0 (f_H - f_L)(f_H^2 + f_H f_L + f_L^2)}{3}$$

**Constant bilateral Energy Spectral Density**

**Special case of an UWB pulse FCC compliant**

- $f_L = 3.1 \text{ GHz}$
- $f_H = 10.6 \text{ GHz}$
- $B = 7.5 \text{ GHz}$
- $2G_0 = 9.86 \cdot 10^{-24} \text{ J/Hz}$
- $N_0 = 2 \cdot 10^{-20} \text{ W/Hz}$

**Mean Standard Deviation of TOA estimation error**

$$\sigma_\hat{\tau} = 2.44 \mu \text{m}$$
• The Cramer-Rao lower bound provides only a theoretical bound for ranging estimation error.

• Ranging accuracy is typically limited by:
  – Receiver hardware limitations
  – Reduced efficiency in the generation of the transmitted signal
  – Presence of multipath propagation
  – Presence of MUI
• **Node-centered positioning** is defined as the action of computing the positions of a set of target nodes with respect to a reference node.

![Diagram showing node-centered positioning]
Relative positioning indicates the action of computing the position of a set of nodes with respect to a common system of coordinates.
• Both node-centered and relative positioning require ranging for retrieving distances
• The degree of accuracy in distance estimation has an impact on positioning accuracy
• The distance estimation technique must be selected according to requirements imposed by the application
• The ranging procedure provides an estimation of distances between pairs of nodes of a given network
• Assume that a node $N_i$ knows its distance from all the other nodes
• Among these nodes, $N_i$ can choose $k$ reference nodes ($N_1, \ldots, N_k$) to form a reference system, in which it estimates its position
Spherical Positioning technique is based on the observation that in a tridimensional space \((x,y,z)\), each distance between \(N_i\) and the reference \(N_j\) determines a sphere of radius \(D_{ji}\) centered in \(N_j\).

Position of \(N_i\) is thus determined by the intersection of the \(k\) spheres of radii \((D_{1i}, \ldots, D_{ki})\) centered in the reference nodes \((N_1, \ldots, N_k)\).

Since the intersection of 4 spheres is required for determining a single point in the tridimensional space, at least 4 reference nodes are required in 3D positioning, (or 3 nodes in 2D positioning).
Example of spherical positioning of \( Ni \) in a bidimensional space with reference nodes \( N1, N2, \) and \( N3 \)
Spherical Positioning (3/5)

System of equations which determines the position \((X_i, Y_i, Z_i)\) of the target node \(N_i\) in 3D-positioning \((k \geq 4)\):

\[
\begin{align*}
\sqrt{(X_1 - X_i)^2 + (Y_1 - Y_i)^2 + (Z_1 - Z_i)^2} \\
\sqrt{(X_2 - X_i)^2 + (Y_2 - Y_i)^2 + (Z_2 - Z_i)^2} \\
\sqrt{(X_k - X_i)^2 + (Y_k - Y_i)^2 + (Z_k - Z_i)^2}
\end{align*}
\]

\[
\begin{bmatrix}
D_{1i} \\
D_{2i} \\
L \\
D_{ki}
\end{bmatrix}
\]

System of equations which determines the position \((X_i, Y_i)\) of the target node \(N_i\) in 2D-positioning \((k \geq 3)\):

\[
\begin{align*}
\sqrt{(X_1 - X_i)^2 + (Y_1 - Y_i)^2} \\
\sqrt{(X_2 - X_i)^2 + (Y_2 - Y_i)^2} \\
\sqrt{(X_k - X_i)^2 + (Y_k - Y_i)^2}
\end{align*}
\]

\[
\begin{bmatrix}
D_{1i} \\
D_{2i} \\
L \\
D_{ki}
\end{bmatrix}
\]
• The spherical positioning technique requires error-free ranging information to provide a position of the target node.
• Ranging estimates, however, are affected by errors due to thermal noise, multipath,…
• In the presence of ranging errors, the analytical solution of the system of equations which determines the position of node \( N_i \) may not exist.
Spherical Positioning (5/5)

Effect of ranging errors on spherical positioning of node $N_i$ in a bidimensional space with reference nodes $N_1$, $N_2$, and $N_3$.
Hyperbolic Positioning 1/3

- Spherical positioning can be used only when a common time reference is available to $N_i$ and all reference nodes $\{N_1, \ldots, N_k\}$.
- Hyperbolic positioning only requires a common time reference to be available between the reference nodes, and compensates for an unknown delay $\delta$ between the common time reference and the time reference of target node $N_i$ by working in time differences:

$$D_{ni} - D_{(n-1)i} = c(\tau_{ni} + \delta) - c(\tau_{(n-1)i} + \delta) = c(\tau_{ni} - \tau_{(n-1)i})$$

- In conditions of perfect distance measurements, hyperbolic positioning leads to the same result of spherical positioning.
- It can be shown however that ranging errors have a stronger effect on hyperbolic positioning.
Hyperbolic Positioning 2/3

Given a target node $N_i$ its position in a tridimensional space is determined as the intersection of hyperboloids in space, as described by the following equations:

$$\begin{align*}
\sqrt{(X_2 - X_i)^2 + (Y_2 - Y_i)^2 + (Z_2 - Z_i)^2} - \sqrt{(X_1 - X_i)^2 + (Y_1 - Y_i)^2 + (Z_1 - Z_i)^2} \\
\sqrt{(X_3 - X_i)^2 + (Y_3 - Y_i)^2 + (Z_3 - Z_i)^2} - \sqrt{(X_2 - X_i)^2 + (Y_2 - Y_i)^2 + (Z_2 - Z_i)^2} \\
\vdots \\
\sqrt{(X_k - X_i)^2 + (Y_k - Y_i)^2 + (Z_k - Z_i)^2} - \sqrt{(X_{k-1} - X_i)^2 + (Y_{k-1} - Y_i)^2 + (Z_{k-1} - Z_i)^2}
\end{align*}$$

$$D_{2i} - D_{1i}$$
$$D_{3i} - D_{2i}$$
$$\vdots$$
$$D_{ki} - D_{(k-1)i}$$

with $k \geq 4$
Hyperbolic Positioning 3/3

In a bidimensional space, one has:

\[
\begin{align*}
\sqrt{(X_2 - X_i)^2 + (Y_2 - Y_i)^2} - \sqrt{(X_1 - X_i)^2 + (Y_1 - Y_i)^2} \\
\sqrt{(X_3 - X_i)^2 + (Y_3 - Y_i)^2} - \sqrt{(X_2 - X_i)^2 + (Y_2 - Y_i)^2} \\
\cdots \\
\sqrt{(X_k - X_i)^2 + (Y_k - Y_i)^2} - \sqrt{(X_{k-1} - X_i)^2 + (Y_{k-1} - Y_i)^2}
\end{align*}
\]

\[
\frac{D_{2i} - D_{1i}}{D_{3i} - D_{2i}} = \cdots = \frac{D_{ki} - D_{(k-1)i}}{with \; k \geq 3}
\]

The solution is thus given by the intersection of two hyperboles in the plane:
The effect of ranging errors on positioning can be reduced by adopting minimization procedures such as the **Least Square Error (LSE)**

\[
-2 \begin{bmatrix}
(X_1 - X_k) & (Y_1 - Y_k) & (Z_1 - Z_k) \\
(X_2 - X_k) & (Y_2 - Y_k) & (Z_2 - Z_k) \\
(X_{k-1} - X_k) & (Y_{k-1} - Y_k) & (Z_{k-1} - Z_k)
\end{bmatrix}
\begin{bmatrix}
X_i \\
Y_i \\
Z_i
\end{bmatrix} = \begin{bmatrix}
D_{i1}^2 - D_{ki}^2 - X_1^2 + X_k^2 - Y_1^2 + Y_k^2 - Z_1^2 + Z_k^2 \\
D_{i2}^2 - D_{ki}^2 - X_2^2 + X_k^2 - Y_2^2 + Y_k^2 - Z_2^2 + Z_k^2 \\
D_{(k-1)i}^2 - D_{ki}^2 - X_{(k-1)}^2 + X_k^2 - Y_{(k-1)}^2 + Y_k^2 - Z_{(k-1)}^2 + Z_k^2
\end{bmatrix}
\]

**Set of linear equations in the example case of 3D-positioning**

\[
P = \begin{bmatrix}
X_i \\
Y_i \\
Z_i
\end{bmatrix} \quad \text{Vector } P \text{ contains the coordinates of node } i
Positioning with LSE minimization (2/7)

The above system of equations can be solved in the sense of LSE minimization

\[ AP = b \]

• The above system of equations can be solved in the sense of LSE minimization

\[ P = A^{-1} b \]

• In this case, the adoption of a redundant set of ranging measurements reduces the variance of the positioning error
Positioning with LSE minimization (3/7)

An example: set of 10 nodes in an area 50x50 m²
Positioning with LSE minimization (4/7)

An example: set of 10 nodes in an area 50x50 m²
An example: set of 10 nodes in an area 50x50 m²

Position Estimation by means of LSE with a ranging error having $\sigma \approx 2$ m

Position Estimation by means of LSE without ranging errors
Positioning with LSE minimization (6/7)

An example: set of 10 nodes in an area 50x50 m²

Average Positioning Error vs. variance of ranging errors

Variance of Ranging Error $\sigma^2$

Ranging error increases
An example: set of 10 nodes in an area 50x50 m²
Average Positioning Error vs. Number of reference nodes

Positioning error decreases as the number of reference nodes increases. With 3 reference nodes, the average error in ranging is approximately 
\(\sigma \approx 2.2\) m.
Positioning with UWB

- **UWB Sapphire tags from Zebra**
  - Designed for in-building positioning (typically hospitals)
  - TOA for ranging measurements
  - TDOA (hyperbolic) positioning
  - Requires installation of ceil-mounted receivers, which are cabled for maintaining a common time reference
  - Calibration is performed at system set-up by means of a tag at known location
  - Position accuracy: better than 30 cm
Positioning with UWB

- Decawave ScenSor DW1000
  - Based on the IEEE802.15.4-2011 standard.
  - Data rates of 110 kbps, 850 kbps and 6.8 Mbps
  - In/Outdoor Localization within 10 cm.
  - TOA/TDOA